Analysis of effective capacity for free-space optical communication systems over gamma-gamma turbulence channels with pointing errors*

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To facilitate the efficient support of quality-of-service (QoS) for promising free-space optical (FSO) communication systems, it is essential to model and analyze FSO channels in terms of delay QoS. However, most existing works focus on the average capacity and outage capacity for FSO, which are not enough to characterize the effective transmission data rate when delay-sensitive service is applied. In this paper, the effective capacity of FSO communication systems under statistical QoS provisioning constraints is investigated to meet heterogeneous traffic demands. A novel closed-form expression for effective capacity is derived under the combined effects of atmospheric turbulence conditions, pointing errors, beam widths, detector sizes and QoS exponents. The obtained results reveal the effects of some significant parameters on effective capacity, which can be used for the design of FSO systems carrying a wide range of services with diverse QoS requirements.
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Up to now, most existing works about capacity analysis focus on the average capacity and outage capacity for free-space optical (FSO) communication systems[1-4]. However, it should be noted that the channel capacity based on the concept of Shannon capacity is not enough to characterize the effective transmission data rate when delay-sensitive service is applied.

Nowadays, a wide range of services with diverse delay requirements have sprung up, leading to a growing need for delay QoS guarantees. Because of taking the QoS metrics into account when applying the prevalent information-theoretic results, the concept of effective capacity has attracted much attention in conventional radio frequency (RF) communications. The analysis of effective capacity covers resource allocation management[5,6], spectral efficiency[7], user scheduling schemes[8], and so on. However, these results are confined to only the area of RF communication. Specifically, the probability density function (PDF) of the received instantaneous signal noise ratio (SNR) in the gamma-gamma (GG) turbulence channels with pointing errors is complex. Thus the effective capacity computation may sometimes suffer from mathematical intractability, which brings about more difficulties.

As a result, investigating the effective capacity with statistical QoS guarantees for FSO communication systems is interesting and essential. In this paper, a novel and concise closed-form expression of QoS-aware effective capacity is deduced based on Meijer G-function. Then we estimate the achievable effective capacity of the FSO link, which is subject to a given delay QoS constraint, for the specific parameters (conditions of atmospheric turbulence, pointing errors, beam width and detector size).

The concept of effective capacity, firstly introduced by D. Wu and Negi[9], is used to characterize the maximum arrival rate that a time-varying fading channel can support under a given delay QoS constraint. Analytically, the effective capacity for block-fading channels is expressed as[9]

$$E_{\mathrm{C}}(\theta)=-\frac{1}{\theta}\log\left\{E\left[\mathrm{e}^{-\theta R[i]}\right]\right\}, \tag{1}$$

where $E[\cdot]$ denotes the expectation, i is the time index, and $R[i]$ (i=1,2,...) refers to the discrete time stochastic service process which is assumed to be stationary and ergodic. The parameter θ is QoS exponent, representing the decaying rate of the QoS violation probability. A greater θ indicates more strict QoS requirements, whereas smaller θ denotes more flexible QoS constraints.

In this paper, we concentrate on a discrete-time system over a point-to-point optical wireless link using intensity modulation/direct detection with on-off keying. The laser

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beams propagate along a horizontal path through a GG turbulence channel with additive white Gaussian noise (AWGN) in the presence of pointing errors. The channel is assumed to be memoryless, stationary and ergodic with independently and identically distributed intensity block fading statistics. We also assume that the channel state information is available at both transmitter and receiver. Furthermore, we omit the detector responsivity and the path loss. In this case, the channel model is given by

$$y = hx + n\,, \tag{2}$$

where y is the electrical signal at the receiver, h is the channel fading coefficient, x is the transmitted intensity taking values of 0 or $2P_{\mathrm{t}}$ (P_{t} is the average transmitted optical power), and n is AWGN with zero mean and variance of σ_n^2. In our model, the channel fading coefficient h is considered to be a product of two independent factors, i.e., $h=h_{\mathrm{a}}h_{\mathrm{p}}$, where h_{a} is the random attenuation due to atmospheric turbulence, and h_{p} is the random attenuation due to pointing errors because of transmitter or receiver sway.

