# Average bit error rate performance analysis of subcarrier intensity modulated MRC and EGC FSO systems with dual branches over M distribution turbulence channels*

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Based on the space diversity reception, the binary phase-shift keying (BPSK) modulated free space optical (FSO) system over Málaga (M) fading channels is investigated in detail. Under independently and identically distributed and independently and non-identically distributed dual branches, the analytical average bit error rate (ABER) expressions in terms of H-Fox function for maximal ratio combining (MRC) and equal gain combining (EGC) diversity techniques are derived, respectively, by transforming the modified Bessel function of the second kind into the integral form of Meijer G-function. Monte Carlo (MC) simulation is also provided to verify the accuracy of the presented models.

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In recent years, free-space optical (FSO) communication has become very popular in commercial communications for its special characteristics, such as effective cost, high bandwidth, free license and perfect security[1,2]. However, the atmosphere conditions will have severe impact on the reliability and availability of FSO links. And the major impairment is the atmosphere turbulence, which will induce the density and phase fluctuations of optical signals[3]. Spatial diversity can avoid this problem by using multiple smaller receiving apertures to create a large aperture at the receiver end, which can improve the performance of FSO system significantly. Up to now, three main diversity combining technologies have been reported, including maximal ratio combining (MRC), equal gain combining (EGC) and selection combining (SC)[4,5]. Among them, both MRC and EGC can fully exploit the amount of the diversity offered by the channels, and achieve substantial power gains.

In order to evaluate the reliability and the effectiveness of the methods for improving the performance of FSO systems, accurate fading distribution models are required. Recently, a new fading model called Málaga (M) distribution was proposed by Antonio Jurado-Navas et al[6], and their study showed that M distribution can achieve excellent agreement with the experimental data under weak-to-strong turbulence conditions. However, the analytical average bit error rate (ABER) performance of FSO systems considering diversity technologies based on the M distribution only has been reported in Ref.[7] so far, in which Liang Yang et al achieved the ABER expressions of SC and optimum combining (OC) by expressing the Meijer-G function in terms of the generalized power series, but they mainly focused on the asymptotic analysis with ABER only from $10^{-1}$ to $10^{-4}$.

In this paper, the ABER performances of dual-branch MRC and EGC FSO systems over M distribution fading channels are investigated systematically. The probability density functions (PDFs) of the combined M distribution in MRC and EGC systems are obtained by expressing the modified Bessel function of the second kind with the integral form of Meijer G-function. The ABER expressions are then derived and represented as the H-Fox function. The ABER performances for independently and identically distributed (the channels are assumed to have the same fading parameters and independent of each other), independently and non-identically distributed (the channels are independent of each other but not necessary to have the same fading parameters)[5] dual channels

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under weak-to-strong turbulence conditions are compared with those of single-in single-out (SISO). The ABERs of MRC and EGC systems with different scattering powers coupled to the line-of-sight (LOS) component $\rho$ for the same intensity of turbulence are further studied. All the analytical results are in excellent agreement with the Monte Carlo (MC) simulations.

A binary phase shift keying (BPSK) subcarrier intensity modulation (SIM) system[8] with dual independent channels is considered here. Fig.1 shows the block diagram of dual-branch system. $a_1$ and $a_2$ are the weight factors. At the $l$th receiver, the received electrical signal can be written as $y=RP_tXI_l+n_l(t)$, where $P_t$ is the average transmitted optical power, $R$ is the detector responsibility, $X\in\{-1,1\}$ represents the transmitted data, $I_l$ is the irradiance induced by atmosphere turbulence which is assumed to follow the M distribution, and $n_l(t)$ is the additive white Gaussian noise in the $l$th receiver with zero mean and variance of $\sigma_n^2$ . According to the definition of the instantaneous signal to noise ratio (SNR) $r$ in Ref.[9], it can be expressed as $r=\left(RP_tI\right)^2/\sigma_n^{\,2}$, and the electrical SNR can be obtained as $\bar{r}=\left(RP_t\right)^2/\sigma_n^{\,2}$.

