

A novel repetition space-time coding scheme for mobile FSO systems*

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Considering the influence of more random atmospheric turbulence, worse pointing errors and highly dynamic link on the transmission performance of mobile multiple-input multiple-output (MIMO) free space optics (FSO) communication systems, this paper establishes a channel model for the mobile platform. Based on the combination of Alamouti space-time code and time hopping ultra-wide band (TH-UWB) communications, a novel repetition space-time coding (RSTC) method for mobile 2×2 free-space optical communications with pulse position modulation (PPM) is developed. In particular, two decoding methods of equal gain combining (EGC) maximum likelihood detection (MLD) and correlation matrix detection (CMD) are derived. When a quasi-static fading and weak turbulence channel model are considered, simulation results show that whether the channel state information (CSI) is known or not, the coding system demonstrates more significant performance of the symbol error rate (SER) than the uncoding. In other words, transmitting diversity can be achieved while conveying the information only through the time delays of the modulated signals transmitted from different antennas. CMD has almost the same effect of signal combining with maximal ratio combining (MRC). However, when the channel correlation increases, SER performance of the coding 2×2 system degrades significantly.

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Free-space optics (FSO) systems establish point-to-point communication links through the atmosphere. Optical signal transmission in free space is affected by atmospheric turbulence and pointing errors, which fade the signal at the receiver and deteriorate the link performance^[1-3]. Multiple-input multiple-output (MIMO) diversity technology is used in free-space optical communication to overcome the problems^[4-6]. Haas et al^[6] proposed space-time coding standards, according to the perspective of information theory and the atmospheric turbulence channel for FSO, and derived the bit error rate (SER) formula. But there is no further scheme applied to atmospheric turbulence channel of space-time coding. Alamouti's space-time coding is used to design a time-frequency code, and its decoding method is introduced in Ref.[6]. Xing Xuefeng put forward a kind of channel model based on quasi orthogonal space-time block coding and space diversity FSO system, and analyzed channel capacity and error rate of the system in Refs.[7,8]. Wang Huiqin^[9-11] proposed space-time block codes and corresponding coding scheme. Ehsan^[12] discussed the coherent differential space-time coding scheme, and pointed out that compared with the direct detection, super heterodyne detection can effectively

improve the performance of the system. Chadi^[13,14] respectively studied the repetition space-time coding (RSTC) schemes, error performance and channel capacity of space optical communication. Ehsan^[15] compared the orthogonal space-time block codes (OSTBCs) and repetition space-time codes, and found out that both of them can achieve full diversity. Therefore, combining Alamouti's space-time code with a time-hopping ultra-wide band (TH-UWB) technology, we put forward a kind of repetition space-time coding method, which is suitable for pulse position modulation (PPM). It makes decoding method simple and the system performance is also improved.

In an $M \times N$ (M is the number of lasers, and N is the number of detectors) MIMO system, the n -th detector signal received can be expressed as^[1]

$$I_n = As(t) \sum_{m=1}^M h_{mn}(t) + I_b, \quad 1 \leq n \leq N, 1 \leq m \leq M, \quad (1)$$

where A is each signal intensity in the case of atmospheric scintillation, h_{mn} is light intensity gain from the m laser to the n -th detector, $s(t)$ is the transmission symbol, and I_b is the background intensity. Assume that the channel gain is $E[h_{mn}(t)] = 1$. The channel state h_{mn}

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models the random attenuation of the propagation channel. In our model, h arises due to three factors: path loss h^l , geometric spread and pointing errors h_{mn}^p , and atmospheric turbulence h_{mn}^a . The channel state can be formulated as^[1,13,14]:

$$h_{mn} = h^l h_{mn}^a h_{mn}^p \quad (2)$$

The attenuation of laser power through the atmosphere is described by the exponential Beers–Lambert law as:

$$h^l(z) = P(z) / P(0) = \exp(-\kappa z) \quad (3)$$

where $P(z)$ is the laser power at z distance, and κ is the attenuation coefficient. The attenuation h^l is considered as a fixed scaling factor during a long period of time, and no randomness exists in its behavior. It depends on the size and distribution of the scattering particles and the wavelength utilized. A Gamma-Gamma distribution is used to model atmospheric fading. In this case, the probability density function (PDF) of h^a is given as:

$$f_{h^a}(h^a) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} h^{(\alpha+\beta)/2-1} K_{\beta-\alpha}(2\sqrt{\alpha\beta h^a}), h > 0, \quad (4)$$

where $K(\bullet)$ is the modified Bessel function of the second kind, and $1/\beta$ and $1/\alpha$ are the variances of the small and large scale eddies, respectively. It is shown that the Gamma-Gamma PDF is in close agreement with measurements under a variety of turbulence conditions.

