# Large-scale spatial angle measurement and the pointing error analysis*

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A large-scale spatial angle measurement method is proposed based on inertial reference. Common measurement reference is established in inertial space, and the spatial vector coordinates of each measured axis in inertial space are measured by using autocollimation tracking and inertial measurement technology. According to the spatial coordinates of each test vector axis, the measurement of large-scale spatial angle is easily realized. The pointing error of tracking device based on the two mirrors in the measurement system is studied, and the influence of different installation errors to the pointing error is analyzed. This research can lay a foundation for error allocation, calibration and compensation for the measurement system.

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Large-scale spatial angle is the angle of two elements separated at a large distance in three-dimensional space[1]. With the rapid development of large equipment, such as aircraft, shipbuilding and heavy duty machine building, large-scale spatial angle measurement has become one of most challenging issues in manufacturing engineering[2,3]. Since the measured objects are distant apart and difficult to move or turn over, it is not easy to establish a common reference for measuring the large spatial angle[4-6]. For some traditional measurement methods based on coordinate measuring of characteristic point, such as coordinate measurement machine[7], laser trackers[8,9] and indoor global positioning system (GPS)[10], most of them need strict calibration in the field, and the measurement procedure is complex. The drawback is due to failing to establish an accurate and operable common measurement reference for each measured object in a large space. In order to realize the delivery of measurement reference, Liu et al[11] proposed a novel instrument for high precision angle measurement. An electronic theodolite was placed on a two-dimensional linear displacement platform, and the electronic theodolite was moved to different positions to measure different objects. The two-dimensional linear displacement platform can transfer the measurement reference effectively, but it has an obvious disadvantage in terms of volume and operability. Hu et al[12] proposed a method for measuring a large-scale spatial angle by using a common light plane. The light plane was used as the measurement reference, and then the light plane was placed on measured elements. The angles between these elements and the projections of light planes were measured, and thereafter the angle between two elements was determined. However, such a method is easily be interfered by the shape of the measured elements. A complex shape structure of measured elements may block the projected light.

In this paper, a novel method for large-scale spatial angle measurement is proposed, and the measurement system is designed. Optical axis tracking device is an important device in the measurement system, and its pointing error affects the accuracy of measurement system directly[13-15]. So the pointing error of optical axis tracking device is also analyzed.

The measurement principle of large-scale spatial angle measurement proposed in this paper can be explained through a practical measurement example. As shown in Fig.1, there are two measured axes separated at a large distance on the main body. One of them is set as reference axis, and the other is set as measured axis. It is required to measure the spatial angle between reference axis and measured axis. Assume that there is no angular motion for the main body in the measuring process. The distance between the reference axis and the measured axis is far apart, and the measurement reference is difficult to transfer. In this paper, the coordinate system is established in inertial space to serve as a measurement reference. The reference axis and the measured axis are seen as two unit vectors in inertial coordinate system. The coordinate values of these two vectors can be measured by the measurement system. And the spatial angle between the reference axis and the measured axis can be calculated by their coordinate values.

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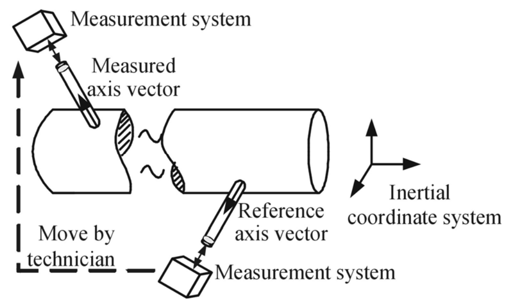


**Fig.1 Schematic diagram of measurement principle**

The measurement system is a portable hand-held measuring device. Its main task is to capture the direction of the measured axis and measure the unit vector coordinate values of the measured axis in inertial coordinate system. Its working principle is shown in Fig.2.

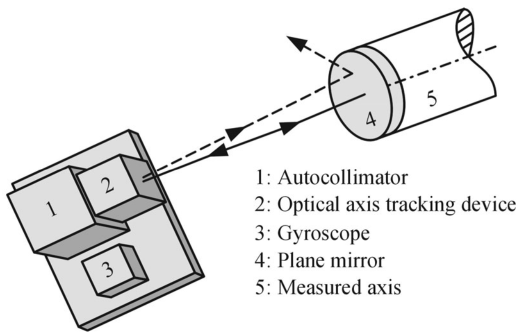


**Fig.2 Schematic diagram of measurement system working principle**

The direction of the measured axis can be captured exactly by autocollimation principle. In order to capture the direction of non-optical measured axis, it is necessary to install a plane mirror in front of the measured axis and ensure the optical axis of the plane mirror parallel to the measured axis. When the measurement system achieves autocollimation, the optical axis of measurement system is parallel to the measured axis, and the direction of measured axis can be transferred to the optical axis of the measurement system by this way. However, the status of the handheld measurement system may change easily due to the interference from technician, which results in the unstable autocollimation. The non-autocollimation optical path is shown by the dash lines in Fig.2.