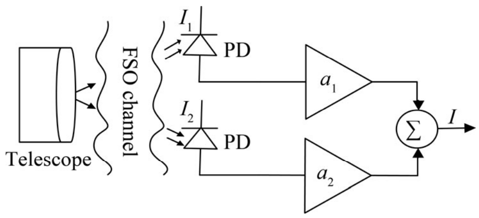



**Fig.1 The structure of MRC and EGC FSO systems with dual branches**

Here, the turbulence fading is modeled by the M distribution, and the PDF of $I$ in each branch is given as

$$f\left(I\right)=A\sum_{k=1}^{\beta}a_kI^{\frac{\alpha+k}{2}-1}K_{\alpha-k}\left(2\sqrt{\frac{\alpha\beta I}{\gamma\beta+\Omega'}}\right), \tag{1}$$

where

$$A=\frac{2\alpha^{\frac{\alpha}{2}}}{\gamma^{1+\frac{\alpha}{2}}\Gamma\left(\alpha\right)}\left(\frac{\gamma\beta}{\gamma\beta+\Omega'}\right)^{\beta+\frac{\alpha}{2}}, \tag{2}$$

$$a_k=\binom{\beta-1}{k-1}\frac{\left(\gamma\beta+\Omega'\right)^{1-\frac{k}{2}}}{(k-1)!}\left(\frac{\Omega'}{\gamma}\right)^{k-1}\left(\frac{\alpha}{\beta}\right)^{\frac{k}{2}}. \tag{3}$$

Moreover, $K_v(\cdot)$ is the second kind of modified Bessel function of order $v$, $\alpha$ is a positive parameter depending on the effective number of large scale cells in the scattering process, and $\beta$ is a natural number, which denotes the amount of fading parameters. $\gamma=\rho\gamma_0$ represents the average power of the classic scattering components received by the off axis, where $\rho(0\le\rho\le1)$ denotes the scattering power coupled to the LOS component, and $\gamma_0$ is the average power of the total scattering components[6]. $\Gamma(\cdot)$ is the Gamma function. The parameter $\Omega'$ denotes the average power from the coherent contribution. The M distribution unifies most of the existing fading models. For instance, setting $\rho$=1, the M distribution will reduce to the Gamma-Gamma (GG) model, and it will reduce to the strong turbulence condition of K distribution[10] with $\rho$=0, $\Omega'$=0 or $\beta$=1.

When MRC technique is employed, the SNR of the system is $r_{\rm MRC}=\sum_{l=1}^{L}r_l/L=\left(\bar{r}/L\right)\sum_{l=1}^{L}I_l^2$ [11], where $L$ means the number of branches, which is taken as 2 here.

To obtain the joint PDF $f_{\rm MRC}(I)$, the moment generation function (MGF) method is employed. Letting $Y=I^2$ and replacing $K_v(\cdot)$ with the integral form of Meijer G-function[12], the PDF of $Y$ is given as

$$f_Y\left(Y\right)=\frac{A}{4}\sum_{k=1}^{\beta}a_kY^{\frac{\alpha+k}{4}-1}\frac{1}{2\pi\mathrm{j}}\int_{\sigma-\mathrm{j}\infty}^{\sigma+\mathrm{j}\infty}\Gamma\left(\mu-s\right)\times$$
$$\Gamma\left(-\mu-s\right)B^sY^{\frac{s}{2}}\mathrm{d}s, \tag{4}$$

where $\mu=(\alpha-k)/2$ and $B=\alpha\beta/(\gamma\beta+\Omega')$.

Applying the Laplace transformation to Eq.(4), the MGF of $Y$ is expressed as

$$M_Y(t)=\frac{A}{4}\sum_{k=1}^{\beta}a_k\frac{1}{2\pi\mathrm{j}}\int_{\sigma-\mathrm{j}\infty}^{\sigma+\mathrm{j}\infty}\Gamma\left(\mu-s\right)\Gamma\left(-\mu-s\right)\times$$
$$\Gamma\left(\omega+\frac{s}{2}\right)B^st^{-\left(\omega+\frac{s}{2}\right)}\mathrm{d}s, \tag{5}$$

where $\omega=(\alpha+k)/4$.