Consider independent identical Gaussian distributions for the elevation and the horizontal displacement (sway), as was done in previous work^[1]. The radial displacement r at the receiver is modeled by a Rayleigh distribution^[1]

$$f_r(r) = \frac{r}{\sigma_s^2} \exp\left(-\frac{r^2}{2\sigma_s^2}\right), r > 0, \quad (5)$$

where σ_s^2 is the jitter variance at the receiver. The probability distribution of h^p can be expressed as^[14]

$$f_{h^p}(h^p) = \frac{\gamma^2}{H_0^2} (h^p)^{\gamma^2-1}, 0 \leq h^p \leq H_0, \quad (6)$$

where $\gamma = w_{z_{eq}} / 2\sigma_s^2$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation at the receiver. The parameter $w_{z_{eq}}$ that is the equivalent beam width can be calculated using the relations of $v = \sqrt{A} / (\sqrt{2}w_z)$, $H_0 = [\text{erf}(v)]$, and $w_{z_{eq}}^2 = w_z^2 \sqrt{\pi} \text{erf}(v) / 2v \exp(-v^2)$, where $\text{erf}(\cdot)$ is the error function and w_z is the beam waist (radius calculated at e^{-2} at distance z).

Using the previous PDF for turbulence and misalignment fading, the combined PDF of h is given as

$$f_h(h) = \int f_{h_{hp}}(h|h^a) f_{h^a}(h^a) dh^a, \quad (7)$$

where $f_{h_{hp}}(h|h^a)$ is the conditional probability given h^a state and is expressed by

$$f_{h_{hp}}(h|h^a) = \frac{1}{h^a h^l} f_{h^p}\left(\frac{h}{h^a h^l}\right) = \frac{\gamma^2}{H_0^2 h^a h^l} \left(\frac{h}{h^a h^l}\right)^{\gamma^2-1}, 0 \leq h \leq H_0 h^a h^l. \quad (8)$$

By substituting Eqs.(6) and (7) into Eq.(8), we can obtain^[1]

$$f_h(h) = \frac{\gamma^2}{(H_0 h^l)^{\gamma^2}} h^{\gamma^2-1} \int_{h/H_0 h^l}^{\infty} (h^a)^{-\gamma^2} f_{h^a}(h^a) dh^a. \quad (9)$$

Alamouti space-time coding method is suitable for dual antennas. Considering that the symbols to be transmitted in some moment are x_1 and x_2 , they are transmitted in two consecutive time slots. In the first time slot, x_1 and x_2 are respectively transmitted from 1 and 2 antennas; in the second time slot, $-x_2^*$ and x_1^* are respectively transmitted, and $[\bullet]^*$ represents conjugate. Two antennas transmit sequences, which are $T_1 = [x_1 \quad -x_2^*]$ and $T_2 = [x_2 \quad x_1^*]$, orthogonal to each other. We can construct an orthogonal space-time code by using the orthogonal properties. In the orthogonal coding scheme, we can use cyclic-shift to represent complex conjugation and the negative sign. The transmission information in Q-PPM can be represented by a vector e_q , so the composed vector space is

$$E = \{e_q; q = 1, 2 \dots Q\}, \quad (10)$$

where e_q is the Q -by- Q identity matrix I_Q in column q . Obviously, the vectors in different columns are pairwise orthogonal. For 4-PPM modulation, we get

$$E = \left\{ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T; \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}^T; \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T; \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T \right\}. \quad (11)$$

The Q-PPM of 2×2 space-time coding can be represented by the following $2Q$ -by- 2 matrix

$$X = \begin{bmatrix} x_1 & \Omega x_2 \\ x_2 & \Omega x_1 \end{bmatrix} \text{ (if } x_1 \geq x_2 \text{) or } X = \begin{bmatrix} x_1 & \Omega^T x_2 \\ x_2 & \Omega^T x_1 \end{bmatrix} \text{ (if } x_1 < x_2 \text{)}, \quad (12)$$

where $x_1, x_2 \in E$, and they are Q vector information symbols. Ω is the following cyclic-shift matrix

$$\Omega = \begin{bmatrix} \mathbf{0}_{b \times (Q-1)} & 1 \\ \mathbf{I}_{Q-1} & \mathbf{0}_{(Q-1) \times 1} \end{bmatrix}, \quad (13)$$

where $\mathbf{0}_{p \times q}$ is all zero matrix of $p \times q$. Eqs.(12) and (13) show that $\Omega x \in E$, Ωx is also Q-PPM code. If the q -th information of PPM is represented as $x = q$, modu-

lation information $\mathcal{Q}x$ represents the $q+1$ to Q mode, that is, the modulation signal will cycle right shift 1 position or the $\mathcal{Q}^T x$ modulation signal will cycle left shift one.

