Spatial modulation characteristics of single-photon frequency up-conversion systems pumped by Gaussian laser beam*

MA Jian-hui (马建辉), **LI Xiong-jie** (李雄杰), **WU Wen-jie** (吴文杰), **HUANG Kun** (黄坤), **PAN Hai-feng** (潘海峰)**, **and WU E** (武愕)

State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai 200062, China

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In a Gaussian laser beam pumped single-photon frequency up-conversion system, the spatial distribution of the conversion efficiency is calculated, which strongly depends on the intensity distribution of the pump beam and leads to a spatial modulation of the output single photons. As a result, by simply varying the Gaussian pump beam intensity and the beam size, the converted photons could be modulated spatially and exhibit a programmable distribution. This will be meaningful for the researches on quantum communication and quantum manipulation based on frequency up-conversion system.

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In the last decade, the technique of frequency up-conversion in a single or a few photons level has been well developed and employed in various applications, such as infrared single-photon detection[1,2], spectral measurement[3], quantum transduction[4-6], quantum key distribution[7], high resolution imaging[8] and so on[9,10]. Single-photon frequency up-conversion can be described as a nonlinear optical process in which signal single-photon is combined with a strongly pumped laser beam in a quadratic nonlinear medium to generate a sum-frequency photon, which has been structured experimentally in many schemes by using periodically poled lithium niobate (PPLN) crystal[11,12] or waveguide[13,14] devices for the large nonlinear coefficient. Cavity resonance enhancement and pulsed pump excitation are some other solutions to realize the strong nonlinear interaction by increasing the pump intensity[15-17]. Usually, the spatial modulation or imaging can only be realized by the structured frequency up-conversion system with a bulk crystal because of the unchanged spatial phase information[18,19]. In this letter, the spatial modulation of the converted photon is numerically stimulated as a function of the intensity distribution of the Gaussian pump beam in a frequency up-conversion system. The quantum conversion efficiencies are also investigated at different pump intensities and pump beam sizes. It is proved to be an efficient method to yield programmable spatial distribution of the converted photons by conveniently changing the intensity and the profile of the pump beam in a single-photon frequency up-conversion system.

The quantum physics of the single-photon frequency up-conversion process in transparent nonlinear materials can be described by the following effective Hamiltonian

$$\hat{H} = \mathrm{i}\hbar\gamma(\hat{a}_1\hat{a}_2\hat{a}_3^\dagger - \hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_3) , \tag{1}$$

where E represents the effective Hamiltonian, $\hbar$ is the reduced Plank constant, and γ is the coupling constant which is determined by the second-order susceptibility of the nonlinear medium. $\hat{a}_{m=1,2,3}$ ($\hat{a}^\dagger_{m=1,2,3}$) annihilates (creates) one photon at frequency $\omega_{i=1,2,3}$. According to the experimental setup, the pump field is considered to be strong enough to be regarded as non-depletion during the interaction, so the amplitude of the pump field can be treated classically as a constant E_2. The interaction process is given by

$$\frac{\mathrm{d}\hat{a}_1}{\mathrm{d}z} = -\gamma E_2\hat{a}_3 , \tag{2}$$

$$\frac{\mathrm{d}\hat{a}_3}{\mathrm{d}z} = -\gamma E_2\hat{a}_1 . \tag{3}$$

The solutions can be described as

$$\hat{a}_1(L) = \hat{a}_1(0)\cos(\gamma E_2 L) - \hat{a}_3(0)\sin(\gamma E_2 L) , \tag{4}$$

