Performance analysis of relay-aided free-space optical communication system over gamma-gamma fading channels with pointing errors*

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The average bit error rate (*ABER*) performance of a decode-and-forward (DF) based relay-assisted free-space optical (FSO) communication system over gamma-gamma distribution channels considering the pointing errors is studied. With the help of Meijer's G-function, the probability density function (PDF) and cumulative distribution function (CDF) of the aggregated channel model are derived on the basis of the best path selection scheme. The analytical *ABER* expression is achieved and the system performance is then investigated with the influence of pointing errors, turbulence strengths and structure parameters. Monte Carlo (MC) simulation is also provided to confirm the analytical *ABER* expression.

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Free space optical (FSO) communication has attracted considerable research attention recently due to its unlimited bandwidth, unlicensed spectrum, excellent security, and low cost[1,2]. Compared with traditional wireless communication, FSO provides a solution for the last mile problem to bridge the bandwidth gap between the end users and fiber optics networks[3]. In spite of the major advantages of FSO communication, its widespread application is hampered by several challenges[4]. Among them, scintillation phenomenon is a major one related to the atmospheric turbulence conditions, causing random fluctuations in the intensity and phase of the optical signal along the transmission path, and it can degrade FSO system performance over a distance of 1 km or longer[5]. Additionally, the pointing errors can severely affect the communication quality. Pointing errors between transmitter and receiver occur mostly due to beam wander, building sway, or errors in the tracking system[6]. To overcome the above-mentioned impairments, relay-aided transmission was proposed by Acampora and Krishnamurthy[7]. Among various relay-aided schemes, parallel and serial relay transmissions (i.e., multi-hop transmission) were investigated extensively[8]. Very recently, a more practical FSO network called multi-hop parallel scheme over log-normal fading channels was proposed with decode-and-forward (DF) protocol[9]. Then, the average bit error rate (*ABER*) of this FSO structure over gamma-gamma fading channels was studied[10]. However, the system performance of multi-hop parallel FSO links over gamma-gamma turbulence channels with pointing error impairment has never been reported so far, to the best of our knowledge.

In this work, the *ABER* performance of a multi-hop parallel DF based FSO communication system over aggregated fading channels considering atmospheric turbulence, pointing errors and structure parameters (I and J) is investigated in detail. The probability density function (PDF) and cumulative distribution function (CDF) are derived in terms of Meijer's G-function. The analytical *ABER* expression of the binary phase shift keying (BPSK) modulated FSO system is obtained with Gauss-Laguerre quadrature rule. The *ABER* performance is then analyzed with different turbulence regimes, normalized beam widths, normalized jitters and structure parameters. Monte Carlo (MC) simulation is also offered for verification.

Fig.1 shows the FSO communication system with DF protocol, assuming that the source node transmits the same information to destination node indirectly with the help of I parallel paths by using J hops, that is, there are J-1 relays in each path. Besides, a direct link (SD) is con-

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sidered from source to destination. BPSK subcarrier intensity modulation is adopted in each link. The received signal at the j-th hop in the i-th path can be given as[10] $y_{i,j}=h_{i,j}Rx_{i,j}+n_{i,j}$, where $h_{i,j}$ represents the aggregated channel gain of each link, R is the photodetector responsivity, $x_{i,j}$ denotes the input signal, and $x_{i,j}\in\{-P_{t_{i,j}},P_{t_{i,j}}\}$, $P_{t_{i,j}}$ is the average transmitted optical power. $n_{i,j}$ represents the signal-independent additive white Gaussian noise (AWGN) with zero mean and variance of $\sigma_n^2=N_0/2$. Here, N_0 is the single sided power spectral density. The instantaneous electrical signal-to-noise ratio (SNR) $\gamma_{i,j}$ of each link is given as

$$\gamma_{i,j}=(RP_{t_{i,j}}h_{i,j})^2/2\sigma_n^2=\overline{\gamma}_{i,j}h_{i,j}{}^2, \quad (1)$$

where $\overline{\gamma}_{i,j}=(RP_{t_{i,j}})^2/2\sigma_n^2$ is the average SNR.

