A novel construction scheme of QC-LDPC codes based on the RU algorithm for optical transmission systems*

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A novel lower-complexity construction scheme of quasi-cyclic low-density parity-check (QC-LDPC) codes for optical transmission systems is proposed based on the structure of the parity-check matrix for the Richardson-Urbanke (RU) algorithm. Furthermore, a novel irregular QC-LDPC(4 288, 4 020) code with high code-rate of 0.937 is constructed by this novel construction scheme. The simulation analyses show that the net coding gain (*NCG*) of the novel irregular QC-LDPC(4 288,4 020) code is respectively 2.08 dB, 1.25 dB and 0.29 dB more than those of the classic RS(255, 239) code, the LDPC(32 640, 30 592) code and the irregular QC-LDPC(3 843, 3 603) code at the bit error rate (*BER*) of 10^{-6}. The irregular QC-LDPC(4 288, 4 020) code has the lower encoding/decoding complexity compared with the LDPC(32 640, 30 592) code and the irregular QC-LDPC(3 843, 3 603) code. The proposed novel QC-LDPC(4 288, 4 020) code can be more suitable for the increasing development requirements of high-speed optical transmission systems.

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The quasi-cyclic low-density parity-check (QC-LDPC) codes have many advantages[1-4], so the research of QC-LDPC codes for optical transmission systems has become a hot topic in recent years[5-10]. For constructing an excellent QC-LPDC code to meet the increasing requirements of optical transmission systems, a novel lower-complexity construction scheme for QC-LDPC codes is proposed based on Richardson-Urbanke (RU) algorithm in this paper. Furthermore, a lower-complexity irregular QC-LDPC(4 288, 4 020) code with the code rate of 0.937 is constructed, and the simulation analyses show that it has better error correction performance.

The LDPC codes are usually long in practical application, and the encoding complexity is proportional to the square of the code length through using generator matrix for encoding directly, namely the encoding complexity is $O(n^2)$, where n is the code length. Therefore, the code is encoded by RU algorithm using the check matrix as shown in Fig.1[11], where the sub-matrix $\boldsymbol{A}$ is a $(m-g)\times(n-m)$ matrix, the sub-matrix $\boldsymbol{B}$ is a $(m-g)\times g$ matrix, the sub-matrix $\boldsymbol{C}$ is a $g\times(n-m)$ matrix, the sub-matrix $\boldsymbol{D}$ is a $g\times g$ matrix, the sub-matrix $\boldsymbol{T}$ is a $(m-g)\times(m-g)$ matrix, and the sub-matrix $\boldsymbol{E}$ is a $g\times(m-g)$ matrix. In addition, it is proved that the complexity of the encoding is $O(n)+O(g^2)$, where n is the code length and g is the dimension of the square matrix $\boldsymbol{D}$. $O(g^2)$ appears because of inversion of the matrix $\boldsymbol{\varphi}$ ($\boldsymbol{\varphi}=\boldsymbol{ET}^{-1}\boldsymbol{B}+\boldsymbol{D}$).

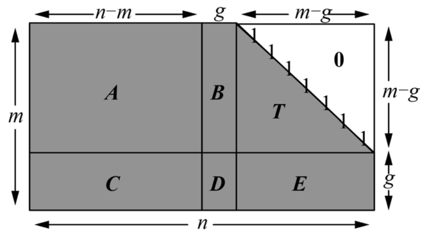

Fig.1 The partition of the check matrix in RU encoding

Set the codeword as $\boldsymbol{c}=(\boldsymbol{s}, \boldsymbol{p}_1, \boldsymbol{p}_1)$, where $\boldsymbol{s}$ corresponds to the information part, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ correspond to the two parts of the check part, and their lengths are g and $m-g$, respectively. Using the check equation $\boldsymbol{Hc}^{\mathrm{T}}=\boldsymbol{0}^{\mathrm{T}}$ and the partition of the matrix shown in Fig.1, it can be inferred that

$$\boldsymbol{As}^{\mathrm{T}}+\boldsymbol{Bp}_1^{\mathrm{T}}+\boldsymbol{Tp}_2^{\mathrm{T}}=\boldsymbol{0}, \tag{1}$$

$$\boldsymbol{Cs}^{\mathrm{T}}+\boldsymbol{Dp}_1^{\mathrm{T}}+\boldsymbol{Ep}_2^{\mathrm{T}}=\boldsymbol{0}. \tag{2}$$

