# Stereo imaging quality evaluation in a full－color three－ dimensional display system＊ 

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#### Abstract

To evaluate the light field display system with a liquid crystal display（LCD）and a holographic functional screen（HFS）， the voxel theory based on the ray tracing is used．By analyzing the voxels defined by the cases of corresponding pixels overlapping completely and partially in the image space，the resolution characteristics of the system are discussed．The theoretical model is verified in the reconstruction experiment of a resolution target and compared with the calculation re－ sult of the presented system．Finally，we give an optimization method for the display image quality．


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Due to the important potential applications，a consider－ able number of three－dimensional（3D）display methods have been presented in recent years ${ }^{[1-10]}$ ．A real－time， large－size，full－color 3D light field display system with the holographic functional screen（HFS）and the projector array was demonstrated in our laboratory ${ }^{[5,6]}$ ．However， complex control equipment and large space for projection are required．Given that the light field display based on liquid crystal display（LCD）panel can be conveniently controlled and is compact，a light field display system with HFS and LCD is demonstrated ${ }^{[9]}$ ．For the purpose of evaluating the display and the reconstruction image，in this paper，the voxel theory based on the ray tracing is used to derive lateral and depth resolutions quantitatively， and their distribution features in image space are dis－ cussed．The size and distribution of the 3D voxel in im－ age space are also derived and confirmed by reconstruc－ tion experiments．

The 3D information can be reconstructed by creating the light field with the relative directions and intensities in the same way as the light originating from a 3D object or scene，which is the basis of our display method．The structure of the system is shown in Fig．1．A 4K LCD panel is used as the display unit to display the elemental images（EIs）which are captured by the camera in differ－ ent angles．In order to improve the display efficiency，an HFS which has the spatial information parallel modula－ tion capability is applied as the terminal display equip－
ment．For efficiently utilizing the LCD panel to display， the Fresnel lens array is used as the light control unit．By adjusting the content of the LCD panel and modulating the Fresnel lens array，the 3D information from different viewpoints is projected to the common area of the HFS． Finally，the 3D space information is modulated and dis－ played on the HFS．


Fig． 1 Schematic diagram of the system structure
20 parallax images are displayed by the LCD，and all the beams from the images with their optical axes con－ verging to a common area are projected onto the HFS at various angles through the Fresnel lens array．The point of the HFS emits multiple light beams with various inten－ sities and colors in different directions in a controlled way，as if they are emitted from the point of the real 3D object at a fixed spatial position ${ }^{[5]}$ ． 20 sub－images with the information of the object are projected into the com－ mon region on the HFS with the special spreading func－

[^0]tion, and the images are arranged in a specific geometry. The HFS makes the necessary optical transformation. Each beam from the corresponding image is spread with the right spatial angle, and all the images contribute to each 3D scene view without sharp boundary between views. So the display provides continuous and smooth change at different view areas. The resolution is one of the important indices of the display system, therefore for the purpose of evaluating the reconstruction of the system precisely, the voxel mode ${ }^{[11]}$ based on the ray tracing is introduced. The model is simple and needs no complicated calculation. According to this mode, the resolution characteristics of the system can be directly demonstrated.

Voxel is the smallest unit of digital data in 3D space. Owing to the limited size of the pixel in the display device, the pixels containing the corresponding points in each EI are regarded as the corresponding pixels overlapping in the image space, which leads to a spatial energy distribution of the light field with 3D information of the object. In fact, this overlapping region cannot be resolved by the viewer unless the signal-to-noise ratio ( $S N R$ ) is high enough. Therefore, the resolved voxel with the HFS is the region in which the overlapping light comes from the corresponding pixel with high $S N R$, which is the minimum resolved unit in the space.

As shown in Fig.2, the reconstructed point $A$ is formed by the pixel from each corresponding EI through the elemental lens center. $d_{1}$ is the size of the EI, $d_{2}$ is the interval of the adjacent EIs, $\delta$ is the size of the pixel, $L$ is the size of the elemental lens, $p$ is the aperture of the elemental lens, and $g$ is the distance between the Fresnel lens array and the EIs.


