A novel TS-EIA-PTS *PAPR* reduction algorithm for optical OFDM systems*

YUAN Jian-guo (袁建国)**, **LI Zhang-chao** (李璋超), **MA Jun** (马骏), **WANG Yong** (王永), **LIN Jin-zhao** (林金朝), **and PANG Yu** (庞宇)

Key Laboratory of Optical Communication and Networks, Chongqing University of Posts and Telecommunications, Chongqing 400065, China

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Because the partial transmit sequence (PTS) peak-to-average power ratio (*PAPR*) reduction technology for optical orthogonal frequency division multiplexing (O-OFDM) systems has higher computational complexity, a novel two-stage enhanced-iterative-algorithm PTS (TS-EIA-PTS) *PAPR* reduction algorithm with lower computational complexity is proposed in this paper. The simulation results show that the proposed TS-EIA-PTS *PAPR* reduction algorithm can reduce the computational complexity by 18.47% in the condition of the original signal sequence partitioned into 4 sub-blocks at the remaining stage of $n-d$=5. Furthermore, it has almost the same *PAPR* reduction performance and the same bit error rate (*BER*) performance as the EIA-PTS algorithm, and with the increase of the subcarrier number, the computational complexity can be further reduced. As a result, the proposed TS-EIA-PTS *PAPR* reduction algorithm is more suitable for the practical O-OFDM systems.

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The optical orthogonal frequency division multiplexing (O-OFDM) technique is the perfect combination of advantages of wireless OFDM technique and optical communication[1], which has received extensive attention owing to its characteristics of anti-multipath fading, anti-intersymbol interference and high spectral efficiency[2]. In general, however, all systems based on OFDM technique inevitably have high peak-to-average power ratio (*PAPR*) that easily leads to the degradation of systems' performance, so some relative techniques are used to reduce the *PAPR* of the system[3,4].

Many *PAPR* reduction techniques[5-10] have been proposed in the past decade. Among them, the partial transmit sequence (PTS) technique is a promising one which has no loss in the data transmission rate, power and bit error rate (*BER*), and can significantly improve the *PAPR* except the transmission need of the side information[8-11]. However, the conventional PTS technique suffers from higher computational complexity[10,11], so some PTS algorithms with low computational complexity have been developed. Ref.[12] proposed a simple iterative flipping algorithm. However, its *PAPR* reduction performance is worse than that of the conventional PTS technique due to less alternative sequences. An improved iterative flipping algorithm for PTS technique called as enhanced-iterative-algorithm PTS (EIA-PTS) is proposed in Ref.[13]. It reduces the computational complexity by flexibly adjusting the parameters. Furthermore, an EIA-PTS-clipping combined *PAPR* reduction technique, which can reach a better tradeoff between *BER* performance and the *PAPR* reduction effect but will cause the degeneration of *BER* performance, is also proposed in Ref.[13].

In this paper, further researches based on the sub-optimal PTS algorithm[12] and EIA-PTS algorithm[13] are processed. A two-stage EIA-PTS (TS-EIA-PTS) algorithm for reducing the computational complexity is proposed. The computational complexity is analyzed, and the *PAPR* reduction performance and the *BER* performance are simulated at the same time.

The O-OFDM signal is the sum of a number of modulated subcarriers, and the *PAPR* of the O-OFDM signal is expressed as[14]

$$PAPR(x_n)=10\log_{10}\frac{\max\{|x_n|^2\}}{E\{|x_n|^2\}}, \quad (1)$$

where x_n denotes the output signal through the inverse fast Fourier transform (IFFT) operation. O-OFDM generally uses complementary cumulative distribution function (*CCDF*) to evaluate the probability of *PAPR* ex-

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** E-mail: yuanjg@cqupt.edu.cn

ceeding the peak threshold $PAPR_0$, which can be expressed as[15]

$$P\{PAPR > PAPR_0\} = 1 - \left(1 - e^{-\lambda}\right)^N . \quad (2)$$

The N input symbol vectors $\boldsymbol{X}=\{\boldsymbol{X}_k, k=0, 1, \cdots, N-1\}$ firstly are partitioned into M disjoint sub-blocks $\{\boldsymbol{X}_m, m=1, 2, \cdots, M\}$ according to some partition methods, which can be expressed as

$$\boldsymbol{X} = \sum_{m=1}^{M} \boldsymbol{X}_m . \quad (3)$$

Let $b=\{b_m=e^{j\varphi_m}, m=1, 2, \cdots, M\}$ ($\varphi_m \in [0, 2\pi]$) be the set of phase factors which are applied to each signal sub-block $\boldsymbol{X}_m$ and combined together to create a set of candidates