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Define $\boldsymbol{\varphi}=\boldsymbol{ET}^{-1}\boldsymbol{B}+\boldsymbol{D}$, and it can be obtained that

$$\boldsymbol{p}_1^{\mathrm{T}}=\boldsymbol{\varphi}^{-1}(\boldsymbol{ET}^{-1}\boldsymbol{A}+\boldsymbol{C})\boldsymbol{s}^{\mathrm{T}}, \tag{3}$$

$$\boldsymbol{p}_2^{\mathrm{T}}=\boldsymbol{T}^{-1}(\boldsymbol{As}^{\mathrm{T}}+\boldsymbol{Bp}_1^{\mathrm{T}}), \tag{4}$$

where $\boldsymbol{s}$ stands for the information part of the codeword, the size of the unit matrix in the check matrix $\boldsymbol{H}$ is $L\times L$, and $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ stand for the front L bits and the last $(m-1)L$ bits of the check part, respectively.

Thus, it can be obtained that

$$\boldsymbol{p}_1^{\mathrm{T}}=\boldsymbol{\varphi}^{-1}(\boldsymbol{ET}^{-1}\boldsymbol{A}+\boldsymbol{C})\boldsymbol{s}^{\mathrm{T}}=\boldsymbol{\varphi}^{-1}(\boldsymbol{ET}^{-1}\boldsymbol{As}^{\mathrm{T}}+\boldsymbol{Cs}^{\mathrm{T}}). \tag{5}$$

As matrix $\boldsymbol{\varphi}^{-1}$ is not a sparse matrix under normal circumstances, the computational complexity of $\boldsymbol{p}_1$ can be represented as $O(N)+O(L^2)$, where $N=nL$. Nevertheless, if matrix $\boldsymbol{\varphi}$ is a unit matrix or a simple circulant permutation matrix, the computational complexity of $\boldsymbol{p}_1$ can be reduced to $O(N)$ in the case of ignoring the size of the circulant permutation matrix. This is also the core concept of the novel construction scheme based on RU algorithm in this paper.

A construction scheme of the check matrix of irregular QC-LDPC codes is proposed based on the structural characteristics of QC-LDPC codes and the theoretical analysis of RU algorithm in this paper. The check matrix is an approximate lower triangular one, and the low-complexity and efficient encoding method of the QC-LDPC code is given. Richardson[11], MacKay[12], Eleftheriou[13] and others have tried to use the approximate lower triangular matrix to construct the check matrix. The novel construction scheme in this paper is an improved method of the RU algorithm.

At first, the check matrix $\boldsymbol{H}$ is divided into $\boldsymbol{H_I}$ and $\boldsymbol{H_P}$. $\boldsymbol{H_I}$ generates the information bit of the codeword, and $\boldsymbol{H_P}$ generates the check bit of the codeword, namely $\boldsymbol{H}=[\boldsymbol{H_I}|\boldsymbol{H_P}]$. $\boldsymbol{H_I}$ is an $mL\times kL$ matrix, and $\boldsymbol{H_P}$ is an $mL\times mL$ matrix. In order to make the encoding more efficient, the matrix of the check part $\boldsymbol{H_P}$ is converted into the approximate lower triangular matrix. The concrete form of the check matrix is shown as