Since the channels are independent of the spatial diversity system for each branch, through the product of the MGFs of all single channels, the MGF of $Y=\sum_{l=1}^{2}Y_l$ can be given as

$$M_Y\left(t\right)=\frac{A_1A_2}{16}\sum_{k_1=1}^{\beta_1}\sum_{k_2=1}^{\beta_2}a_{k_1}a_{k_2}\left(\frac{1}{2\pi\mathrm{j}}\right)^2\times$$
$$\int_{\sigma_1-\mathrm{j}\infty}^{\sigma_1+\mathrm{j}\infty}\int_{\sigma_2-\mathrm{j}\infty}^{\sigma_2+\mathrm{j}\infty}\prod_{l=1}^{2}\Gamma\left(\mu_l-s_l\right)\times$$
$$\Gamma\left(-\mu_l-s_l\right)\Gamma\left(\omega_l+\frac{s_l}{2}\right)B_l^{s_l}t^{-\left(\omega_l+\frac{s_l}{2}\right)}\mathrm{d}s_1\mathrm{d}s_2. \tag{6}$$

Moreover, by applying the inverse Laplace transform to Eq.(6), the final joint PDF for M fading model in the case of MRC can be expressed as

$$f_{\rm MRC}\left(I\right)=\frac{A_1A_2}{16}\sum_{k_1=1}^{\beta_1}\sum_{k_2=1}^{\beta_2}a_{k_1}a_{k_2}\left(\frac{1}{2\pi\mathrm{j}}\right)^2\times$$
$$\int_{\sigma_1-\mathrm{j}\infty}^{\sigma_1+\mathrm{j}\infty}\int_{\sigma_2-\mathrm{j}\infty}^{\sigma_2+\mathrm{j}\infty}\prod_{l=1}^{2}\Gamma\left(\mu_l-s_l\right)\Gamma\left(-\mu_l-s_l\right)\Gamma\left(\omega_l+\frac{s_l}{2}\right)\times$$
$$B_l^{s_l}\frac{2I^{\sum_{l=1}^{2}(2\omega_l+s_l)-1}}{\Gamma\left[\sum_{l=1}^{2}\left(\omega_l+\frac{s_l}{2}\right)\right]}\mathrm{d}s_1\mathrm{d}s_2, \tag{7}$$

and thus the ABER expression of MRC system can be obtained as

$$P_{\rm MRC}=\frac{1}{2}\int_0^\infty erfc(\sqrt{\frac{r_{\rm MRC}}{2}})f_{\rm MRC}(I)\,{\rm d}I=$$

$$\frac{A_1A_2}{32\pi^{1/2}}\sum_{k_1=1}^{\beta_1}\sum_{k_2=1}^{\beta_2}a_{k_1}a_{k_2}\left(\frac{1}{2\pi{\rm j}}\right)^2\int_{\sigma_1-{\rm j}\infty}^{\sigma_1+{\rm j}\infty}\int_{\sigma_2-{\rm j}\infty}^{\sigma_2+{\rm j}\infty}\prod_{l=1}^{2}\Gamma(\mu_l-s_l)\times$$

$$\Gamma(-\mu_l-s_l)\Gamma\left(\omega_l+\frac{s_l}{2}\right)B_l^{s_l}\times$$

$$\frac{\Gamma\left\{\frac{1}{2}+\frac{1}{2}\left[\sum_{l=1}^{2}(2\omega_l+s_l)\right]\right\}}{\Gamma\left[1+\sum_{l=1}^{2}\left(\omega_l+\frac{s_l}{2}\right)\right]}\left(\frac{\bar{r}}{4}\right)^{-\sum_{l=1}^{2}\left(\omega_l+\frac{s_l}{2}\right)}{\rm d}s_1{\rm d}s_2. \quad (8)$$

For EGC, the SNR of EGC systems is given as $r_{\rm EGC}=(\sum_{l=1}^{L}\sqrt{r_l})^2/L^2=(\bar{r}/L^2)\left(\sum_{l=1}^{L}I_l\right)^2$ [11]. Following the similar procedure of deriving Eq.(7), the joint PDF $f_{\rm EGC}(I)$ can be expressed as

$$f_{\rm EGC}(I)=\frac{A_1A_2}{4}\sum_{k_1=1}^{\beta_1}\sum_{k_2=1}^{\beta_2}a_{k_1}a_{k_2}\left(\frac{1}{2\pi{\rm j}}\right)^2\times$$

