Construction and performance research on variable-length codes for multirate OCDMA multimedia networks*

LI Chuan-qi (李传起)**, **YANG Meng-jie** (杨梦婕), **LUO De-jun** (罗德俊), **LU Ye** (陆叶), **KONG Yi-pu** (孔一卜), **and ZHANG Dong-chuang** (张东闯)

College of Electronic Engineering, Guangxi Normal University, Guilin 541004, China

(Received 17 May 2014; Revised 16 June 2014)

A new kind of variable-length codes with good correlation properties for the multirate asynchronous optical code division multiple access (OCDMA) multimedia networks is proposed, called non-repetition interval (NRI) codes. The NRI codes can be constructed by structuring the interval-sets with no repetition, and the code length depends on the number of users and the code weight. According to the structural characteristics of NRI codes, the formula of bit error rate (BER) is derived. Compared with other variable-length codes, the NRI codes have lower BER. A multirate OCDMA multimedia simulation system is designed and built, the longer codes are assigned to the users who need slow speed, while the shorter codes are assigned to the users who need high speed. It can be obtained by analyzing the eye diagram that the user with slower speed has lower BER, and the conclusion is the same as the actual demand in multimedia data transport.

Document code: A **Article ID:** 1673-1905(2014)05-0360-5

DOI 10.1007/s11801-014-4086-7

Optical code division multiple access (OCDMA) technology has been regarded as an attractive way to support many users to share a common channel resource simultaneously. So far, the conventional constructions of codes are restricted to generate sequences with the same length in each code set, based on the assumption that there is only one type of media and one signaling rate in the system[1]. However, on the same network, different users need different signaling rates and qualities of service (QoS)[2].

The conventional codes with the same length, such as prime codes (PCs)[3] and optical orthogonal codes (OOCs)[4,5], are not able to meet the requirements of diverse QoS [6,7]. There are two major approaches to provide multirate services in OCDMA, which are multicode technique and variable-length spreading technique. Compared with the multicode CDMA, the variable-length spreading CDMA can make full use of resources and reduce the number of codecs, in which the high-speed users employ the short codes, the low-speed users employ the long codes, and just one codec for each user in different rates[8-12]. So the variable-length codes are getting a lot of attention.

A new algorithm to generate multilength OOCs has been developed[13]. However, these codes become very long to support many simultaneous users needing variable rates because of the multiplied relationship of code lengths. In this paper, a new kind of variable-length codes with ideal correlation property is proposed, whose code length depends on the number of actual users and the code weight, and they can adapt to users' QoS more flexibly.

The OOC of (L, w, λ_a, λ_c) is a code family with code length L, code weight w, auto-correlation λ_a and cross-correlation λ_c. A code C_1 can be denoted as (a_0, $a_1,\cdots,a_i$), where a_i (i=0,1,$\cdots$,w–1) is the location of '1' in the (0,1) sequences. The all-interval-set of C_1 can be denoted as ($b_0^n,b_1^n,\cdots,b_{w-1}^n$), where n=1,2,$\cdots$,w–1 is the distance between two '1's in the sequence, in which $b_0^n=a_n-a_0$, $b_1^n=a_{n+1}-a_1$, $\cdots$, $b_{w-n-1}^n=a_{w-1}-a_{w-n-1}$, $b_{w-n}^n=a_0+L-a_{w-n}$, $\cdots$, and $b_{w-1}^n=a_{n-1}+L-a_{w-1}$. If $b_i^j\neq b_{i'}^{j'}$, when $i\neq i'$ or $j\neq j'$, the auto-correlation of C_1 is 1. The other code C_2 has the all-interval-set ($c_0^n,c_1^n,\cdots,c_{w-1}^n$). If $b_i^j\neq c_{i'}^{j'}$, when $i\neq i'$ or $j\neq j'$, the cross-correlation between C_1 and C_2 is 1.

The non-repetition interval (NRI) codes can be constructed by structuring the non-repetition interval-sets, and they have the ideal auto-correlation and cross-correlation, namely, there is no repetitive number in the all-interval-set.

Use matrix $\boldsymbol{A}_N^w$ and set $\boldsymbol{B}_N^w$ to denote the all-interval-set of NRI codes. To facilitate the calculation of $\boldsymbol{B}_N^w$, divide it into w–1 parts, namely, $\boldsymbol{B}_N^w$=[$\boldsymbol{B}_1$ $\boldsymbol{B}_2$ $\boldsymbol{B}_3$ $\cdots$ $\boldsymbol{B}_{w-1}$]. $\boldsymbol{A}_N^w$ and $\boldsymbol{B}_n$ are defined as

* This work has been supported by the University Nature Science Fund of Guangxi Province (No.2010 MS018), and the Guilin Scientific Research and Technological Development Program Topics (No.20120104-18).