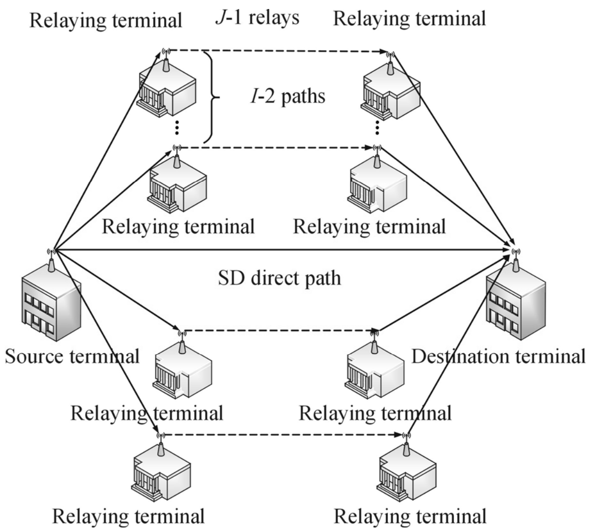

Fig.1 Structure of the multi-hop parallel DF FSO communication system

Considering the combined effects of path loss $h_{i,j}^{\mathrm{l}}$, atmospheric turbulence $h_{i,j}^{\mathrm{a}}$ and pointing errors $h_{i,j}^{\mathrm{p}}$, the aggregated channel model in each link is proposed in this work. Since $h_{i,j}^{\mathrm{l}}$ is deterministic, and $h_{i,j}^{\mathrm{a}}$ and $h_{i,j}^{\mathrm{p}}$ are independent, the channel gain can be described as $h_{i,j}=h_{i,j}^{\mathrm{l}}h_{i,j}^{\mathrm{a}}h_{i,j}^{\mathrm{p}}$. Since $h_{i,j}^{\mathrm{l}}$ is deterministic, without loss of generality, $h_{i,j}^{\mathrm{l}}=1$ is assumed throughout the work.

In this work, gamma-gamma distribution is used to model the fading due to atmospheric turbulence, and the PDF of $h_{i,j}^{\mathrm{a}}$ in each link is given as[5,10]

$$f_{h_{i,j}^{\mathrm{a}}}\left(h_{i,j}^{\mathrm{a}}\right)=\frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)}(h_{i,j}^{\mathrm{a}})^{(\alpha+\beta)/2-1}K_{\alpha-\beta}(2\sqrt{\alpha\beta h_{i,j}^{\mathrm{a}}}), \quad (2)$$

where $K_{\alpha-\beta}(\cdot)$ is the Bessel function of the second order. Assuming plane wave propagation, terms α and β are defined as

$$\alpha=\left\{\exp\left[0.49\sigma_{\mathrm{R}}^2/(1+1.11\sigma_{\mathrm{R}}^{12/5})^{7/6}\right]-1\right\}^{-1},$$

$$\beta=\left\{\exp\left[0.51\sigma_{\mathrm{R}}^2/(1+0.69\sigma_{\mathrm{R}}^{12/5})^{5/6}\right]-1\right\}^{-1}, \quad (3)$$

where σ_{R}^2 is the Rytov variance defined as $\sigma_{\mathrm{R}}^2=1.23C_n^2k^{7/6}z_{i,j}^{11/6}$, k is optical wave number decided by $k=2\pi/\lambda$, λ is the wavelength, $z_{i,j}$ is the propagation distance of each link, and C_n^2 is the refractive index structure parameter. In general, weak, moderate, and strong turbulence conditions are represented by the values of C_n^2 (in $m^{-2/3}$) as 8.4×10^{-15}, 1.7×10^{-14}, and 5×10^{-14}, respectively.