$$\boldsymbol{H}=\left[\boldsymbol{H_I}\,|\,\boldsymbol{H_P}\right]=\begin{bmatrix} & \boldsymbol{P}^{b_1} & \boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ & \boldsymbol{0} & \boldsymbol{P}^{b_2} & \boldsymbol{I} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ & \vdots & \boldsymbol{0} & \boldsymbol{P}^{b_3} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{H_I} & \boldsymbol{P}^{y} & \vdots & \vdots & \cdots & \vdots & \vdots \\ & \vdots & \vdots & \vdots & \cdots & \boldsymbol{I} & \boldsymbol{0} \\ & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{P}^{b_{m-1}} & \boldsymbol{I} \\ & \boldsymbol{P}^{x} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{P}^{b_m} \end{bmatrix}, \tag{6}$$

where $\boldsymbol{H_I}$ is composed of $\boldsymbol{0}$ matrices or circulant shift matrices. In $\boldsymbol{H_I}$, the size of each $\boldsymbol{0}$ matrix or circulant shift matrix is $L\times L$, and the number of the $\boldsymbol{0}$ matrices or circulant permutation matrices is $m\times k$. $\boldsymbol{0}$ in $\boldsymbol{H_P}$ stands for $\boldsymbol{0}$ matrix, $\boldsymbol{I}$ in $\boldsymbol{H_P}$ stands for the unit matrix, and the size of them is also $L\times L$. $\boldsymbol{P}^{bi}$ with $b_i=0, 1, 2, \ldots, L-1$ stands for the second circulant shift matrix of the unit matrix, and $\boldsymbol{P}^{y}$ is selected in l ($l\neq 1$, m) row, where l is selected as half of m generally. Based on the construction scheme of RU algorithm, the check matrix can be divided as

$$\boldsymbol{H}=\begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{T} \\ \boldsymbol{C} & \boldsymbol{D} & \boldsymbol{E} \end{pmatrix}, \tag{7}$$

where $\boldsymbol{A}$ is a $(m-1)L\times kL$ matrix, $\boldsymbol{B}$ is a $(m-1)L\times L$ matrix, $\boldsymbol{T}$ is a $(m-1)L\times(m-1)L$ matrix, $\boldsymbol{C}$ is an $L\times kL$ matrix, $\boldsymbol{D}=\boldsymbol{P}^{x}$ is an $L\times L$ matrix, and $\boldsymbol{E}$ is an $L\times(m-1)L$ matrix. Assuming $\boldsymbol{H_P}$ is full rank, namely each row of $\boldsymbol{H_P}$ is linearly independent, this linearly independent condition obtained by using the Gaussian elimination method means that the matrix $\boldsymbol{\varphi}=\boldsymbol{ET}^{-1}\boldsymbol{B}+\boldsymbol{D}$ is full rank. Let $\boldsymbol{c}$ be the codeword to be transmitted, and then $\boldsymbol{Hc}^{\mathrm{T}}=\boldsymbol{0}^{\mathrm{T}}$ can be obtained.

According to Eqs.(6) and (7), the following formula can be deduced and obtained

$$\boldsymbol{T}^{-1}=\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{P}^{b_2} & \boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{P}^{(2,3)} & \boldsymbol{P}^{b_3} & \boldsymbol{I} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{P}^{(2,m-2)} & \boldsymbol{P}^{(3,m-2)} & \boldsymbol{P}^{(4,m-2)} & \cdots & \boldsymbol{P}^{b_{m-2}} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{P}^{(2,m-1)} & \boldsymbol{P}^{(2,m-1)} & \boldsymbol{P}^{(4,m-1)} & \cdots & \boldsymbol{P}^{(m-2,m-1)} & \boldsymbol{P}^{b_{m-1}} & \boldsymbol{I} \end{bmatrix}, \tag{8}$$