An optical wireless channel is a randomly time-variant channel, and the received instantaneous electrical signal-to-noise ratio (SNR) $\gamma[i]$ is a random variable as

$$\gamma[i] = \frac{2P_{\mathrm{t}}^2 h^2[i]}{\sigma_n^2}\,, \tag{3}$$

where i is the time index of the frame, and P_{t} is the average transmitted optical power. We assume that the system's instantaneous capacity can be achieved. Thus, the instantaneous service rate of the frame i, denoted by $R[i]$, can be expressed as

$$R[i] = T_{\mathrm{f}} B \log_2\{1+\gamma[i]\}\,, \tag{4}$$

where B denotes the system spectral bandwidth, and T_{f} represents the frame duration. In the following discussions, we omit the discrete time-index of i for simplicity.

For moderate-to-strong turbulence conditions, the GG distribution can accurately characterize the channel fading caused by scintillation. In the GG turbulence model, the PDF of h_{a} is given by

$$f_{h_{\mathrm{a}}}(h_{\mathrm{a}}) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)}\left(h_{\mathrm{a}}\right)^{\frac{(\alpha+\beta)}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_{\mathrm{a}}}\right), \tag{5}$$

where $\Gamma(\cdot)$ is the gamma function, and $K_v(\cdot)$ is the modified Bessel function of the second kind of order v. The parameters of α and β can be directly related to the atmospheric conditions for the case of plane wave by

$$\alpha = \left\{\exp\left[\frac{0.49\sigma_{\mathrm{R}}^2}{\left(1+0.18d^2+0.56\sigma_{\mathrm{R}}^{12/5}\right)^{7/6}}\right]-1\right\}^{-1}, \tag{6}$$

$$\beta = \left\{\exp\left[\frac{0.51\sigma_{\mathrm{R}}^2}{\left(1+0.9d^2+0.62d^2\sigma_{\mathrm{R}}^{12/5}\right)^{5/6}}\right]-1\right\}^{-1}, \tag{7}$$

where $d=\sqrt{kD^2/4L}$, $\sigma_{\mathrm{R}}^2 = 1.23C_n^2 k^{7/6} L^{11/6}$ is the Rytov variance, k=2π/λ is the optical wave number, D is the aperture diameter of the receiver, λ is the communication wavelength, L is the distance between transmitter and receiver, and C_n^2 is the index of refraction structure parameter which represents the atmospheric turbulence condition. In general, C_n^2 varies from 10^{-17} m$^{-2/3}$ to 10^{-13} m$^{-2/3}$ for weak to strong turbulence cases, respectively.

In line-of-sight FSO communication links, the pointing accuracy is an important factor in determining link performance and reliability. Considering independent identical Gaussian distributions for the elevation and the horizontal displacement, and assuming a circular detection aperture with radius of r and a Gaussian beam, the PDF of h_{p} can be expressed as

$$f_{h_{\mathrm{p}}}(h_{\mathrm{p}}) = \frac{\eta^2}{A_0^{\eta^2}} h_{\mathrm{p}}^{\eta^2-1}, \quad 0 \le h_{\mathrm{p}} \le A_0\,, \tag{8}$$

where A_0=[erf(v)]^2 is the fraction of the collected power at r=0, erf(·) is the error function, $v=\sqrt{\pi}r/\sqrt{2}W_Z$, W_Z is the beam width at distance z, $\eta=W_{\mathrm{Zeq}}/2\sigma_{\mathrm{s}}$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation at the receiver, σ_{s}^2 is the jitter variance at the receiver, and W_{Zeq} is the equivalent beam width with $W_{\mathrm{Zeq}}^2 = W_Z^2\sqrt{\pi}\mathrm{erf}(v)/[2v\exp(-v^2)]$.

The PDF of the channel states $h=h_{\mathrm{a}}h_{\mathrm{p}}$ for the combined effects of GG turbulence and pointing errors can be expressed as[10]

$$f_h(h) = \int \frac{\eta^2}{A_0^{\eta^2} h_{\mathrm{a}}}\left(\frac{h}{h_{\mathrm{a}}}\right)^{\eta^2-1} f_{h_{\mathrm{a}}}(h_{\mathrm{a}})\mathrm{d}h_{\mathrm{a}} =$$

$$\frac{\alpha\beta\eta^2}{A_0\Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left[\frac{\alpha\beta h}{A_0}\middle|\begin{matrix}\eta^2\\ \eta^2-1,\alpha-1,\beta-1\end{matrix}\right],$$

$$0 \le h_{\mathrm{p}} \le A_0 h_{\mathrm{a}}\,. \tag{9}$$

Thus, we can obtain the following PDF with respect to γ for the GG distribution model with pointing errors as