Fig. 2 Voxel defined by complete overlapping of the corresponding pixels

The voxel is approximated as a cuboid whose lateral size is decided by the projection of the corresponding pixel and longitudinal size is decided by the projection of the corresponding pixel in the marginal EI. According to the geometric relationship, the lateral size and the longitudinal size of the voxel can be expressed as

$$
\begin{align*}
& Z_{\mathrm{c}}=\frac{L g}{L_{\mathrm{p}}-L}\left(L_{\mathrm{p}}=d_{1}+d_{2}+m \delta, m=1,2, \ldots, \frac{d_{1}}{2 \delta}\right),  \tag{1}\\
& H_{\mathrm{c}}=\frac{Z_{\mathrm{c}} \delta}{g}=\frac{L \delta}{L_{\mathrm{p}}-L},  \tag{2}\\
& D_{\mathrm{c}}=\frac{2 L g \delta}{\left(L_{\mathrm{p}}-L\right)\left[(N-1)\left(L_{\mathrm{p}}-L\right)-\delta\right]}, \tag{3}
\end{align*}
$$

where $Z_{\mathrm{c}}$ is the coordinate of the point $A$ along $z$ axis, $N$ is the number of the elemental lenses, $L_{\mathrm{p}}$ is the disparity of the corresponding image points, $H_{\mathrm{c}}$ and $D_{\mathrm{c}}$ are the lateral size and longitudinal size of the voxel, respectively.

In Fig.3, $A$ and $B$ are two neighboring reconstruction points in the image space. The differences between $A$ and $B$ are the corresponding pixels which are adjacent in the marginal EI due to different recording angles. As shown in Fig.3, point $B$ is the region where the light beams from corresponding pixels overlap partially, and it has a local maximum of the $S N R$. The partially overlapping region exits until the distance between $A$ and $B$ is large enough, which means all the corresponding pixels of point $B$ in each EI can be distinguished from those of point $A$. Because of the local maximum of the $S N R$, the region can be regarded as a voxel which overlaps partially with corresponding pixels. The voxel is treated as a cuboid whose lateral size is decided by the projection of half of the corresponding pixel on the middle plane of the partially overlapping region, and the longitudinal size is decided by the length of the partially overlapping region.


Fig. 3 Voxel defined by partial overlapping of the corresponding pixels

In order to make full use of the horizontal parallax, each EI has an offset of $\Delta d$ compared with the corresponding EI in the top row. As shown in Fig.4, $\Delta d$ can be illustrated as

$$
\begin{equation*}
\Delta d=\frac{p\left(Z_{\mathrm{c}}+g\right)}{Z_{\mathrm{c}}}=\frac{L_{\mathrm{p}} p}{L}, \tag{4}
\end{equation*}
$$

so the lateral size and the longitudinal size of the voxel can be represented as

$$
\begin{align*}
& Z_{\mathrm{u}}=\frac{\frac{5}{2} g L}{2 L_{\mathrm{p}}+\Delta d-\frac{5}{2} L}=\frac{5 g L^{2}}{2 L_{\mathrm{p}}(2 L+p)-5 L^{2}},  \tag{5}\\
& H_{\mathrm{u}}=\frac{5 \delta L^{2}}{4 L_{\mathrm{p}}(2 L+p)-10 L^{2}},  \tag{6}\\
& D_{\mathrm{u}}= \\
& {\left[2 L_{\mathrm{p}}(2 L+p)-5 L^{2}\right]^{2}(N-1)-5 \delta L\left[2 L_{\mathrm{p}}(2 L+p)-5 L^{2}\right]} \tag{7}
\end{align*}
$$

where $Z_{\mathrm{u}}$ is the coordinate of the middle plane of the partially overlapping region, $H_{\mathrm{u}}$ and $D_{\mathrm{u}}$ are the lateral size and the longitudinal size of the voxel, respectively.


Fig. 4 The offset $\Delta d$ compared with the corresponding El in the top row

In order to demonstrate the resolution characteristics of the system, the voxel size and the distribution along the $z$ axis are calculated. In the system, the pixel size $\delta$ of each EI used here is 0.096 mm . The Fresnel lens array has $4 \times 5$ elemental lenses with the size of $L=105 \mathrm{~mm}$ and the aperture of $p=25 \mathrm{~mm}$. The distance between the Fresnel lens array and the EIs is $g=135 \mathrm{~mm}$. The number of the elemental lenses in each line is $N=5$. And the distance between HFS and Fresnel lens array is 270 mm . The voxel is the minimum unit that can be resolved in the space, so the reciprocal of the voxel size can be defined as the lateral and longitudinal resolutions. The lateral and longitudinal voxel sizes and their resolutions are shown in Fig.5.