For an MIMO system, the received signal can be expressed as

$$\mathbf{R} = \mathbf{H}\mathbf{X} + \mathbf{N}, \quad (14)$$

where \mathbf{X} is the $QM \times T$ -dimensional coding matrix. The element in the matrix ($x_{(m-1)Q+q} = 1$) is transmitter pulse of antenna m in the T slot at q position. \mathbf{R} is the received $QM \times T$ -dimensional signal matrix. \mathbf{N} is the receiving $QM \times T$ -dimensional additive noise matrix. \mathbf{H} is the $QM \times QN$ -dimensional channel gain matrix, which can be expressed as a column vector as

$$\mathbf{H} = [\mathbf{H}_1^T \cdots \mathbf{H}_m^T \cdots \mathbf{H}_M^T]^T \quad (m=1,2,\dots,M), \quad (15)$$

where \mathbf{H} is an $M \times N$ matrix. $\mathbf{H}_m = [h_{m,1} \cdots h_{m,n} \cdots h_{m,N}]$. For a simple 2×2 MIMO system, Eq.(15) can be rewritten as (assuming $x_1 \leq x_2$)

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 & \mathcal{Q}^T x_2 \\ x_2 & \mathcal{Q}^T x_1 \end{bmatrix} + \mathbf{N}, \quad (16)$$

where $r_{n,j}$ is the received signal of the n receiving antenna in the j slot. According to the maximum likelihood criterion, we get

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in E^2} \|\mathbf{R} - \mathbf{H}\mathbf{X}\|^2. \quad (17)$$

Only when $M \geq N$, we can have a reliable decision by Eq.(17). When the receiver has accurately obtained the channel gain of \mathbf{H} , we need to search Q^2 times to obtain $\hat{\mathbf{X}}$.

If the receiver can't accurately obtain the channel gain \mathbf{H} , the estimation $\hat{\mathbf{X}}$ can also be obtained by means of the following blind detection. Suppose transmission symbol sequences x_1, x_2 respectively send the q -th and q' -th light pulse modulations (assuming $q \leq q'$). The vectors are respectively represented as $\begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \\ 1 & & & & \end{bmatrix}^T$ and $\begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \\ 1 & & & & \end{bmatrix}^T$. From Eq.(10), the received vectors are respectively expressed as follows

$$\begin{bmatrix} r_{1,1}^T \\ r_{2,1}^T \\ r_{1,2}^T \\ r_{2,2}^T \end{bmatrix} = \begin{bmatrix} 0 & \cdots & h_{11} & \cdots & h_{21} & \cdots & 0 \\ & & & & & & \\ 0 & \cdots & h_{12} & \cdots & h_{22} & \cdots & 0 \\ & & & & & & \\ 0 & \cdots & h_{21} & \cdots & h_{11} & \cdots & 0 \\ & & & & & & \\ 0 & \cdots & h_{22} & \cdots & h_{12} & \cdots & 0 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix}, \quad (18)$$

where n_1 and n_2 are respectively the Gaussian noises of receivers 1 and 2 in the time slot 1, and they are independent of each other. Meanwhile, n_3 and n_4 are respec-

tively the Gaussian noises in the time slot 2. Receiving signal is cyclic shift, so the new receiving sequence constructed by equal gain combining can quickly estimate q and q' . The receiving sequence is structured as follows

$$\tilde{\mathbf{r}}_1 = \mathbf{r}_{1,1}^T + \mathbf{r}_{2,1}^T + (\mathbf{r}_{1,2}^T + \mathbf{r}_{2,2}^T)\mathcal{Q} = \begin{bmatrix} \cdots & \sum_{m=1}^2 \sum_{n=1}^2 h_{m,n} & \cdots & \cdots & \sum_{m=1}^2 \sum_{n=1}^2 h_{m,n} & \cdots \end{bmatrix} + \tilde{\mathbf{n}}_1. \quad (19)$$