$$\boldsymbol{X}_k^{'} = \sum_{m=1}^{M} b_m \boldsymbol{X}_k^{(m)}, k = 0,1,\ldots,N-1 , \quad (4)$$

where $\boldsymbol{X}_k^{(m)}$ is the data symbol of the sub-block $\boldsymbol{X}_m$. Taking the IFFT of Eq.(4) and using the linearity property of the inverse discrete Fourier transform (IDFT), the time domain partial transmit sequences can be expressed as

$$x_n^{'} = \sum_{m=1}^{M} b_m x_n^{(m)}, n = 0,1,\ldots,N-1 , \quad (5)$$

where $x_n^{(m)}$ is a partial transmission sequence of x_n. Consequently, for finding the optimal phase factor for each input data sequence, assume that there are W phase vectors, and W^{M-1} combinations should be computed and compared in order to obtain the minimum $PAPR$, which should meet the condition of

$$\left\{\hat{b}_1,\ldots,\hat{b}_m\right\} = \underset{[b_1,\ldots,b_m]}{\arg\min}\left\{\max\left\{\left|x_n^{'}\right|^2\right\}\right\} . \quad (6)$$

Finally, the candidate with the lowest $PAPR$ is chosen by exhaustive search of the candidates for transmission.

In conventional iterative flipping algorithm[12], each sub-block only retains one phase factor after each iterative flipping, which does not necessarily show a minimum $PAPR$ in the other sub-blocks. Thus, an improved EIA-PTS technique is firstly proposed in Ref.[13], and four phase factors $b_m \in \{\pm 1, \pm j\}$ are used to reduce the $PAPR$ of signals. The adjustable parameter G is set as the reserved phase factor in this algorithm, where $1 \le G \le W$. So the larger the value of G, the better the obtained performance, while the more calculation amount is needed.

The steps of calculating candidates in the EIA-PTS algorithm after partitioning are summarized as follows. ①Select adjustable parameter G ($1 \le G \le W$) which determines the reserving number of phase factors in each sub-block and needs to compromise the system performance and the computational complexity. ②Retain $G+1$ phase factors when the first sub-block is calculated, and the W phase factors are retained when $G+1 \ge W$. ③ Apply iterative flipping algorithm to each node which is retained from the first sub-block, and then retain G phase factors in the second sub-block. ④Apply iterative flipping algorithm to each node which is retained from the second sub-block, and retain $G-1$ phase factors in the third sub-block. Repeat this operation until the last sub-block. ⑤Find the phase sequence with the minimum $PAPR$ and obtain the candidate signal to transmit after finding phase sequence to each sub-block.

According to Ref.[13], the iteration number of the EIA-PTS algorithm is expressed as

$$C = 2W + (W-1)(M-1)G - 1 , \quad (7)$$

where the iteration number of the conventional PTS algorithm is W^{M-1}.

Based on the EIA-PTS algorithm, an improved TS-EIA-PTS algorithm with the low computational complexity is proposed. Schematic diagram of the $PAPR$ reduction technique based on the TE-EIA-PTS algorithm is shown in Fig.1. Different from the conventional PTS algorithm and the EIA-PTS algorithm which partitions input symbol sequences at the initial stage, the proposed algorithm partitions the input OFDM signals after the first d stages of IFFT, where $1 \le d \le n-1$. That is to say, the mapping operation $2^n \rightarrow 2^d$ is applied to the input OFDM signals, and the 2^n-point IFFT operation of the input symbol sequence X_{data} in the conventional algorithm is divided into two parts. The first part is the first d stages of IFFT, and the second part is the remaining $n-d$ stages (assume that the number of subcarriers is $N=2^n$). The IFFT operations are partially applied to the input symbol sequence X_{data} to form a temporary signal sequence x_{data}, and the temporary signal sequence is partitioned into M temporary signal subsequences. Then, the remaining $n-d$ stages of IFFT are applied to each of the temporary signal subsequence, and the EIA-PTS algorithm of G adjustable parameters is applied to find the phase sequence with the minimum $PAPR$. Finally, obtain the candidate signal to transmit after finding phase sequence to each sub-block.