As for the impact of pointing errors, the mathematical model derived by Farid and Hranilovic[11] is adopted in this study wherein the effects of beam width, detector size and jitter variance are taken into account. Assuming a Gaussian spatial intensity profile with beam waist $\omega_{Z_{i,j}}$ on the receiver plane at distance $z_{i,j}$ from the transmitter and a circular aperture of radius, the PDF of $h_{i,j}^{\mathrm{p}}$ is given in as[11]

$$f_{h_{i,j}^{\mathrm{p}}}\left(h_{i,j}^{\mathrm{p}}\right)=\frac{\rho_{i,j}^2}{A_{0_{i,j}}^{\rho_{i,j}^2}}(h_{i,j}^{\mathrm{p}})^{\rho_{i,j}^2-1},\quad 0\le h_{i,j}^{\mathrm{p}}\le A_{0_{i,j}}, \quad (4)$$

where $\rho_{i,j}=\omega_{\mathrm{Zeq}_{i,j}}/2\sigma_{\mathrm{s}}$ is the ratio between the equivalent beam radius ($\omega_{\mathrm{Zeq}_{i,j}}$) at the receiver and the pointing error displacement standard deviation (jitter) at the receiver, $v_{i,j}=\sqrt{\pi}r/\sqrt{2}\omega_{Z_{i,j}}$, $\omega_{\mathrm{Zeq}_{i,j}}^2=\omega_{Z_{i,j}}^2\sqrt{\pi}\mathrm{erf}(v_{i,j})/2v_{i,j}\exp(-v_{i,j}^2)$, $A_{0_{i,j}}=[\mathrm{erf}(v_{i,j})]^2$, and $\mathrm{erf}(\cdot)$ is the error function. Independent identical Gaussian distributions for the elevation and horizontal displacement with jitter variance σ_{s}^2 are considered[4].

Because of the deterministic nature of the path loss $h_{i,j}^{\mathrm{l}}$, PDF of the channel gain $h_{i,j}$ can be expressed as[11]

$$f_{h_{i,j}}\left(h_{i,j}\right)=\int f_{h_{i,j}|h_{i,j}^{\mathrm{a}}}(h_{i,j}|h_{i,j}^{\mathrm{a}})f_{h_{i,j}^{\mathrm{a}}}(h_{i,j}^{\mathrm{a}})\mathrm{d}h_{i,j}^{\mathrm{a}}, \quad (5)$$

where $f_{h_{i,j}|h_{i,j}^{\mathrm{a}}}(h_{i,j}|h_{i,j}^{\mathrm{a}})$ is the conditional probability of $h_{i,j}^{\mathrm{a}}$ turbulence state and is given as

$$f_{h_{i,j}|h_{i,j}^{\mathrm{a}}}(h_{i,j}|h_{i,j}^{\mathrm{a}})=\frac{1}{h_{i,j}^{\mathrm{a}}h_{i,j}^{\mathrm{l}}}f_{h_{i,j}^{\mathrm{p}}}\left(\frac{h_{i,j}}{h_{i,j}^{\mathrm{a}}h_{i,j}^{\mathrm{l}}}\right)=$$

$$\frac{\rho_{i,j}^2}{h_{i,j}^{\mathrm{a}}h_{i,j}^{\mathrm{l}}A_{0_{i,j}}^{\rho_{i,j}^2}}\left(\frac{h_{i,j}}{h_{i,j}^{\mathrm{a}}h_{i,j}^{\mathrm{l}}}\right)^{\rho_{i,j}^2-1},\quad 0\le h_{i,j}\le A_{0_{i,j}}h_{i,j}^{\mathrm{a}}h_{i,j}^{\mathrm{l}}. \quad (6)$$

Since $h_{i,j}^{\mathrm{l}}=1$ is assumed, the conditional PDF $f_{h_{i,j}|h_{i,j}^{\mathrm{a}}}(h_{i,j}|h_{i,j}^{\mathrm{a}})$ is given by

$$f_{h_{i,j}|h_{i,j}^{\mathrm{a}}}(h_{i,j}|h_{i,j}^{\mathrm{a}})=\frac{\rho_{i,j}^2}{h_{i,j}^{\mathrm{a}}A_{0_{i,j}}^{\rho_{i,j}^2}}\left(\frac{h_{i,j}}{h_{i,j}^{\mathrm{a}}}\right)^{\rho_{i,j}^2-1},\quad 0\le h_{i,j}\le A_{0_{i,j}}h_{i,j}^{\mathrm{a}}. \quad (7)$$