** E-mail: lcq@mailbox.gxnu.edu.cn

$$A_N^w=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1w} \\ a_{21} & a_{22} & \cdots & a_{2w} \\ \vdots & \vdots & & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{Nw} \end{bmatrix}=\begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \\ \vdots \\ \boldsymbol{A}_N \end{bmatrix}, \quad (1)$$

$$\boldsymbol{B}_n=\begin{bmatrix} b_{11}^n & b_{12}^n & \cdots & b_{1y}^n \\ b_{21}^n & b_{22}^n & \cdots & b_{2y}^n \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1}^n & b_{N2}^n & \cdots & b_{Ny}^n \end{bmatrix}, \quad (2)$$

where $\boldsymbol{A}_i$ (i=1,2,$\cdots$,N) is the 1 interval-set of code C_i, and $y=\sum_{k=1}^{n}\binom{n}{k}=\sum_{k=1}^{n}\frac{n!}{k!(n-k)!}$, where n=1,2,$\cdots$,w–1.

The next step is to get all the data in A_N^w and B_N^w under the condition of $a_{ij}\neq a_{i'j'}$, $a_{ij}\neq b_{ij}^k$ and $b_{ij}^k\neq b_{i'j'}^{k'}$, where $i\neq i'$ or $j\neq j'$ or $k\neq k'$, i=1,2,$\cdots$,N, j=1,2,$\cdots$,N, and k=1,2,$\cdots$,w–1.

$$a_{i1}=a_{(i-1)w}+b\,, \quad (3)$$

$$a_{ij}=a_{i(j-1)}+a\,, \quad (4)$$

$$\boldsymbol{C}=[a_{i1},a_{i2},\cdots,a_{i(j-1)}]\,, \quad (5)$$

$$b_{im}^{j\text{-}1}=a_{ij}+\sum \mathrm{nchoose}k(\boldsymbol{C},k)\,, \quad (6)$$

where a is the first positive integer which makes a_{ij} not belong to $\boldsymbol{B}_N^w$, b is the first positive integer which makes a_{i1} not belong to $\boldsymbol{B}_N^w$, nchoose$k$($\boldsymbol{C}$, k) is the function in Matlab as the algorithm that picks up k elements in set $\boldsymbol{C}$, and $\sum$nchoose$k$($\boldsymbol{C}$,k) is the sum of the k elements, where i=2,3,$\cdots$,N, j=2,3,$\cdots$,w, k=1,2,$\cdots$,j–1, and m=1,2,$\cdots$,y. The main idea of the construction is to use the non-repetition interval-sets to construct the variable-length codes, meanwhile maintaining the correlation properties. The method for constructing the NRI codes is as follows.

First, set N as the number of users, and pick positive integer w as the code weight. Second, let a_{11}=1 and $a_{i1}=a_{(i-1)w}+b$, and substitute them into Eq.(3). Then let b=1 and if $a_{i1}\notin \boldsymbol{B}_N^w$, let b plus 1 until $a_{i1}\notin \boldsymbol{B}_N^w$. Third, in Eq.(4), a is the first positive integer which makes a_{ij} not belong to $\boldsymbol{B}_N^w$, where i=2,3,$\cdots$,N and j=2,3,$\cdots$,w. Forth, in Eqs.(5) and (6), if b_{im}^{j-1} belongs to $\boldsymbol{B}_N^w$, make a plus 1 and steps 2 and 3 should be repeated until b_{im}^{j-1} doesn't belong to $\boldsymbol{B}_N^w$. Fifth, take i=1,2, $\cdots$,N and j=2,3,$\cdots$,w, and repeat the steps 2–4 to get all the data in $\boldsymbol{A}_N^w$ and $\boldsymbol{B}_N^w$. Sixth, each row of $\boldsymbol{A}_N^w$ represents a set of 1 interval-set of a code with weight of w and volume of N, which is called as $\boldsymbol{A}_i$. Finally, an NRI code can be drawn as {0, a_{i1}, $a_{i1}+a_{i2}$,…, $\sum_{k=1}^{w-1}a_{ik}$ }, and $L_i=\sum_{k=1}^{w}a_{ik}$, where i=1,2,$\cdots$, N.

For example, we construct the NRI codes with code weight of 3 and capacity of 10. Using the algorithm above, programming in Matlab and giving w=3 and N=10, the code family is shown in Tab.1.