$$\int_{\sigma_1-{\rm j}\infty}^{\sigma_1+{\rm j}\infty}\int_{\sigma_2-{\rm j}\infty}^{\sigma_2+{\rm j}\infty}\prod_{l=1}^{2}\Gamma(\mu_l-s_l)\Gamma(-\mu_l-s_l)\Gamma(2\omega_l+s_l)\times$$

$$B_l^{s_l}\frac{I^{\sum_{l=1}^{2}(2\omega_l+s_l)-1}}{\Gamma\left[\sum_{l=1}^{2}(2\omega_l+s_l)\right]}{\rm d}s_1{\rm d}s_2. \quad (9)$$

Therefore, the ABER of EGC system can be given as

$$P_{\rm EGC}=\frac{1}{2}\int_0^\infty erfc(\sqrt{\frac{r_{\rm EGC}}{2}})f_{\rm EGC}(I)\,{\rm d}I=$$

$$\frac{A_1A_2}{8\pi^{1/2}}\sum_{k_1=1}^{\beta_1}\sum_{k_2=1}^{\beta_2}a_{k_1}a_{k_2}\left(\frac{1}{2\pi{\rm j}}\right)^2\int_{\sigma_1-{\rm j}\infty}^{\sigma_1+{\rm j}\infty}\int_{\sigma_2-{\rm j}\infty}^{\sigma_2+{\rm j}\infty}\prod_{l=1}^{2}\Gamma(\mu_l-s_l)\times$$

$$\Gamma(-\mu_l-s_l)\Gamma(2\omega_l+s_l)B_l^{s_l}\times$$

$$\frac{\Gamma\left\{\frac{1}{2}+\frac{1}{2}\left[\sum_{l=1}^{2}(2\omega_l+s_l)\right]\right\}}{\Gamma\left[1+\sum_{l=1}^{2}(2\omega_l+s_l)\right]}\left(\frac{\bar{r}}{8}\right)^{-\sum_{l=1}^{2}\left(\omega_l+\frac{s_l}{2}\right)}{\rm d}s_1{\rm d}s_2. \quad (10)$$

Finally, the ABER expressions of MRC and EGC systems under M fading channels in terms of H-Fox function[13] are obtained as

$$P_{\rm MRC}=\frac{A_1A_2}{32\pi^{1/2}}\sum_{k_1=1}^{\beta_1}\sum_{k_2=1}^{\beta_2}a_{k_1}a_{k_2}\left(\frac{\bar{r}}{4}\right)^{-\sum_{l=1}^{2}\omega_l}H_{1,1:1,2;1,2}^{0,1:2,1;2,1}\left[\begin{matrix}B_1\left(\frac{\bar{r}}{4}\right)^{-\frac{1}{2}}\\ B_2\left(\frac{\bar{r}}{4}\right)^{-\frac{1}{2}}\end{matrix}\middle|\begin{matrix}\left(\frac{1}{2}-\sum_{l=1}^{2}\omega_l;\frac{1}{2},\frac{1}{2}\right):\left(1-\omega_1,\frac{1}{2}\right),\left(1-\omega_2,\frac{1}{2}\right)\\ \left(-\sum_{l=1}^{2}\omega_l;1,1\right):(\mu_1,1),(-\mu_1,1);(\mu_2,1),(-\mu_2,1)\end{matrix}\right], \quad (11)$$

$$P_{\rm EGC}=\frac{A_1A_2}{8\pi^{1/2}}\sum_{k_1=1}^{\beta_1}\sum_{k_2=1}^{\beta_2}a_{k_1}a_{k_2}\left(\frac{\bar{r}}{8}\right)^{-\sum_{l=1}^{2}\omega_l}H_{1,1:1,2;1,2}^{0,1:2,1;2,1}\left[\begin{matrix}B_1\left(\frac{\bar{r}}{8}\right)^{-\frac{1}{2}}\\ B_2\left(\frac{\bar{r}}{8}\right)^{-\frac{1}{2}}\end{matrix}\middle|\begin{matrix}\left(\frac{1}{2}-\sum_{l=1}^{2}\omega_l;\frac{1}{2},\frac{1}{2}\right):(1-2\omega_1,1),(1-2\omega_2,1)\\ \left(-\sum_{l=1}^{2}2\omega_l;1,1\right):(\mu_1,1),(-\mu_1,1);(\mu_2,1),(-\mu_2,1)\end{matrix}\right], \quad (12)$$

which can be readily evaluated based on the above two-fold Mellin-Barnes representations.