Then, substituting Eq.(7) into Eq.(5), the PDF of $h_{i,j}$ can be obtained as

$$f_{h_{i,j}}(h_{i,j})=\frac{2\rho_{i,j}^2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{A_{0_{i,j}}^{\rho_{i,j}^2}\Gamma(\alpha)\Gamma(\beta)}h_{i,j}^{\rho_{i,j}^2-1}\times$$

$$\int_{h_{i,j}/A_{0_{i,j}}}^{\infty}\left(h_{i,j}^{\mathrm{a}}\right)^{\frac{\alpha+\beta}{2}-\rho_{i,j}^2-1}K_{\alpha-\beta}(2\sqrt{\alpha\beta h_{i,j}^{\mathrm{a}}})\mathrm{d}h_{i,j}^{\mathrm{a}}. \quad (8)$$

With the help of Ref.[12], $f_{h_{i,j}}(h_{i,j})$ can be finally expressed as

$$f_{h_{i,j}}(h_{i,j})=\frac{\alpha\beta\rho_{i,j}^2}{A_{0_{i,j}}\Gamma(\alpha)\Gamma(\beta)}\times$$

$$G_{1,3}^{3,0}\left(\frac{\alpha\beta h_{i,j}}{A_{0_{i,j}}}\middle|\begin{matrix}\rho_{i,j}^2\\ \rho_{i,j}^2-1,\alpha-1,\beta-1\end{matrix}\right), \quad (9)$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer's G-function.

The PDF of instantaneous *SNR* $\gamma_{i,j}$ in the *j*-th hop of the *i*-th path can be derived by the relationship of $f_{\gamma_{i,j}}(\gamma_{i,j})=f_{h_{i,j}}(\sqrt{\gamma_{i,j}/\overline{\gamma}_{i,j}})/2\sqrt{\gamma_{i,j}\overline{\gamma}_{i,j}}$ as

$$f_{\gamma_{i,j}}(\gamma_{i,j})=$$

$$A_{i,j}B_{i,j}\gamma^{-\frac{1}{2}}G_{1,3}^{3,0}\left(A_{i,j}\gamma_{i,j}^{\frac{1}{2}}\middle|\begin{matrix}\rho_{i,j}^2\\ \rho_{i,j}^2-1,\alpha-1,\beta-1\end{matrix}\right), \quad (10)$$

where $A_{i,j}=\alpha\beta\overline{\gamma}_{i,j}^{-1/2}/A_{0_{i,j}}$, $B_{i,j}=\rho_{i,j}^2/[2\Gamma(\alpha)\Gamma(\beta)]$.

The CDF of each link can be achieved with regard to $\gamma_{i,j}$ as

$$F_{\gamma_{i,j}}(\gamma_{i,j})=$$

$$A_{i,j}B_{i,j}\int_0^{\gamma_{i,j}}x^{-\frac{1}{2}}G_{1,3}^{3,0}\left(A_{i,j}x^{\frac{1}{2}}\middle|\begin{matrix}\rho_{i,j}^2\\ \rho_{i,j}^2-1,\alpha-1,\beta-1\end{matrix}\right)\mathrm{d}x. \quad (11)$$

Eq.(11) can be further simplified according to Ref.[12] as

$$F_{\gamma_{i,j}}(\gamma_{i,j})=2B_{i,j}G_{2,4}^{3,1}\left(A\gamma_{i,j}^{\frac{1}{2}}\middle|\begin{matrix}1,\rho_{i,j}^2+1\\ \rho_{i,j}^2,\alpha,\beta,0\end{matrix}\right). \quad (12)$$

In this system, the minimum *SNR* in each path is achieved first, and then the maximum value γ_{est} of these lowest-values is selected by using the max-min selection criterion scheme as the best path selection scheme[10].