According to kinematics theory, the status change of measurement system in inertial space can be decomposed into three vertically axial angular motions. Therefore, to stabilize the autocollimation, more than two axial angular motions (except the motion around its own optical axis) of the measurement system must be compensated by means of optical axis tracking device. The measurement system can maintain autocollimation state after such a compensation, and the optical path is indicated by the solid lines in Fig.2.

After the measurement system captures the direction of measured axis successfully, the measurement system's optical axis unit vector coordinates are equal to the measured axis unit vector coordinates in inertial coordinate system. Once gyroscope is connected with the optical axis of the measurement system, it can directly measure the optical axis unit vector coordinates in inertial coordinate system. However, this installation is complex and bulky. In order to reduce the volume of the measurement system, gyroscope is connected with the shell of measurement system, and thus the measurement is practiced by mathematical model in an indirect way.

The traditional optical axis tracking device is realized by a two-axis gimbal. However, the optical axis tracking device based on gimbal has a large size and weight generally, and its hand-held motion is not convenient. Therefore, the optical axis tracking device is realized by two mirrors in this paper, and its structure is shown in Fig. 3.

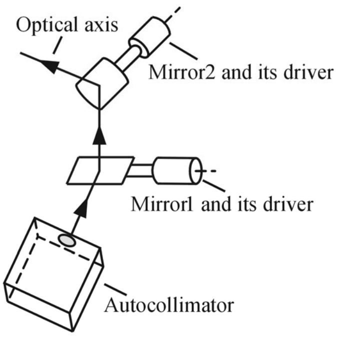


**Fig.3 Schematic diagram of the optical axis tracking device**

The optical beam is reflected twice in the optical axis tracking device, so the law of optical axis pointing is more complicated. In order to research the law of optical axis pointing, a coordinate system is set. The $x$-axis points to the front along the longitudinal axis of the measurement system, the $z$-axis points to the bottom of measurement system, and the $y$-axis points to the right of the measurement system. The light path in the coordinate system is shown in Fig.4. The angle between initial reflective surface of mirror1 and $xoy$ plane is 45°, the angle between initial reflective surface of mirror2 and $yoz$ plane is also 45°, and these two mirrors can rotate freely along their rotation axes. The azimuth angle of optical axis ($\alpha$) is the angle between the projection of optical axis in $xoy$ plane and $x$-axis, the pitch angle of optical axis ($\beta$) is the angle between the optical axis and $xoy$ plane.

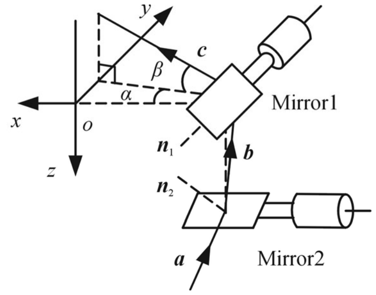


**Fig.4 Light path of the optical axis tracking device**

The optical beam (***a***) transmitted from autocollimator is parallel to *yoz* plane, so its unit vector is

$$\boldsymbol{a}=\begin{bmatrix}0 & 1 & 0\end{bmatrix}^{\mathrm{T}}. \tag{1}$$

Assuming that the rotation angle of mirror1 is $\lambda_1$, its normal unit vector is

$$\boldsymbol{n}_1=\begin{bmatrix}0\\ \sin(45°+\lambda_1)\\ -\cos(45°+\lambda_1)\end{bmatrix}. \tag{2}$$

The reflection matrix of mirror1 is

$$\boldsymbol{M}_1=\boldsymbol{I}-2\boldsymbol{n}_1\cdot\boldsymbol{n}_1^{\mathrm{T}}=\begin{bmatrix}1 & 0 & 0\\ 0 & -\sin 2\lambda_1 & \cos 2\lambda_1\\ 0 & \cos 2\lambda_1 & \sin 2\lambda_1\end{bmatrix}. \tag{3}$$

According to the law of reflection, the unit vector of reflected beam (***b***) is

$$\boldsymbol{b}=\boldsymbol{M}_1\cdot\boldsymbol{a}=\begin{bmatrix}0 & -\sin 2\lambda_1 & \cos 2\lambda_1\end{bmatrix}^{\mathrm{T}}. \tag{4}$$