where $\boldsymbol{P}^{(i,j)}=\prod_{k=i}^{j}\boldsymbol{P}^{b_k}=\boldsymbol{P}^{b_i+b_{i+1}+\cdots+b_j}$, the following formula can be obtained by $\boldsymbol{B}^{\mathrm{T}}=[(\boldsymbol{P}^{b1})^{\mathrm{T}}\ \boldsymbol{0}\ \ldots\ (\boldsymbol{P}^{y})^{\mathrm{T}}\ \boldsymbol{0}\ \ldots\boldsymbol{0}]$ and $\boldsymbol{E}=[\boldsymbol{0}\ \boldsymbol{0}\ \ldots\boldsymbol{0}\ \ \boldsymbol{P}^{b_m}]$ as

$$\begin{aligned} &\boldsymbol{ET}^{-1}\boldsymbol{B}=\boldsymbol{P}^{b_m}\left[\boldsymbol{P}^{(2,m-1)}\boldsymbol{P}^{(3,m-1)}\ldots\boldsymbol{P}^{(b,m-1)}\boldsymbol{I}\right]\boldsymbol{B}= \\ &\boldsymbol{P}^{b_m}\boldsymbol{P}^{(2,m-1)}\boldsymbol{P}^{b_1}+\boldsymbol{P}^{b_m}\boldsymbol{P}^{(l+1,m-1)}\boldsymbol{P}^{y}= \\ &\boldsymbol{P}^{(1,m)}+\boldsymbol{P}^{(l+1,m)}\boldsymbol{P}^{y}. \end{aligned} \tag{9}$$

When the structure of the check matrix $\boldsymbol{H}$ is shown as Eqs.(6) and (7), if x and y meet the following formulas as

$$x=\sum_{i=1}^{m}b_i \bmod L, \tag{10}$$

$$y=-\sum_{i=l+1}^{m}b_i \bmod L, \tag{11}$$

the matrix $\boldsymbol{\varphi}=\boldsymbol{ET}^{-1}\boldsymbol{B}+\boldsymbol{D}$ is a unit matrix.

The detailed encoding process is shown as follows:

Step1: Calculate $\boldsymbol{As}^{\mathrm{T}}$ and $\boldsymbol{Cs}^{\mathrm{T}}$.

Step2: Calculate $\boldsymbol{ET}^{-1}\boldsymbol{As}^{\mathrm{T}}=[\boldsymbol{P}^{(2,m)}\ \ldots\ \boldsymbol{P}^{m}]\boldsymbol{As}^{\mathrm{T}}$.

Step3: Calculate $\boldsymbol{p}_1^{\mathrm{T}}$, $\boldsymbol{p}_1^{\mathrm{T}}=\boldsymbol{ET}^{-1}\boldsymbol{As}^{\mathrm{T}}+\boldsymbol{Cs}^{\mathrm{T}}$.

Step4: Calculate $\boldsymbol{p}_2^{\mathrm{T}}$, $\boldsymbol{Tp}_2^{\mathrm{T}}=\boldsymbol{As}^{\mathrm{T}}+\boldsymbol{Bp}_1^{\mathrm{T}}$.

Ignore the circulant shift operation in the whole encoding process, and only focus on the condition of exclusive or operation. Then the encoding complexity is shown in Tab.1. Here, $R=1-m/n$ stands for the code rate, $N=nL$ stands for the code length, supposing c stands for the average weight of each column of the check matrix, and the weight of the check matrix is expressed as $w(\boldsymbol{x})$, namely, the number of nonzero elements in check matrix. According to Eqs.(6) and (7), $w(\boldsymbol{A})+w(\boldsymbol{C})=cN-(2m+1)L$ can be obtained, and the calculation amount of $\boldsymbol{As}^{\mathrm{T}}$ and $\boldsymbol{Cs}^{\mathrm{T}}$ in step 1 is $cN-(3m+1)L$. The calculation amount of $\boldsymbol{ET}^{-1}\boldsymbol{As}^{\mathrm{T}}$ in step 2 is $(m-2)L$ because of $w(\boldsymbol{ET}^{-1})=(m-1)L$. And the calculation amount of L in step 3 can be obtained according to the calculation amounts in step 1 and step 2. As the calculation amounts of $\boldsymbol{As}^{\mathrm{T}}+\boldsymbol{Bp}_1^{\mathrm{T}}$ and $\boldsymbol{p}_2^{\mathrm{T}}$ are respectively $2L$ and $(m-2)L$, the total encoding complexity of the novel construction scheme proposed in this paper for QC-LDPC codes is $(c-1+R)N-2L$, namely the encoding complexity of $O(N)$.