Tab.1 NRI codes with code weight of 3 and capacity of 10

NRI codes	Code length
(0,1,3)	7
(0,8,17)	27
(0,11,23)	36
(0,14,29)	45
(0,20,41)	63
(0,26,54)	86
(0,33,67)	102
(0,37,75)	114
(0,40,84)	131
(0,48,97)	147

During the transmission of the system, the analysis is focused on the exact influence due to multiuser and the true performance of NRI codes, in which the negative effects of the shot noise and thermal noise in the photodetection process, types of receiver and photodetector structures are neglected. When there are K users in the system at the same time, the codes L_1,L_2,$\cdots$,L_k are assigned. According to the characteristics of NRI codes, each code is assigned to a user, so different users have various code lengths. The lengths are denoted by L_1,L_2,$\cdots$,L_k from short to long, where the first i–1 codes are shorter than the ith, and the last K–i–1 codes are longer than the ith.

The performance of the ith user affected by other k–1 users is analyzed using the following method. Firstly, the collision probability between the shorter codes and the ith code is analyzed. L_m (1≤m≤i) is defined as one of the short codes. $r_m=\left\lceil \frac{L_i}{L_m}\right\rceil+1$, where $\lceil\ \rceil$ is the rounded up sign. L_i can be divided into three parts, which are called as sections 1, 2 and 3, respectively. τ (0≤τ≤L_m) indicates the time delay, and there are two cases as shown in Fig.1 according to the different values of τ. When $(r_m-2)L_m<L_i-\tau$, there are r_m–2 copies of L_m in section 2, and let $r_m'=r_m-2$. When $(r_m-2)L_m>L_i-\tau$, there are r_m–3 copies of L_m in section 2, and let $r_m'=r_m-3$. The different values of r_m' are taken depending on those of τ. As the discussion above, the hit probabilities of the three sections between the two codes are as follows,

$$p_1=\frac{1}{2}\cdot\frac{w}{L_m}\cdot w\frac{\tau}{L_i}=\frac{w^2\tau}{2L_mL_i}\,, \quad (7)$$

$$p_2=\frac{1}{2}\cdot\frac{w}{L_m}\cdot w\frac{L_m}{L_i}=\frac{w^2}{2L_i}\,, \quad (8)$$

$$p_3=\frac{1}{2}\cdot\frac{w}{L_m}\cdot\left(w\frac{L_i-\tau-r_m'L_m}{L_i}\right)=\frac{w^2(L_i-\tau-r_m'L_m)}{2L_mL_i}\,, \quad (9)$$

where the factor $\frac{1}{2}$ is due to the assumption of equiprobable binary data bit.

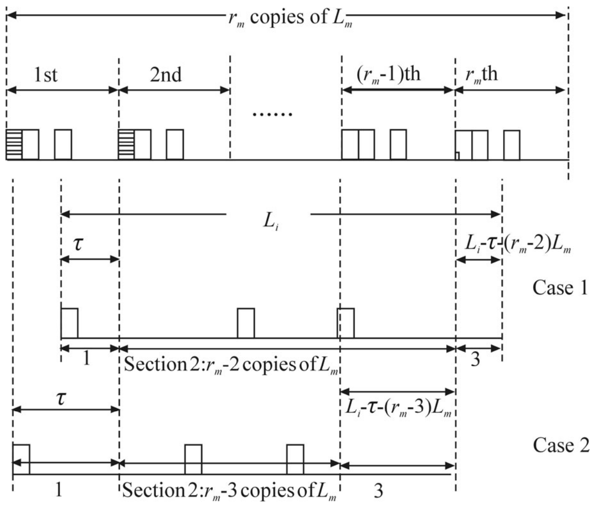


Fig.1 Inter-cross-correlation of L_i and r_m copies of L_m

Because the hit number between the single L_m and L_i is 1, the hit probability between r_m copies of L_m and L_i is

$$q_{w,1}^{m}=\sum_{\tau=0}^{L_m-1}\frac{1}{L_m}[p_1(1-p_2)^{r_m'}(1-p_3)+ r_m'(1-p_1)p_2\cdot(1-p_2)^{r_m'-1}(1-p_3)+ (1-p_1)(1-p_2)^{r_m'}p_3], \tag{10}$$

where the three product terms inside the summation account for the three cases which generate the cross-correlation value of 1. The first term represents the case that there is no hit in sections 2 and 3 but one hit in section 1. The second term represents the case that there is one hit in section 2 but no hit in sections 1 and 3, and r_m' represents the hit probability in one L_m and r_m' copes of L_m. The third term represents the case that there is one hit in section 3 but no hit in sections 1 and 2.