Assuming $\overline{\gamma}_{i,j}=\overline{\gamma}_{s,d}=\overline{\gamma}$, $\overline{\gamma}_{s,d}$ is the average *SNR* for SD link. Besides, let $A_{i,j}=A_{s,d}=A$, $B_{i,j}=B_{s,d}=B$ for each link in the current system. Thus, the CDF of γ_{est} can be derived based on Ref.[13] as follows

$$F_{\gamma_{\text{est}}}(\gamma)=\left\{1-\left[1-F_{\gamma_{i,j}}(\gamma)\right]^J\right\}^I=$$

$$\left\{1-\left[1-2BG_{2,4}^{3,1}\left(A\gamma^{\frac{1}{2}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right)\right]^J\right\}^I. \quad (13)$$

Considering the SD link, the larger one between γ_{est} and $\gamma_{s,d}$ is selected as γ_{eq}, where $\gamma_{s,d}$ denotes the instantaneous *SNR* of SD link, and the CDF of $\gamma_{s,d}$ can be expressed as

$$F_{\gamma_{s,d}}(\gamma)=2BG_{2,4}^{3,1}\left(A\gamma^{\frac{1}{2}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right). \quad (14)$$

By using Eqs.(13), (14) and Ref.[13], the CDF of γ_{eq} can be derived as

$$F_{\gamma_{\text{eq}}}(\gamma)=F_{\gamma_{\text{est}}}(\gamma)\cdot F_{\gamma_{s,d}}(\gamma)=2BG_{2,4}^{3,1}\left(A\gamma^{\frac{1}{2}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right)\times$$

$$\left\{1-\left[1-2BG_{2,4}^{3,1}\left(A\gamma^{\frac{1}{2}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right)\right]^J\right\}^I. \quad (15)$$

Then, the PDF of γ_{eq} can be obtained by differentiating Eq.(15) as follows

$$f_{\gamma_{\text{eq}}}(\gamma)=AB\gamma^{-\frac{1}{2}}G_{1,3}^{3,0}\left(A\gamma^{\frac{1}{2}}\middle|\begin{matrix}\rho^2\\ \rho^2-1,\alpha-1,\beta-1\end{matrix}\right)\times$$

$$\left\{\left[1-2BG_{2,4}^{3,1}\left(A\gamma^{\frac{1}{2}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right)\right]^{J-1}\times\right.$$

$$2BG_{2,4}^{3,1}\left(A\gamma^{\frac{1}{2}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right)\times$$

$$IJ\left\{1-\left[1-2BG_{2,4}^{3,1}\left(A\gamma^{\frac{1}{2}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right)\right]^J\right\}^{I-1}+$$

$$\left.\left\{1-\left[1-2BG_{2,4}^{3,1}\left(A\gamma^{\frac{1}{2}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right)\right]^J\right\}^I\right\}. \quad (16)$$

Considering the difficulty of determining the accurate end-to-end *SNR* of the present system, γ_{eq} is used as the approximate end-to-end *SNR*[10]. In Ref.[14], the *ABER* expression of BPSK over an AWGN channel can be written as

$$P_{\text{e}}(\gamma)=Q(\sqrt{2\gamma}), \quad (17)$$

where $Q(\cdot)$ is the *Q*-function which can be given as $Q(x)=(1/\sqrt{2\pi})\int_x^\infty\exp(-t^2/2)\mathrm{d}t$.

Here, the analytical *ABER* expression can be obtained from Eqs.(16) and (17) as

$$P_{\text{e}}=\int_0^\infty P_{\text{e}}(\gamma)\times f_{\gamma_{\text{eq}}}(\gamma)\mathrm{d}\gamma=-\int_0^\infty F_{\gamma_{\text{eq}}}(\gamma)\mathrm{d}P_{\text{e}}(\gamma)=$$

$$\frac{1}{2\sqrt{\pi}}\int_0^\infty\gamma^{-1/2}\mathrm{e}^{-\gamma}F_{\gamma_{\text{eq}}}(\gamma)\mathrm{d}\gamma. \quad (18)$$