The analytical ABER expressions of MRC and EGC systems over M distribution are obtained based on Eqs.(8) and (10), respectively. In Fig.2, the ABER performances of FSO systems are plotted against the electrical SNR $\bar{r}$ over independently and identically distributed M turbulence channels under weak-to-strong turbulence conditions. The performance of SISO system is also provided for comparison[14]. It can be seen that the analytical results have excellent agreement with MC simulations for all the turbulence conditions, which shows the correctness of the presented ABER expressions of MRC and EGC. In addition, the ABER performances of both the MRC and EGC systems are substantially improved compared with that of the SISO system, and the performance of MRC outperforms that of EGC over M fading channels under the same turbulence conditions.

Fig.3 presents the ABER performances of both MRC and EGC FSO systems over independently and non-identically distributed M turbulence channels under weak-to-strong turbulence conditions. The scintillation index $\sigma_I^2$ is used to characterize the amount of scintillation induced by the atmospheric turbulence, which is defined as

$$\sigma_I^2=E[I^2]/E^2[I]-1. \quad (13)$$

For the M model, the scintillation index can be obtained with the help of the expression as[6]

$$m_n(I)=E[I^n]=\frac{\Gamma(\alpha+k)}{\Gamma(\alpha)\alpha^n}\frac{1}{\gamma}\left(\frac{\gamma\beta}{\gamma\beta+\Omega'}\right)^\beta\times$$

$$\sum_{k=0}^{\beta}\binom{\beta-1}{k}\frac{1}{k!}\left[\frac{\Omega'}{\gamma(\gamma\beta+\Omega')}\right]^k\frac{\Gamma(k+n+1)}{\left(\frac{\beta}{\gamma\beta+\Omega'}\right)^{k+n+1}}. \quad (14)$$

For MRC, each turbulence condition includes two sets of parameters, from which the $\sigma_I^2$ [(0.32, 0.36), (0.36, 0.4)], [(0.840 1, 0.705), (0.840 1, 1.011)] and [(1.95, 1.487 8), (1.664 1, 1.95)] can be obtained. While for EGC, the corresponding scintillation indices $\sigma_I^2$ are [(0.30, 0.406 3), (0.43, 0.406 3)], [(0.702 4, 0.893 8), (0.91, 0.893 8)] and [(2, 1.5), (1.56, 1.5)]. Besides, with the increase of $\sigma_I^2$, the ABER performances of both systems get worse. For instance, to achieve the ABER of $10^{-7}$ in MRC system, SNR of about 25 dB is needed with $\sigma_I^2$ of (0.36, 0.4), while SNR of only 22 dB is needed for the system with $\sigma_I^2$ of (0.36, 0.32) over M distribution. For EGC, the worst ABER performance is obtained when $\sigma_I^2$ are (0.91, 0.893 8), whereas the best performance is obtained when $\sigma_I^2$ are (0.702 4, 0.893 8) in moderate turbulence regime.