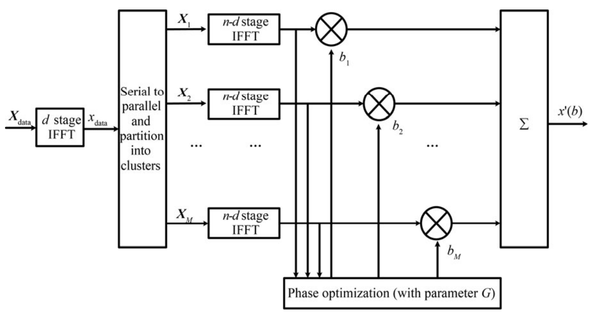

Fig.1 Schematic diagram of the *PAPR* reduction technique based on the TE-EIA-PTS algorithm

The conventional optimal algorithm requires M 2^n-point IFFT operations with M sub-blocks. Therefore, the numbers of the complex multiplications and complex additions for the conventional PTS algorithm are $n_{\text{mul}}=2^{n-1}nM$ and $n_{\text{add}}=2^n nM$, respectively. In addition, $2^n W^{M-1}(M-1)$ complex additions are required for combining the M sub-block signals to obtain the W^{M-1} candidates and searching for the lowest *PAPR* out of them. Thus, the total computational complexity of the conventional PTS algorithm is expressed as

$$C_{\text{C-PTS}} = nM(2^{n-1}+2^{n})+W^{M-1}(M-1)2^{n} . \tag{8}$$

Thus, according to Eq.(8), the total computational complexity of the EIA-PTS algorithm is expressed as

$$C_{\text{EIA-PTS}} = nM(2^{n-1}+2^{n})+C(M-1)2^{n} . \tag{9}$$

Whereas for the proposed algorithm, the total computational complexity is expressed as

$$C_{\text{TS-EIA-PTS}} = C_{d} + C_{n-d} , \tag{10}$$

where $C_d=2^{n-1}n+2^n n$ is the computational complexity of the first d stages, and $C_{n-d}=(M-1)(2^{n-l-1}+2^{n-l})(n-l)+C'(M-2)2^{n-l}$ is the computational complexity of the remaining $n-d$ stages. Here $C'=2W+(W-1)(M-1)G-1$.

It has been proved in Ref.[13] that the EIA-PTS algorithm can reduce the computational complexity of the conventional PTS algorithm. Consequently, in order to demonstrate the complexity reduction ability of the proposed algorithm, the computational complexity reduction ratio (*CCRR*) is defined as

$$CCRR = (1-\frac{C_{\text{TS-EIA-PTS}}}{C_{\text{EIA-PTS}}})\times 100\% . \tag{11}$$

The value of *CCRR* can be computed by Eq.(11) under conditions with different N, M and $n-d$, and the result is shown in Tab.1. As shown in Tab.1, compared with the EIA-PTS algorithm, the proposed algorithm can further reduce the computational complexity. In addition, some results can also be found in Tab.1. Firstly, when we keep M and N constant, the complexity reduction ability of the proposed algorithm is degraded with the increase of the remaining $n-d$ stages, because the proposed algorithm has more IFFT operations with the increase of the remaining stage $n-d$. Secondly, when we keep M and $n-d$ constant, the computational complexity of the proposed algorithm is reduced rapidly with the increase of N. That is to say, the proposed algorithm is suitable for the systems with a large number of subcarriers. Thirdly, when we keep N and $n-d$ constant, the complexity reduction ability of the proposed algorithm is enhanced with the increase of M.

Tab.1 Comparison of *CCRR* with G=2 and W=4

$n-d$	*CCRR* (%)					
	N=128 (n=7)		N=512 (n=9)		N=2 048 (n=11)	
	M=2	M=4	M=2	M=4	M=2	M=4
2	31.25	46.88	32.63	48.72	36.47	54.15
3	25.12	37.50	26.82	39.51	31.81	47.73
4	18.75	28.13	21.25	31.27	27.33	38.92
5	12.50	18.47	16.37	23.13	22.52	30.16

The simulation results of the *PAPR* reduction performance for the TS-EIA-PTS algorithm are shown in Fig.2. In this simulation, the quadrature phase shift keying (QPSK) modulation scheme is applied, the number of oversampling factors is 4, the number of the subcarriers is N=128, the number of the sub-blocks is M=4, and the set of the phase factors is $W\in\{\pm 1, \pm j\}$. Moreover, the pseudo-random partition is applied to the partitioning process, and select the adjustable parameter G=2 as a sample.