The generalized Gauss-Laguerre quadrature function[15], $\int_0^\infty x^\beta\mathrm{e}^{-x}f(x)\mathrm{d}x=\sum_{t=1}^m H_t f(x_t)$, can be used to approximate Eq.(18). Thus, Eq.(18) can be simplified by a truncated series as

$$P_{\text{e}}\approx\sum_{t=1}^m H_t\frac{\rho^2}{2\sqrt{\pi}\Gamma(\alpha)\Gamma(\beta)}G_{2,4}^{3,1}\left[\frac{\alpha\beta}{A_0}\sqrt{\frac{x_t}{\overline{\gamma}}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right]\times$$

$$\left\{1-\left[1-\frac{\rho^2}{\Gamma(\alpha)\Gamma(\beta)}G_{2,4}^{3,1}\left[\frac{\alpha\beta}{A_0}\sqrt{\frac{x_t}{\overline{\gamma}}}\middle|\begin{matrix}1,\rho^2+1\\ \rho^2,\alpha,\beta,0\end{matrix}\right]\right]^J\right\}^I, \quad (19)$$

where x_t is the t-th root of the generalized Laguerre polynomial $L_m^{(-1/2)}(x)$ and the weight H_t can be calculated by $H_t = \Gamma[m+(1/2)]x_t / \{m!(m+1)^2[L_{m+1}^{(-1/2)}(x_t)]^2\}$.

The analytical *ABER* of multi-hop parallel FSO system is obtained from Eq.(19) and m is chosen to be 30 as computing the generalized Gauss-Laguerre approximations. The acceptance rejection method and inverse transform method are adopted to generate random values from gamma-gamma turbulence and pointing errors in MC simulation. Without loss of generality, the structure parameters (I=2, J=3), (I=2, J=5) and (I=4, J=3) are selected to avoid entanglement. Additionally, the link distance between source and destination is 7.5 km and the equal length of z=(7.5/J) km in each hop is assumed. For the number of hops in each path, J=3 is adopted and the corresponding parameters (α, β) of gamma-gamma distribution are (4.5, 2.8), (4.0, 1.8) and (4.7, 1.2) in weak (C_n^2 =8.4×10^{-15}), moderate (C_n^2 =1.7×10^{-14}) and strong (C_n^2 =5×10^{-14}) turbulence conditions, respectively, whereas the parameters for J=5 are (7.5, 6.0), (5.0, 3.3) and (4.0, 1.7) in weak, moderate and strong turbulence conditions, respectively.

In Fig.2, the *ABER* performance of multi-hop parallel FSO system against transmitted optical power over the aggregated fading channel in different turbulence conditions is presented. The normalized beam width ω_z/r and normalized jitter σ_s/r are 10 and 2, respectively. As seen, the analytical *ABER* results have excellent agreement with MC simulation, which confirms the accuracy of our *ABER* model. Compared with the case without pointing errors, to achieve the same *ABER*, a higher transmitted optical power is required in the same turbulence conditions. For instance, in strong regime, to achieve the *ABER* of 10^{-7}, about 5 dBm of transmitted optical power is required for ω_z/r=10, σ_s/r=2, while only −13 dBm is needed for turbulence only. In addition, at a given turbulence strength, *ABER* value is the largest in the strong condition, and decreases in the moderate and weak conditions. It is also found that the *ABER* increases with the increase of the refractive index structure parameter (from weak to strong turbulence). For example, at a given transmitted optical power of −5 dBm, the *ABERs* are approximately 10^{-6}, 5×10^{-5} and 4×10^{-4} in weak, moderate and strong turbulence conditions, respectively. Actually, this phenomenon can be also observed for the case without pointing errors.

The *ABER* versus transmitted optical power in moderate turbulence condition with different normalized beam widths (ω_z/r=10, 12) and fixed normalized jitter (σ_s/r =2) is illustrated in Fig.3. It can be found that for a fixed normalized jitter, the *ABER* values increase with the increasing normalized beam width at given values of I and J, indicating that the *ABER* performance can be enhanced by a narrow beam width. For example, for the system (I=2, J=5) with the transmitted optical power of −10 dBm, when the normalized beam width decreases from ω_z/r=12 to ω_z/r=10, the *ABER* decreases from 3×10^{-3} to 5×10^{-4}.