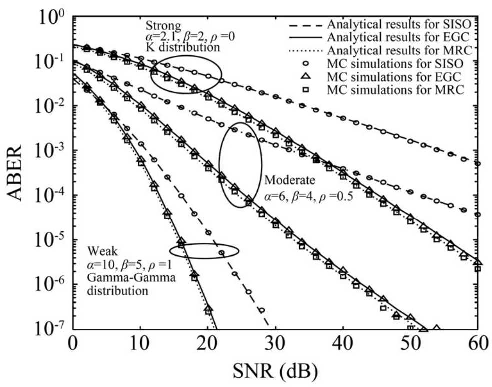
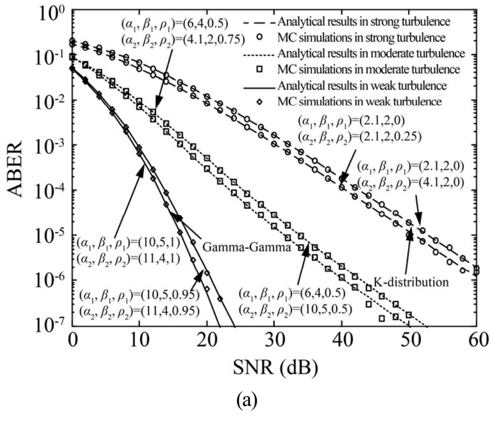
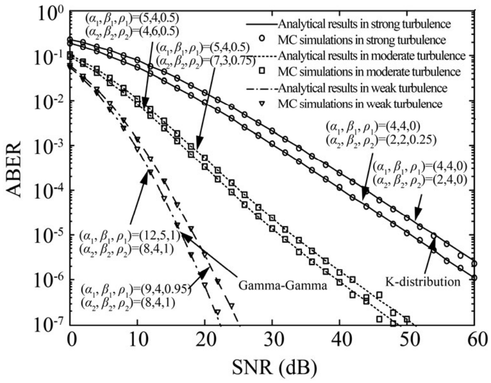



**Fig.2 The ABER performances of SISO, MRC and EGC dual-branch systems against electrical SNR under weak-to-strong turbulence conditions**

The ABER performances of MRC and EGC systems are shown for the same intensity of turbulence ($\sigma_I^2=1$) in Fig.4. The ABER performances of MRC and EGC systems over M fading channels with higher values of $\rho$ are better. For MRC system, to achieve the ABER of $10^{-6}$, SNR of about 34 dB is needed for $\rho$=1, while SNR of 44 dB is needed for $\rho$=0.6. For EGC system, SNR of 35 dB is needed for $\rho$=1, while SNR of 46 dB is needed for $\rho$=0.6. This is caused by the increase of the scattered optical power coupled to the coherent LOS component. It can also be found from Fig.4 that all the ABER curves tend to the same diversity order[15] over M distribution.

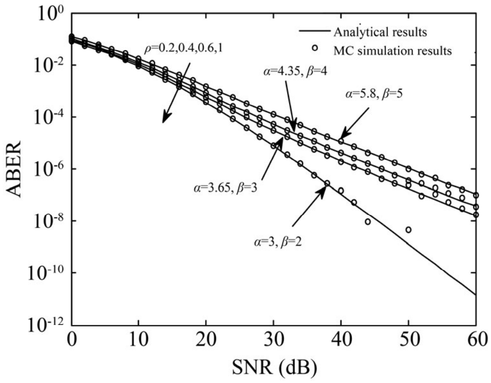



(a)

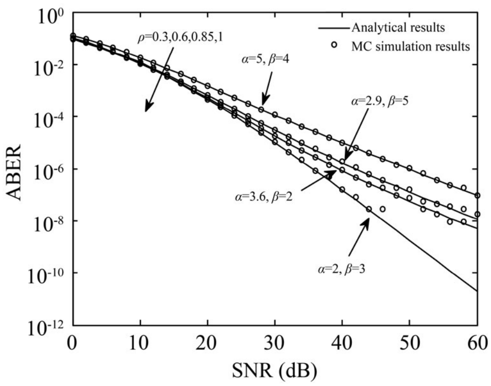



(b)

**Fig.3 The ABER performances of subcarrier BPSK-modulated (a) MRC and (b) EGC dual-branch systems against electrical SNR over independently and non-identically distributed M turbulence channels under different turbulence conditions**




(a)




(b)

**Fig.4 The ABER performances against average SNR of (a) MRC and (b) EGC systems for different $\rho$ with the same intensity of turbulence (moderate turbulence) with $\sigma_I^2=1$**

In summary, the ABER performances of both MRC and EGC FSO systems over M distributed fading channels are investigated in detail. The analytical ABER expressions of H-Fox function for dual-branch systems based on MRC and EGC are developed, respectively. All the analytical results are verified by MC simulation. The performances of dual-branch MRC, EGC systems and those of SISO system over M distribution channels are compared under different turbulence conditions. The results show that both MRC and EGC technologies are efficient to improve the FSO system performance, while MRC outperforms EGC under the same channel conditions. The study provides efficient algorithms for improving the ABER performances of both MRC and EGC systems over M distributed turbulence fading channels, which can be used to design the FSO systems with diversity.

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