Given that the voxel is formed by the projection of the corresponding pixel, the voxel size and its invariable range will increase along the $z$ axis. So the lateral and longitudinal resolutions will decrease. The lateral size originated from the partially overlapping region is smaller, and the longitudinal size is larger than the voxel size which is decided by the completely overlapping region. So this distribution determines the lateral and the longitudinal resolutions with a contrary tendency as shown in Fig.5, and these voxels will distribute nonhomogeneously because of the alternative distributions of completely and partially overlapping regions, as shown by the step-shape curves of voxel size in Fig.5. The lateral size of the voxel determined by the completely overlapping region is shown as the up-convex part in the curve in Fig.5(a). The lateral size of the voxel determined by the partially overlapping region is shown as the downconvex part in the curve in Fig.5(a). The curve of the longitudinal size of the voxel is shown in reversed pattern in Fig.5(b). For the resolution part, the curve has reverse variation to the curve of voxel size.

To verify the model and the above calculation results, a digital reconstruction experiment is presented. As shown in Fig.6, a resolution test card is used as the target in which the different line densities represent different numerical resolutions. The numerical value of the dense line which is distinguishable in the reconstructed image represents the resolution of the reconstruction. The HFS
in our system is 270 mm far from the Fresnel lens array. With the system parameters mentioned above, we use the computer-generated integral imaging method ${ }^{[12]}$ to generate the reconstructed images located within the range from $z=269.4 \mathrm{~mm}$ to $z=270.8 \mathrm{~mm}$. Through the reconstruction of the original image in different positions, the resolution curve can be obtained in the prescribed range. Due to the limited precision of the reconstruction method and the approximation used in the definition of the voxel, there are differences in voxel size between the experimental result and the calculation result. However, the trend of the variation in voxel size in the experiment agrees well with the calculation result, which verifies the voxel model.


Fig. 5 Voxel sizes and resolutions along the $z$ axis
According to Eqs.(2), (3), (6) and (7), the disparity of the corresponding image points $L_{\mathrm{p}}$ is the decisive factor of $H_{\mathrm{c}}, D_{\mathrm{c}}, H_{\mathrm{u}}$ and $D_{\mathrm{u}}$, so it directly determines the resolution of the final display. With the increase of $L_{\mathrm{p}}$, the lateral and longitudinal resolutions will be improved in both of the completely and partially overlapping situations. To avoid the crosstalk between adjacent elemental images, the offset $\Delta d$ is limited as
$\Delta d<d_{1}+d_{2}$.
Combined with Eq.(4), $L_{\mathrm{p}}$ can be inferred as
$L_{\mathrm{p}}<\frac{\left(d_{1}+d_{2}\right) L}{p}$.


Fig. 6 Results of the numerical reconstruction experiment: (a) Resolution test card used as the target; (b) The reconstructed image with the distance of $z=270 \mathrm{~mm}$; Experimental and calculated (c) lateral and (d) longitudinal sizes of voxel with their distributions along $z$ axis from 269.4 mm to 270.6 mm

Thus, within a reasonable range, increasing the value of Lp can appropriately improve the lateral and longitudinal resolutions of the image. Fig. 7 shows the observed


Fig. 7 The observed 3D images with different $L_{p}$ values

3D images with different Lp values. The second row has a larger Lp compared with the top row. As shown in the white circle, the resolution of the reconstructed image in the second row is higher than that in the first row.

In summary, a light field display system based on an LCD and an HFS is an effective 3D display method. We use the voxel theory based on the ray tracing to evaluate the system. By analyzing the corresponding pixels overlapping partially and completely, the voxel size and the resolution of the reconstructed image and their distribution features are derived. The voxel model is verified through the reconstruction experiment. Finally, we give the relationship between the resolution of the reconstructed image and the disparity of the corresponding image points $L_{\mathrm{p}}$, which provides a discriminant condition to the quality of the display image.

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