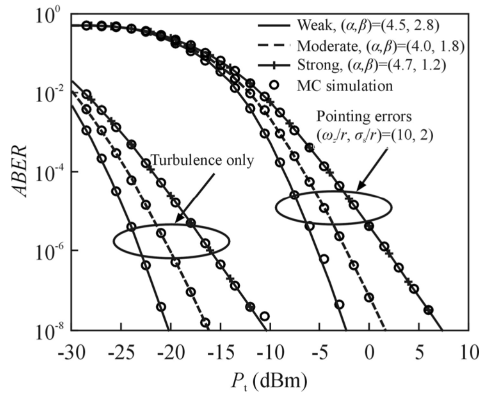

Fig.2 *ABER* performance versus transmitted optical power for the multi-hop parallel FSO system with *I*=2, *J*=3 in different turbulence conditions

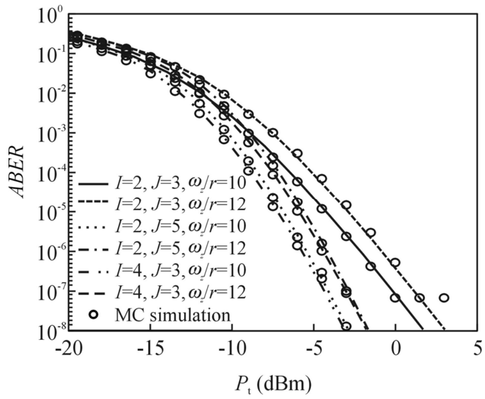

Fig.3 *ABER* performance of the multi-hop parallel FSO system with normalized beam width of ω_z/r=10, 12 and fixed normalized jitter of σ_s/r=2 in moderate turbulence condition

Fig.4 shows the *ABER* performance in strong turbulence condition with different normalized jitters (σ_s/r=2, 4) for normalized beam width (ω_z/r=12) against transmitted optical power. As seen, the *ABER* performance is worse with a higher normalized jitter. This is because the effect of pointing errors between source and destination will become more obvious with the increase of normalized jitter, leading to a degradation of the *ABER*. However, this negative effect can be mitigated by increasing the cooperative paths (I). For example, when the transmitted optical power is equal to 0 dBm, for the normalized jitter of 4, the *ABERs* of (I=2, J=3) and (I=4, J=3) are about 10^{-4} and 10^{-6}, respectively, as shown in Fig.4. Actually, this method can be also adopted to improve the *ABER* degradation caused by the increasing beam width (as shown in Fig.3). Furthermore, the *ABER* performance improves as the value of J increases for a fixed distance between source and destination. For instance, when the link distance between source and destination equals 7.5 km, to achieve the *ABER* of 10^{-8}, the transmitted optical power values of (I=2, J=3) and (I=2, J=5) required are about 9 dBm and 4.5 dBm, respectively for σ_s/r =2 in Fig.4.

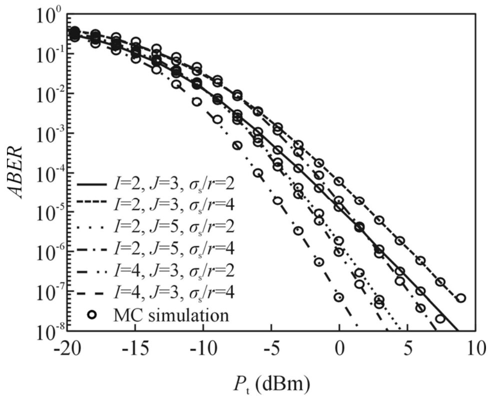

Fig.4 *ABER* performance of the multi-hop parallel FSO system with normalized jitter of σ_s/r=2, 4 and fixed normalized beam width of ω_z/r=12 in strong turbulence condition

In summary, the *ABER* performance of multi-hop parallel FSO communication system over an aggregated fading model with gamma-gamma distribution and pointing errors is presented. On the basis of the best path selection scheme, the CDF and PDF are derived and the analytical *ABER* expression of BPSK modulation is obtained in terms of Meijer's G-function and Gauss-Laguerre quadrature rule. Combined with MC simulation, the *ABER* performance is further discussed with different turbulence strengths, pointing errors and structure parameters. The results show that the relay-assisted FSO system performance could be improved by lower normalized beam width and normalized jitter. Increasing the number of paths (*I*) and hops (*J*) can mitigate the degradation caused by atmospheric turbulence and pointing errors as well. This work benefits the design of multi-hop parallel FSO systems.

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