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** E-mail: hfpan@phy.ecnu.edu.cn

$$\hat{a}_3\left(L\right)=\hat{a}_1\left(0\right)\sin\left(\gamma E_2 L\right)+\hat{a}_3\left(0\right)\cos\left(\gamma E_2 L\right), \quad (5)$$

where L is the interaction length. The single-photon frequency up-conversion can be treated as a sum-frequency generation (SFG) process, where the input state of the SFG photon is a vacuum state as $|\Psi_1\rangle=|n_1 0_3\rangle$. When the condition of $\gamma L=\pi/2$ is fulfilled, Eqs.(4) and (5) can be rewritten as

$$\hat{a}_1\left(\gamma L=\pi/2\right)=-\hat{a}_3\left(0\right), \quad (6)$$

$$\hat{a}_3\left(\gamma L=\pi/2\right)=\hat{a}_1\left(0\right). \quad (7)$$

The quantum conversion efficiency can be given as

$$\eta=\frac{\langle\Psi|\hat{n}_3(L)|\Psi\rangle}{\langle\Psi|\hat{n}_1(0)|\Psi\rangle}=$$

$$\frac{|\langle\Psi\hat{a}_3^\dagger(L)\hat{a}_3(L)|\Psi\rangle}{\langle\Psi|\hat{a}_1^\dagger(0)\hat{a}_1(0)|\Psi\rangle}=\sin^2\left(\gamma E_2 L\right). \quad (8)$$

When γ and L are normalized to be 1 in Eq.(8), the quantum conversion efficiency is simplified as a function of the pump power as follows,

$$\eta=\sin^2(\sqrt{I_2}). \quad (9)$$

As shown in Fig.1, the conversion efficiency of the SFG photon shows a periodic oscillation according to the pump intensity, and could reach a peak value of 100%, which means a complete quantum state transfer.

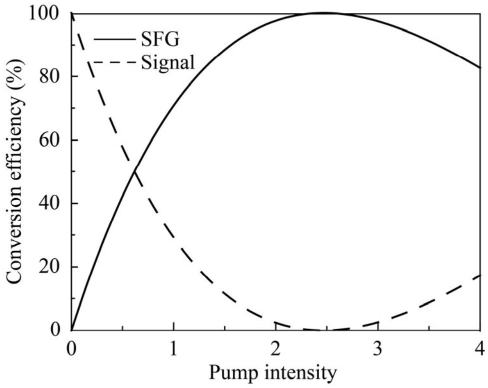


Fig.1 Conversion efficiencies of SFG and signal photons as a function of the pump intensity

Experimentally, the spatial intensity of the pump laser beam profile is always in Gaussian distribution, which makes the spatial modulation of the SFG photon possible due to the fact of the conversion efficiency in single-photon frequency up-conversion system dependent on the pump intensity. The intensity profile in the cross-section of pump Gaussian beam can be written as follows,

$$I=E^2\exp(-2r^2/\omega_0), \quad (10)$$

where E and ω_0 are the peak electric field amplitude and the radius size of the pump beam cross-section, respectively. Together with Eq.(9), the spatial distributions of the conversion efficiency are stimulated with different values of E as π/2, π and 3π/2, respectively. Because the conversion efficiency distribution is directly related to the probability distribution of the SFG photon, in case of single-photon frequency up-conversion, the stimulation can show how the pump intensity varying leads to the spatial modulation of the SFG photon.

The normalized probability distributions of the SFG photon and unconverted photon are shown in Fig.2(a) and (b) with the plane coordinate of the pump beam cross-section, respectively. Since the electric field amplitude E is fixed to be π/2, the overall quantum conversion efficiency is close to the peak value, which means most of the signal photons are converted to the SFG photons. As the radius increases from the center of the plane coordinate, the probability of the SFG photon decreases due to the decline of the pump field amplitude. After the frequency up-conversion interaction, the proportion of the SFG photons converted from the signal single-photon can be weighted by the overall quantum conversion efficiency. In experimental point of view, the conversion efficiency can be determined by measuring the intensity of the SFG photons. The proportion of the SFG photons is calculated to be 86.0%, while the proportion of the unconverted photons is 14.0% due to the unity of the input and output quantum states in Eqs.(4) and (5).