Tab.1 The complexity of the encoding process (Module 2 operation)

Encoding steps	The times of exclusive or operation
Step1	$cN-(3m+1)L$
Step2	$(m-2)L$
Step3	L
Step4	mL
Total	$(c-1+R)N-2L$

Using the original RU algorithm, suppose that the structures of sub-matrices of the check matrix are all approximately lower triangular and the sub-matrices have the same weight as that of the matrix in Eq.(8). As the weight of $\boldsymbol{\varphi}^{-1}$ is $L^2/2$, it is easy to infer that the times of module 2 addition operations in the whole encoding process are $(c-1+R)N-3L+L^2/2$. Define ρ as

$$\rho=\frac{(c-1+R)N-2L}{(c-1+R)N-3L+L^2/2}=\frac{(c-1+R)n-2}{(c-1+R)n-3+L/2}. \tag{12}$$

When $L>2$ and $\rho<1$, it can be noticed that with the increase of L, ρ decreases constantly. Therefore, when the code length is certain, with the increase of the dimension L of the sub-matrix in check matrix, the encoding complexity of the novel construction scheme based on RU algorithm for QC-LDPC codes in this paper will be much smaller than that of the original RU algorithm. Meanwhile, the efficiency of the encoding is improved, and the time delay of the encoding is reduced.

In order to adequately demonstrate the performance of the QC-LDPC code constructed by the proposed novel construction scheme, the check matrix of the QC-LDPC code with m=4, n=64 and L=67 is considered and constructed emphatically under the premise that the code rate and the redundancy of the constructed QC-LDPC code are the same as those of the classic RS(255, 239) code in ITU-T G.975[14], the LDPC(32 640, 30 592) code in ITU-T G.975.1[15] and the irregular QC-LDPC(3 843, 3 603) code constructed based on the Galois field ($GF(q)$) multiplicative group[5]. Since the QC-LDPC code is constructed by using the check matrix to encode indirectly, in this process, the key references are the parameters produced after the check matrix $\boldsymbol{H}$ is converted into the generator matrix $\boldsymbol{G}$. These parameters are the actual parameters of the final QC-LDPC code type, namely the row number and column number of the generator matrix correspond to the code length and the information bit, respectively. So constructed with the above parameters, the row number and the column number of generator matrix $\boldsymbol{G}$ are 4 288 and 4 020, respectively. Moreover, the row weight and the column weight of the check matrix are different, so the constructed code is an irregular QC-LDPC(4 288, 4 020) code with the code rate of 0.937, which can meet the requirements of the error correcting code for optical transmission systems.

The better decoding performance of QC-LDPC codes can be gotten by using the belief propagation (BP) decoding algorithm (namely, the sum product algorithm (SPA) decoding) in the simulation process. So the log likelihood ratio belief propagation (LLR-BP) decoding algorithm is applied to decode the QC-LDPC code, while the Berlekamp-Massey iteration decoding algorithm is applied to decode RS(255, 239) code in this paper. Constructing LDPC codes is easier, and the encoding and decoding complexity is lower in $GF(2)$, compared with that in the non-binary field. The corresponding modulation in $GF(2)$ is binary phase shift keying (BPSK), and QC-LDPC codes are applied in optical transmission systems. Therefore, the transmission channel is addictive white Gaussian noise (AWGN) channel approximately. The girth-4 occurrence ought to be avoided in the process of constructing the check matrix. The girth-4 of the check matrix for the irregular QC-LDPC(4 288, 4 020) code is tested by MATLA, and no girth-4 occurrence is found while a small quantity of girth-6 occurrences are found. Meanwhile, the error correction performance of the QC-LDPC(4 288, 4 020) code is simulated at the 16th iteration time, and the curves of the relationship between bit error rate (BER) of the code type and signal-to-noise ratio (SNR) can be obtained as shown in Fig.2. The number of the iteration times is 16 which can achieve a better balance among the iteration times, the error correction performance and the decoding complexity.