Using Eq.(19), the PPM pulse position q and q' can be estimated as follows

$$\hat{x}_1 = \arg \max_q |\tilde{\mathbf{r}}_1(q)|, \hat{x}_2 = \arg \max_{q'} |\tilde{\mathbf{r}}_2(q')|. \quad (20)$$

If signal correlation is calculated for the matrix of Eq.(12), where the matrix element is r_{ij} ($i=1,2,3,4$; $j=1,2,\dots,Q$), the correlation matrix is noted as \mathbf{C} , whose element can be represented as follows

$$c_{ij} = r_{1i}r_{3(j-1)} + r_{2i}r_{4(j-1)} + r_{3(i-1)}r_{1j} + r_{4(i-1)}r_{2j} \quad (x_1 < x_2), \\ c_{ij} = r_{1i}r_{3(j+1)} + r_{2i}r_{4(j+1)} + r_{3(i+1)}r_{1j} + r_{4(i+1)}r_{2j} \quad (x_1 \geq x_2). \quad (21)$$

Now assuming $x_1 < x_2$, we discuss the following four cases in Eq.(21).

(1) When $i \neq q, j \neq q'$, the correlation matrix can be expressed as follows

$$c_1 = n_{1i}n_{3(j-1)} + n_{2i}n_{4(j-1)} + n_{3(i-1)}n_{1j} + n_{4(i-1)}n_{2j}. \quad (22)$$

(2) When $i = q, j \neq q'$, the correlation matrix can be expressed as follows

$$c_2 = h_{11}n_{3(j-1)} + h_{12}n_{4(j-1)} + h_{21}n_{1j} + h_{22}n_{2j} + c_1. \quad (23)$$

(3) When $i \neq q, j = q'$, the correlation matrix can be expressed as follows

$$c_3 = n_{1i}h_{11} + n_{2i}h_{12} + n_{3i}h_{21} + n_{4i}h_{22} + c_1. \quad (24)$$

(4) When $i = q, j = q'$, the correlation matrix can be expressed as follows

$$c_4 = h_{11}^2 + h_{12}^2 + h_{21}^2 + h_{22}^2 + c_1. \quad (25)$$

Eqs.(22)-(25) show that c_{ij} is the maximum when $i = q, j = q'$, and it can be expressed as

$$\hat{\mathbf{x}} = \arg \max_{q,q'} c_{ij}. \quad (26)$$

When the total power is constant, simulations are performed to discuss the symbol error rate (SER) of system.

We define signal-to-noise ratio as $\gamma_s = \frac{E_s}{2 \sum_{n=1}^N \sigma_n^2}$.

When IPI and symbol appear equally, the conditional SER performance (conditioned on channel irradiance

vector \mathbf{H}) is then given by

$$P_s(\mathbf{x} \rightarrow \mathbf{e}) = \frac{1}{2} (Q-1) \operatorname{erfc} \left(\sqrt{\|\mathbf{H}(\mathbf{x} \rightarrow \mathbf{e})\|^2 \gamma_s / 2} \right), \quad (27)$$

where $\operatorname{erfc}(x) = \frac{2}{\pi} \int_0^x e^{-t^2} dt$ is Gaussian error function. If it is encoded by correlation matrix, the minimum code distance is as follows

$$\|\mathbf{H}(\mathbf{x} \rightarrow \mathbf{e})\|_{\min}^2 = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2, \quad (28)$$

where the channel gain \mathbf{H} is satisfied in Eq.(9). To get a complete analytic expression of Eq.(27) is difficult. We can use Monte Carlo's numerical calculation method to simulate the performance of repetition space-time codes for FSO systems. The analytical conditions are as follows: Total power of the system is fixed; Quasi-static fading channel is used. The simulation parameters are as shown in Tab.1. The simulation results are shown in Figs.1-3.