Assuming that the rotation angle of mirror2 is $\lambda_2$, its normal unit vector is

$$\boldsymbol{n}_2=\begin{bmatrix}\cos(45°+\lambda_2)\\ 0\\ \sin(45°+\lambda_2)\end{bmatrix}. \tag{5}$$

The reflection matrix of mirror2 is

$$\boldsymbol{M}_2=\boldsymbol{I}-2\boldsymbol{n}_2\cdot\boldsymbol{n}_2^{\mathrm{T}}=\begin{bmatrix}\sin 2\lambda_2 & 0 & -\cos 2\lambda_2\\ 0 & 1 & 0\\ -\cos 2\lambda_2 & 0 & -\sin 2\lambda_2\end{bmatrix}. \tag{6}$$

According to the law of reflection, the unit vector of reflected beam (***c***) is

$$\boldsymbol{c}=\boldsymbol{M}_2\cdot\boldsymbol{b}=$$

$$\begin{bmatrix}\cos 2\lambda_1\cos 2\lambda_2 & \sin 2\lambda_1 & \cos 2\lambda_1\sin 2\lambda_2\end{bmatrix}^{\mathrm{T}}. \tag{7}$$

According to the Eq.(7), the azimuth angle and pitch angle of optical axis can be calculated by

$$\begin{cases}\alpha=\arctan(\dfrac{c_y}{c_x})=\arctan\left[\dfrac{\tan(2\lambda_1)}{\cos(2\lambda_2)}\right]\\ \beta=\arctan(\dfrac{c_z}{\sqrt{c_x^2+c_y^2}})=\arctan\left[\dfrac{\sin(2\lambda_2)\cos(2\lambda_1)}{\sqrt{1-\sin^2(2\lambda_2)\cos^2(2\lambda_1)}}\right]\end{cases}. \tag{8}$$

According to Eq.(8), the relationship between optical axis pointing and the rotation angle of mirrors is nonlinear. However, the optical axis pointing rule is calculated in ideal case, which must meet two conditions: the incident light and the rotation axis of mirror1 are perpendicular, and the rotation axes of mirror1 and mirror2 are perpendicular. Due to the influence of installation errors in actual assembly process, it is difficult to fully satisfy the conditions mentioned above. So it is necessary to research the influence of installation errors on optical axis pointing. The installation errors of optical axis tracking device include the nonperpendicularity of the incident light and the rotation axis of mirror1, the nonperpendicularity of the rotation axes of mirror1 and mirror2 and the zero errors of encoders.

The nonperpendicularity of the incident light and the rotation axis of mirror1 can be equivalent to that the incident light rotates $\Delta_{x1}$ around $x$-axis and rotates $\Delta_{z1}$ around $z$-axis. So the unit vector of incident light is

$$\boldsymbol{a}'=\begin{bmatrix}\cos\Delta_{x1}\sin\Delta_{z1} & \cos\Delta_{x1}\cos\Delta_{z1} & \sin\Delta_{x1}\end{bmatrix}^{\mathrm{T}}. \tag{9}$$

The unit vector of reflected beam changes to

$$\boldsymbol{c}'=\boldsymbol{M}_2\cdot\boldsymbol{M}_1\cdot\boldsymbol{a}'. \tag{10}$$

The nonperpendicularity of the rotation axes of mirror1 and mirror2 can be equivalent to that the rotation axis of mirror2 rotates $\Delta_{x2}$ around $x$-axis and rotates $\Delta_{z2}$ around $z$-axis. So the normal unit vector of mirror2 is

$$\boldsymbol{n}_2'=\begin{bmatrix}\cos\Delta_{z2}\cos(45°+\lambda_2)\\ \cos\Delta_{x2}\sin\Delta_{z2}\cos(45°+\lambda_2)+\sin\Delta_{x2}\sin(45°+\lambda_2)\\ -\sin\Delta_{x2}\sin\Delta_{z2}\cos(45°+\lambda_2)+\cos\Delta_{x2}\sin(45°+\lambda_2)\end{bmatrix}. \tag{11}$$

The reflection matrix of mirror2 changes into

$$\boldsymbol{M}_2'=\boldsymbol{I}-2\boldsymbol{n}_2'\cdot\boldsymbol{n}_2'^{\mathrm{T}}. \tag{12}$$