Fig.2 shows the simulation results as the stage of block partition is varied for $n-d$=2 and 5. It is obviously shown in Fig.2 that compared with the original OFDM signal and the sub-optimal algorithm in Ref.[12], the TS-EIA-PTS algorithm significantly reduces the probability of high *PAPR*. And it can further improve the *PAPR* reduction performance of the system, and the curve for TS-EIA-PTS algorithm gradually approaches the EIA-PTS curve with the increase of the remaining stage $n-d$. When *CCDF* is 10^{-4} and $n-d$=5, the $PAPR_0$ of TS-EIA-PTS algorithm is reduced by 3.6 dB compared with the original signal, while it is only degraded by 0.2 dB compared with the EIA-PTS algorithm.

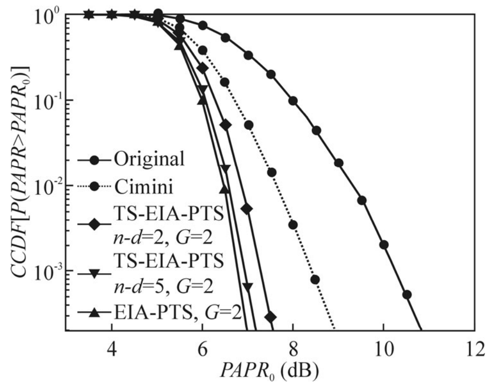

Fig.2 *PAPR* reduction performances of the sub-optimal algorithm in Ref.[12], the TS-EIA-PTS algorithm with $n-d$=2 and 5 and EIA-PTS algorithm

Fig.3 illustrates the comparison of the *BER* performances of the proposed TS-EIA-PTS algorithm, clipping technique proposed in Ref.[5], EIA-PTS algorithm and EIA-PTS-clipping algorithm proposed in Ref.[13]. In order to make the simulation more conducive for comparative analysis, the relatively strict clipping condition (γ=2) is considered. As shown in Fig.3, the TS-EIA-PTS algorithm almost has the same *BER* performance with the EIA-PTS algorithm, because both algorithms don't have distortion of the signal processing and can easily recover the original signal. The *BER* performance of the clipping technique is the worst due to this process causes a serious distortion of the signal, which makes the signal recovery more difficult in the receiver. And the EIA-PTS-clipping technique, which combines both clipping technology and EIA-PTS technology, is able to reduce the influence of noise generated by clipping to some extent.

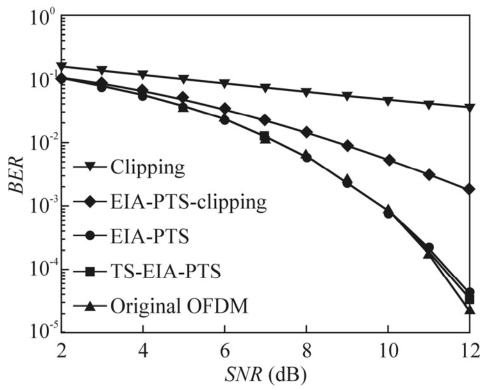

Fig.3 *BER* performances of the TS-EIA-PTS algorithm, the EIA-PTS algorithm, the EIA-PTS-clipping technique and the clipping technique

The *PAPR* performance of the TS-EIA-PTS algorithm is worse than that of EIA-PTS algorithm, not to mention that of the EIA-PTS-clipping algorithm, but the *PAPR* performance of the TS-EIA-PTS algorithm will gradually approach the EIA-PTS curve with the increase of the remaining stage $n-d$, and the computational complexity is reduced by about 18.47% when $n-d=5$. The EIA-PTS-clipping algorithm indeed has the best *PAPR* performance among the three *PAPR* reduction algorithms, but its *BER* performance is the worst, while the *BER* performance is the most important index to evaluate the systems and the purpose of reducing *PAPR* is to optimize the *BER* performance of the systems. Therefore, the TS-EIA-PTS technique for *PAPR* reduction is more feasible to practical optic-fiber communication system when this system needs low *BER* and low computational complexity.

The *PAPR* reduction technique based on PTS algorithm for O-OFDM system is investigated in this paper. An improved TS-EIA-PTS algorithm with low computational complexity based on the EIA-PTS algorithm is proposed. When the original signal sequence is partitioned into 4 sub-blocks with the remaining stage $n-d=5$, the simulation results show that the proposed algorithm can reduce the computational complexity by 18.47% with the *PAPR* performance degradation less than 0.2 dB at the *CCDF* of 10^{-4}. Moreover, it has almost the same *BER* performance as the EIA-PTS algorithm, and the *CCRR* increases as the number of subcarriers increases. Therefore, the proposed algorithm has more applications in practical high data rate O-OFDM systems.

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