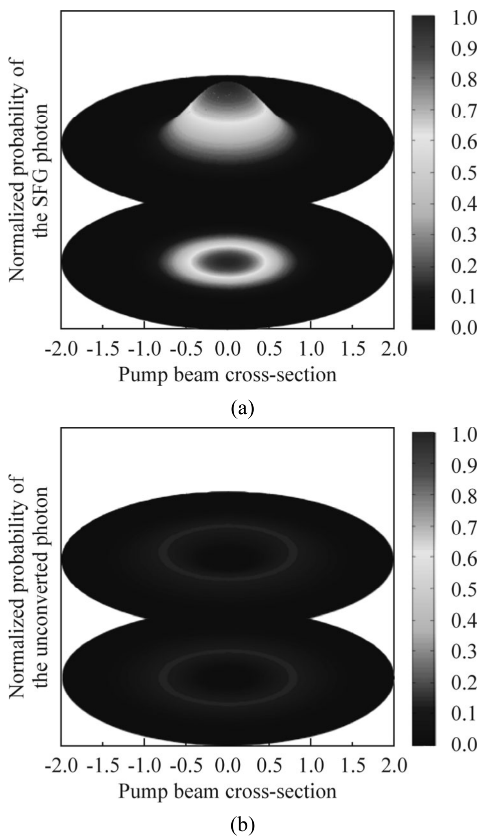


Fig.2 When the peak electric field amplitude *E* is fixed to be π/2, the two-dimensional spatial probability distributions of the (a) SFG and (b) unconverted photons, respectively

When the electric field amplitude is π, the probability distribution of the SFG photon shows a donut shape as shown in Fig.3(a). Although the pump intensity increases, the conversion efficiency at the center of the pump beam decreases to zero due to the oscillation of the conversion efficiency. As the pump intensity varying, the SFG photon and the signal photon switches to each other in a modulated spatial profile. The overall quantum conversion efficiency is calculated to be 28.5%.

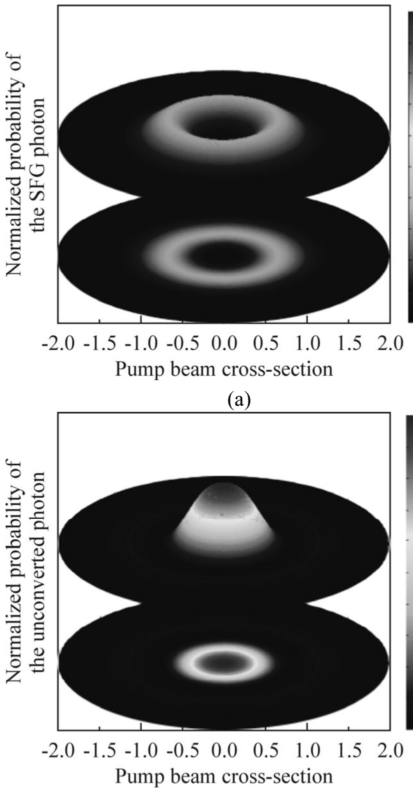


Fig.3 When the peak electric field amplitude *E* is fixed to be π, the two-dimensional spatial probability distributions of the (a) SFG and (b) unconverted photons, respectively

When the electric field amplitude E increases to 3π/2, as shown in Fig.4, the spatial modulated probability distribution of the SFG photon exhibits a pattern as a peak at the center together with bright and dark rings around. The quantum conversion efficiency increases to 67.9% in this case.

The overall quantum conversion efficiency is further investigated as a function of the electric field amplitude whose value changes from 0 to 2π with intervals of π/20. As shown in Fig.5(a), the quantum conversion efficiency reaches the first maximum value 88.8% when E equals 0.55π. It can be predicted that the quantum conversion efficiency varies periodically with the increase of pump intensity, and then the oscillation recedes gradually and finally tends to be a constant. Considering the nonlinear crystal in frequency up-conversion system cannot survive in over strong pump electric field, the variation of quantum conversion efficiency is only simulated with E lower than 2π.