Tab.1 System parameters

Parameter	Symbol	Value
Transmission rate	Rate	1 Gbit/s
Optical transmitted power	P_t	40 mW (16 dBm)
Threshold rate	R_0	0.5 bit/channel
Atmospheric transmittance	κ	0.7
Receiver responsivity	η	0.6
Noise standard deviation	σ_n	10^{-7} A/Hz
Distance between T_x and R_x	z	1 km
Transmit divergence at $1/e$	θ_T	2.5 mrad
Receiver diameter	$2a$	20 cm
PPM order	Q	8
Pointing error	θ_e	1 mrad

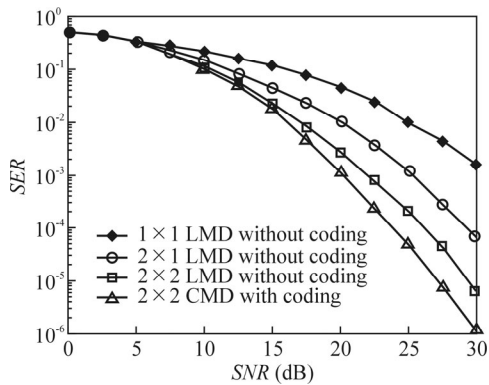


Fig.1 Comparison of SER performance for the RSTC scheme and the uncoding case, assuming unknown CSI

In Fig.1, uncoded 2×1 and 2×2 systems use equal

gain combining (EGC). Because the uncoded 2×1 system acquires transmitting gains, its SER performance is more than 8 dB lower than 1×1 uncoded system when $\gamma_s = 30$ dB. As the 2×2 uncoded system acquires transmitting gains, its SER performance is more than 6 dB lower than 2×1 uncoded system when $\gamma_s = 30$ dB. As it is a very small probability for $|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2$ to deeply fade in 2×2 encoded system, its SER performance is more than 5 dB lower than 2×2 uncoded system. The results show that correlation-matrix detection is equivalent to the optimal gain combining to further improve the receiving diversity gain.

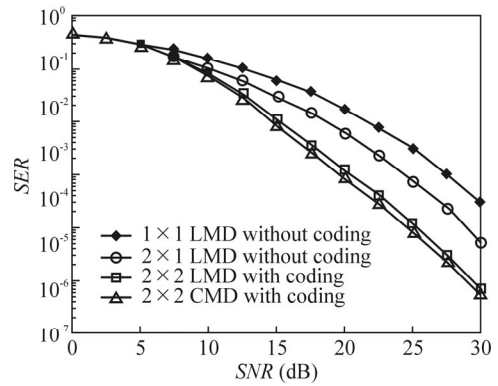


Fig.2 Comparison of SER performance for the RSTC scheme and the uncoding case, assuming known CSI

In Fig.2, the SER performance of the uncoded 2×1 system is more than 8 dB lower than 1×1 encoded system when $\gamma_s = 30$ dB, because it acquires part of diversity gain. For the same reason, the SER performance of the uncoded 2×2 system is more than 11 dB lower than 2×1 uncoded system when $\gamma_s = 30$ dB. But the error rate of 2×2 encoded system is slightly higher than that of the uncoded 2×2 system. Both of 2×2 systems are provided with the same space diversity.

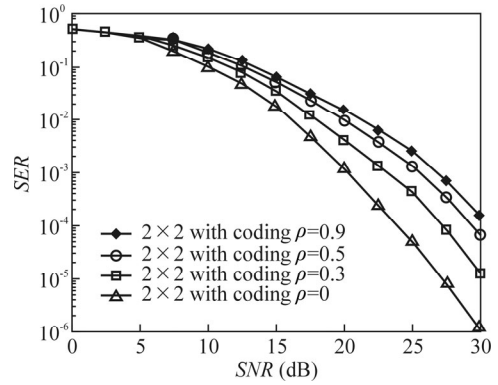


Fig.3 SER performance for the RSTC relative to correlated channels

In Fig.3, when the correlation coefficients ρ of space-diversity channels respectively are 0.3, 0.5 and 0.9,

compared with independent space-diversity channels, the error rates of corresponding 2×2 repetition orthogonal space-time coding system fall respectively 10 dB, 7 dB and 2.6 dB. When the correlation coefficient increases, the possibility that the channel shows deep fading has increased. Because system diversity gain also decreases significantly, the possibility of error is increased.

The repetition orthogonal space-time coding method is proposed which is suitable for the 2×2 free-space optical communication system, and the correlation-matrix detection algorithm is given. The SER performance of the system under different conditions is analyzed, and finally the simulation further proves the effectiveness of the coding method. When the total transmitting power is constant, regardless of the channel information is known or unknown, the simulation results show that the correlation-matrix detection method can effectively improve the receiving diversity gain, and can achieve full rate transmission, but the channel correlation has obvious influence on the system performance. We can use the equal-gain combining likelihood detection for the initial decoding, in order to reduce search space in correlation matrix that decreases the decoding computation complexity.

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