And the unit vector of reflected beam changes into

$$\boldsymbol{c}'=\boldsymbol{M}_2'\cdot\boldsymbol{b}. \tag{13}$$

Because the zero positions of encoders and mirrors do not coincide, the measurement results of encoders can be expressed as

$$\lambda_{\mathrm{m}i}=\lambda_i+B_i, i=1,2, \tag{14}$$

where $\lambda_i$ is the rotation angle of mirror, and $B_i$ is the zero errors of encoders. Substitute Eq.(14) into Eq.(7) and get

$$\boldsymbol{c}'=\begin{bmatrix}\cos 2\lambda_{\mathrm{m1}}\cos 2\lambda_{\mathrm{m2}} & \sin 2\lambda_{\mathrm{m1}} & \cos 2\lambda_{\mathrm{m1}}\sin 2\lambda_{\mathrm{m2}}\end{bmatrix}^{\mathrm{T}}. \tag{15}$$

The ideal unit vector of reflected beam is ***c***, and the actual unit vector of reflected beam is ***c***′, so the pointing error is defined as

$$\Delta\theta=\arccos(\boldsymbol{c},\boldsymbol{c}'). \tag{16}$$

In order to express simply, we set $e_1=\Delta_{x1}$, $e_2=\Delta_{z1}$, $e_3=\Delta_{x2}$, $e_4=\Delta_{z2}$, $e_5=B_1$ and $e_6=B_2$. Through the above analyses, the pointing error model caused by $e_1$—$e_6$ is very complex, and it is difficult to know their influence to pointing error directly. For example, if $e_2=1''$ and $e_3=1''$ respectively and two mirrors rotate in the range of −15°—+15°, the pointing errors caused by $e_2$ and $e_3$ are shown in Fig.5(a) and (b), respectively. It can be seen from Fig.5 that when the rotation angles of mirrors are different, the influence of error terms on pointing error are also different.

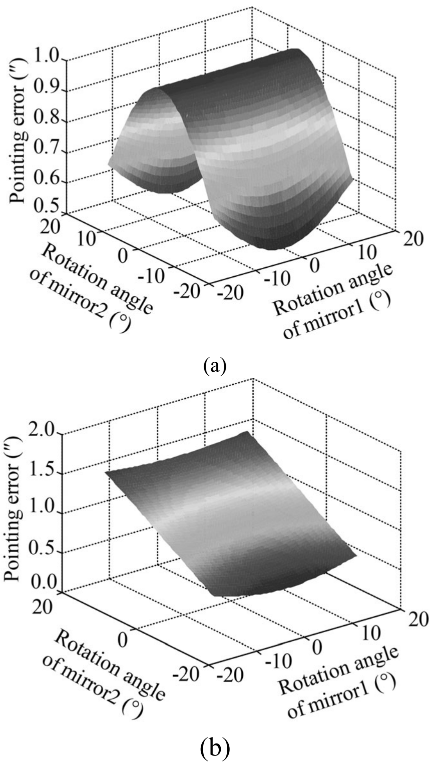


**Fig.5 The pointing error caused by (a) $e_2$ and (b) $e_3$**

In order to quantify the degree of pointing error influenced by each installation error, the error sensitivity is proposed in the paper. The error sensitivity is defined as an average value of pointing error caused by one installation error in the maximum range of mirror rotation when the installation error is 1″, which can be expressed as

$$s_k = \frac{1}{N}\sum_{\Omega}\Delta\theta_{i,k}, (i \in \Omega, k = 1, \cdots, 6) , \tag{17}$$

where $\Omega$ is the maximum range of mirror rotation, $\Delta\theta_{i,k}$ is the pointing error caused by the installation error ($k$) when the mirror rotates at position ($i$), and $N$ is the number of sampling points in the maximum range of mirror rotation.

The pointing error sensitivities for six kinds of installation error are calculated and shown in Fig.6. It can be seen from Fig.6 that the pointing errors caused by $e_5$ and $e_6$ are much larger than those caused by other installation errors. Therefore, it is necessary to calibrate and eliminate the installation errors of encoders in the system assembly.

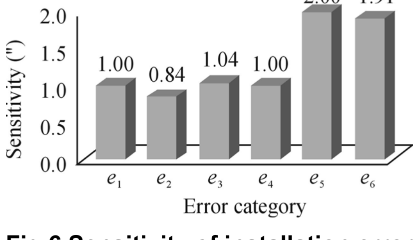


**Fig.6 Sensitivity of installation error**

For establishing an accurate and operable common measurement reference for each measured object in the large-scale spatial angle measurement, a novel method based on inertial reference is proposed, and the measurement system is designed. The pointing error of the optical axis tracking device based on two mirrors in the measurement system is researched, and the pointing error model is established. Error sensitivity is proposed peculiarly to quantify the degree of pointing error influenced by each installation error. It lays a foundation for the following work, like the allocation or compensation of system error.

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