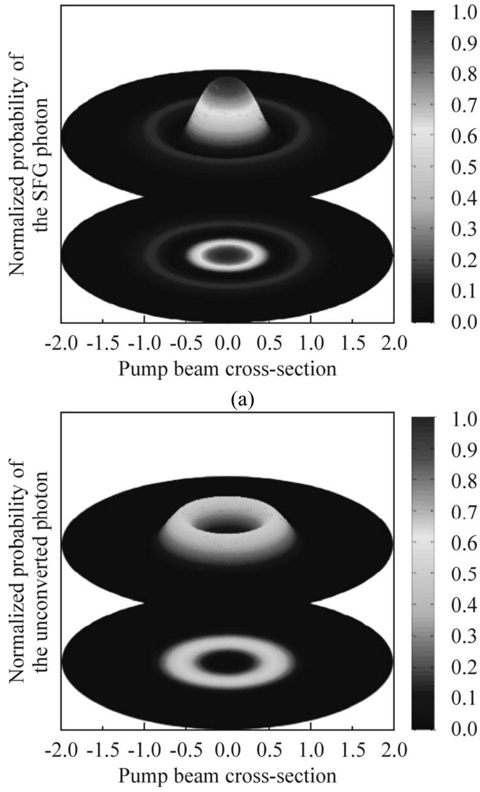
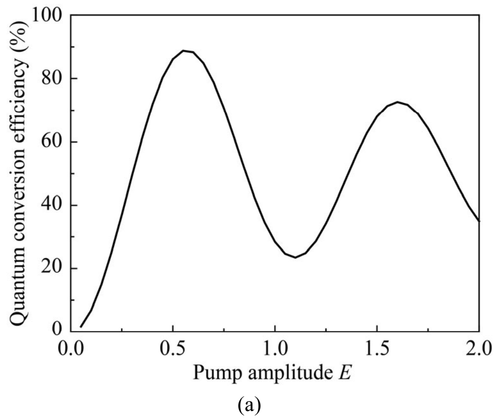
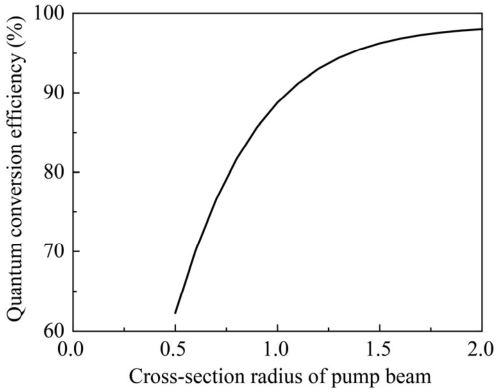


Fig.4 When the peak electric field amplitude *E* is fixed to be 3π/2, the two-dimensional spatial probability distributions of the (a) SFG and (b) unconverted photons, respectively

Additionally, the spatial mode overlap between the pump beam and the signal photon is significant for constructing a single-photon up-conversion system, where the beam size of the pump laser has to be optimized for improving the conversion efficiency. By fixing the pump electric field amplitude at the center of the beam cross-section to be 0.55π, the variation of the quantum conversion efficiency is calculated in Fig.5(b) by changing



(a)



(b)

Fig.5 Quantum conversion efficiency as a function of (a) the pump electric field amplitude and (b) pump beam cross-section radius

the size of the pump beam cross-section radius ω_0 from 0.5 to 2, while the radius of the signal beam is sustained to be 1. The quantum conversion efficiency increases sharply and approaches 98.0% as a saturation value after pump beam radius larger than 2. The simulation results can be used as references in the design of the experimental setup.

The spatial modulation of the converted single-photon is theoretically analyzed by means of frequency up-conversion technique in single-photon or a few photons level. The intensity distribution of the pump beam cross-section is in Gaussian profile. The results indicate that the spatial distribution of the quantum conversion efficiency as well as the output SFG photons strongly depend on the intensity and the cross-section size of the pump beam. By simply and conveniently manipulating the pump laser beam, a programmable spatial modulation in single-photon level could be executed, which will lead to some further important applications in quantum communication and quantum manipulating system, such as optical tweezers, quantum routing and encoding.

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