In order to demonstrate the error correction performance of the irregular QC-LDPC(4 288, 4 020) code fully, the comparison among the irregular QC-LDPC(4 288, 4 020) code, the RS(255, 239) code, the LDPC(32 640, 30 592) code and the irregular QC-LDPC(3 843, 3 603) code is performed. In addition, these four codes have the same code rate of 0.937. The results are shown in Fig.2 when the irregular QC-LDPC(4 288, 4 020) code is at the 16th iteration by applying the LLR-BP decoding algorithm, and the numerical results are listed in Tab.2 at BER of 10^{-6}.

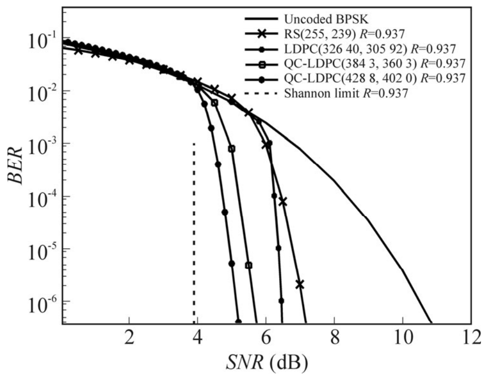

Fig.2 The comparison of the error correction performance among the irregular QC-LDPC(4 288,4 020) code and other codes

Tab.2 The comparison of the error correction performance among the irregular QC-LDPC(4 288, 4 020) code and other three codes at *BER*=10^{-6}

Code type	*NCG* (dB)	Distance from the Shannon limit (dB)
RS(255, 239)	3.28	3.34
LDPC(32 640, 30 592)	4.11	2.51
QC-LDPC(3 843, 3 603)	5.07	1.55
QC-LDPC(4 288, 4 020)	5.36	1.26

From Tab.2, it can be known that the net coding gain (*NCG*) of the irregular QC-LDPC(4 288, 4 020) code is respectively 2.08 dB, 1.25 dB and 0.29 dB more than those of the RS(255, 239) code, the LDPC(32 640, 30 592) code and the irregular QC-LDPC(3 843, 3 603) code at *BER* of 10^{-6}. Therefore, the error correction performance of the irregular QC-LDPC(4 288, 4 020) code constructed by the proposed construction scheme is obviously superior.

A novel lower-complexity construction scheme of QC-LDPC codes with the approximate lower triangular structure is proposed to better meet the increasing development requirements of optical transmission systems in this paper. Furthermore, an irregular QC-LDPC (4 288,4 020) code is constructed by this novel construction scheme. The simulation results show that the error correction performance of the irregular QC-LDPC(4 288, 4 020) code is improved more significantly than those of the RS(255, 239) code in ITU-T G.975, the LDPC (32 640, 30 592) code in ITU-T G.975.1 and the irregular QC-LDPC(3 843, 3 603) code constructed based on the Galois field multiplicative group. Furthermore, compared with the LDPC(32 640, 30 592) code and the irregular QC-LDPC(3 843, 3 603) code, the irregular QC-LDPC (4 288, 4 020) code has lower encoding/decoding complexity. Therefore, the irregular QC-LDPC(4 288, 4 020) code can be better used in high-speed long-haul optical transmission systems.

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