$$f_\gamma(\gamma) = \frac{\alpha\beta\eta^2\sigma_n^2}{2\sqrt{2}A_0 P_{\mathrm{t}}\Gamma(\alpha)\Gamma(\beta)}\gamma^{-\frac{1}{2}} \times$$

$$G_{1,3}^{3,0}\left[\frac{\alpha\beta\sigma_n}{\sqrt{2}A_0P_{\mathrm{t}}}\gamma^{\frac{1}{2}}\middle|\begin{matrix}\eta^2\\ \eta^2-1,\alpha-1,\beta-1\end{matrix}\right], \tag{10}$$

where $G_{p,q}^{m,n}[\cdot]$ describes the Meijer G-function[11]. Note that this function is a standard built-in function in most of the well-known mathematical software packages, such as Mathematica and Maple.

Substituting Eqs.(4) and (10) into Eq.(1), the effective capacity of a GG modeled optical channel is given as

$$E_{\mathrm{C}} = -\frac{1}{\theta}\log\left[\int_0^\infty \mathrm{e}^{-\theta T_{\mathrm{f}} B\log(1+\gamma)} f_\gamma(\gamma)\mathrm{d}\gamma\right] =$$

$$-\frac{1}{\theta}\log\left\{\frac{\alpha\beta\eta^2\sigma_n^2}{2\sqrt{2}A_0P_\mathrm{t}\Gamma(\alpha)\Gamma(\beta)}\int_0^\infty \gamma^{-\frac{1}{2}}\left(1+\gamma\right)^{-\varphi}\times\right.$$
$$\left. G_{1,3}^{3,0}\left[\frac{\alpha\beta\sigma_n}{\sqrt{2}A_0P_\mathrm{t}}\gamma^{\frac{1}{2}}\left|\begin{matrix}\eta^2\\ \eta^2-1,\alpha-1,\beta-1\end{matrix}\right.\right]\mathrm{d}\gamma\right\}, \tag{11}$$

where $\varphi\triangleq\theta T_\mathrm{f}B/\ln 2$ is the normalized QoS exponent. By expressing $(1+\gamma)^{-\varphi}$ as that in Ref.[12], the closed-form mathematical expression for effective capacity under the combined effects of GG turbulence and pointing errors is derived as

$$E_\mathrm{C}=-\frac{1}{\theta}\log\left\{\frac{\eta^2 2^{\alpha+\beta-3}}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\varphi)\pi}\times\right.$$
$$\left. G_{3,7}^{7,1}\left[\frac{(\alpha\beta)^2\sigma_n^2}{32(A_0P_\mathrm{t})^2}\left|\begin{matrix}\frac{\eta^2+1}{2},\frac{\eta^2+2}{2},1\\ \frac{\eta^2}{2},\frac{\eta^2+1}{2},\frac{\alpha}{2},\frac{\alpha+1}{2},\frac{\beta}{2},\frac{\beta+1}{2},\varphi\end{matrix}\right.\right]\right\}. \tag{12}$$

By using Eq.(12), we can evaluate the effective capacity of the FSO communication system under the combined effects of GG turbulence, pointing errors and statistical delay QoS constraint. Moreover, the closed-form expression takes the operational parameters of the system into account, such as link lengths L, operation wavelength λ, receiver aperture diameter D, beam width W_Z, jitter variable σ_s, and delay constraints θ.

Simulation results are based on the parameters of λ=850 nm, L=1 000 m, P_t=10 mW, B=10^5 Hz, T_f=2 ms, σ_n^2=10^{-14} A/Hz. In Fig.1, the refractive index structure parameter C_n^2 varies from 6.5×10^{-15} m$^{-2/3}$ to 6.5×10^{-14} m$^{-2/3}$. Then the Rytov variances can be calculated as σ_R^2=0.26 and σ_R^2=2.6, which represent the moderate and strong turbulence conditions, respectively. It is observed that the effective capacity decreases slightly when the atmospheric turbulence condition varies from moderate to strong.

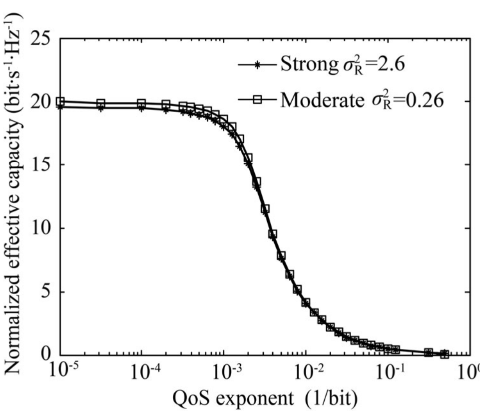

Fig.1 Effective capacity versus QoS exponent for different turbulence conditions with *D*=0.04 m, *W*$_Z$=0.4 m and σ_s=0.2 m

By changing the jitter variable as σ_s=0.1 m, σ_s=0.2 m and σ_s=0.3 m in Fig.2, we can see that an increased jitter degrades the average capacity of FSO communication systems, and it induces a larger decrease for effective capacity when the delay QoS constraint becomes looser.

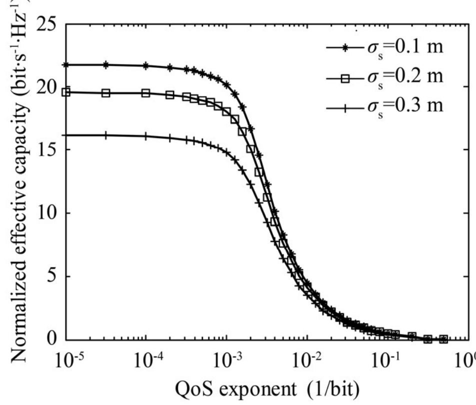

Fig.2 Effective capacity versus QoS exponent for different pointing error displacement jitters with σ_R^2=2.6, *D*=0.04 m and *W*$_Z$=0.4 m

Furthermore, we analyze how effective capacity is affected by different receiver aperture diameters of D=0.04 m and D=0.08 m. From Fig.3, it can be seen that the effective capacity increases with an enlarged receiver aperture.

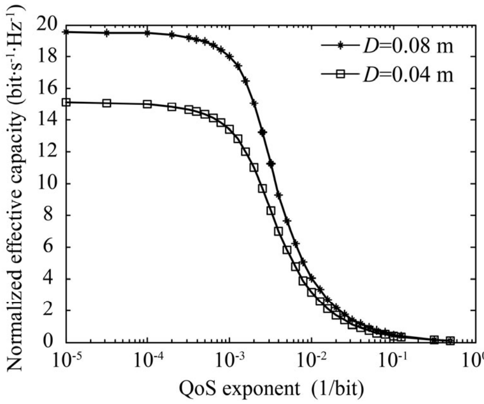

Fig.3 Effective capacity versus QoS exponent for different receiver aperture diameters with σ_R^2=2.6, *W*$_Z$=0.4 m and σ_s=0.2 m

Fig.4 illustrates the effective capacity versus the QoS exponent θ for different receiver beam widths of W_Z=0.2 m, W_Z=0.4 m and W_Z=0.8 m. It is noted that neither larger nor smaller W_Z is preferred, and we can investigate the optimum receiver beam width in the future.

From Figs.1–4, it is obvious that a smaller QoS exponent θ, which implies a looser QoS guarantee, corresponds to larger change in effective capacity when considering the four factors mentioned above. Just observing the QoS exponent θ in the range from 10^{-5} to 10^0, as expected, the effective capacity converges to zero as $\theta\to\infty$ and to the average capacity as $\theta\to 0$. The results are consistent with our theoretical analysis.

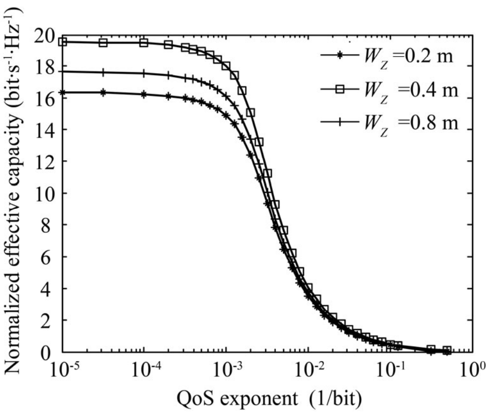

Fig.4 Effective capacity versus QoS exponent for different receiver beam waists with σ_R^2=2.6, D=0.04 m and σ_s=0.2 m

In summary, we consider the QoS provisioning for time-varying atmosphere channels, and deduce the concise closed-form expression of effective capacity. Thanks to Meijer G, the expression of effective capacity is concise and can be easily calculated. The effects of atmospheric turbulence condition, pointing errors, beam width, detector size and QoS exponent on effective capacity are investigated. Our work is a design guide for FSO systems carrying a wide range of services with diverse QoS requirements.

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