Based on the similar arguments, the situation for $2\le j\le[r_m'+2,w]_{\min}$ is discussed, where $[r_m'+2,w]_{\min}$ represents the minimum of $r_m'+2$ and w. When the number of L_m is greater than w, the hits of the two codes are w mostly. When the number of L_m is less than w, the hit between each L_m and L_i is just once, so the hits of the two codes are $r_m'+2$ at most.

$$q_{w,j}^{m}=\sum_{\tau=0}^{L_m-1}\frac{1}{L_m}[\binom{r_m'}{j-1}p_1p_2^{j-1}(1-p_2)^{r_m'-j+1}(1-p_3)+ \binom{r_m'}{j-2}p_1p_2^{j-2}(1-p_2)^{r_m'-j+2}p_3+ \binom{r_m'}{j-1}(1-p_1)p_2^{j-1}(1-p_2)^{r_m'-j+1}p_3+ \binom{r_m'}{j}(1-p_1)p_2^{j}(1-p_2)^{r_m'-j}(1-p_3)]\ , \tag{11}$$

where the four product terms inside the summation account for the four cases which generate the cross-correlation value of j. The analytic method is similar to that of Eq.(11). In order to facilitate the calculation in Eq.(11), the symbol $\binom{\ }{\ }$ is introduced, which is expressed as

$$\binom{y}{x}=\begin{cases}1 & y=x=0 \text{ or } x=0\\ 0 & y<x\\ \dfrac{y!}{x!(y-x)!} & \text{else}\end{cases}\ . \tag{12}$$

For the convenient calculation in Eqs.(11) and (12), the situations of $x>y$ and $x=y=0$ are added in Eq.(12). The condition of $x>y$ indicates that the number of L_m is less than that of hits. Obviously, this case does not exist, so it is in the condition $\binom{y}{x}$ =0. The condition of $x=y=0$ indicates the case that there is no section 2 and no hit in section 2. This case is allowed, so $\binom{y}{x}$=1. While for the other case, the result is the same as the permutation and combination calculation in mathematics. There is no need to discuss the different cases when using the additional-definition symbol $\binom{y}{x}$. $\binom{y}{x}$ shown in Eq.(12) can meet all the demands.

The average probability of hit between L_m and L_i is $q_w^m=\dfrac{\sum_{j=1}^{[r_m'+2,w]_{\min}}jq_{w,j}^m}{\sum_{j=1}^{[r_m'+2,w]_{\min}}j}$, and that between L_n and L_i is $q_w^n=\dfrac{w^2}{2L_n}$.

The average probabilities of hit, which are between the shorter codes and L_i and between the longer codes L_i, are respectively given as

$$q_w^{\mathrm{S}}=\frac{\sum_{m=1}^{i-1}q_w^m}{i-1}=\frac{1}{i-1}\sum_{m=1}^{i-1}\frac{\sum_{j=1}^{[r_m'+2,w]_{\min}}jq_{w,j}^m}{\sum_{j=1}^{[r_m'+2,w]_{\min}}j}\ , \tag{13}$$

$$q_w^{\mathrm{L}}=\frac{\sum_{n=i+1}^{K}q_w^n}{K-i-1}=\frac{1}{K-i-1}\sum_{n=i+1}^{K}\frac{w^2}{2L_n}\ . \tag{14}$$

Now the BER of the ith user is analyzed. For the K simultaneous users, there is k users hitting the ith user in the other K–1 users. And in the k users, there are j users' codes shorter than L_i, while the other k–j codes are longer than L_i. Among the shorter codes, the hit probability between j users and the ith user is $\binom{i-1}{j}(q_w^{\mathrm{S}})^j(1-q_w^{\mathrm{S}})^{i-j-1}$. Similarly, the hit probability between k–j users and the ith user is $\binom{K-i-1}{k-j}(q_w^{\mathrm{L}})^{k-j}(1-q_w^{\mathrm{L}})^{K-i-1-k+j}$. So we get the result as

$$BER=\frac{1}{2}\sum_{k=Th}^{K-1}\sum_{j=0}^{k}\left\{\binom{i-1}{j}(q_w^{\mathrm{S}})^{j}(1-q_w^{\mathrm{S}})^{i-j-1}\cdot\right.$$
$$\left.\binom{K-i-1}{k-j}(q_w^{\mathrm{L}})^{k-j}(1-q_w^{\mathrm{L}})^{K-i-1-k+j}\right\}, \qquad (15)$$

where the factor $\frac{1}{2}$ is also due to the assumption of equiprobable binary data bit.

The BER of the fourth user versus the number of simultaneous users is shown in Fig.2, where three curves represent the codes with weights of 6, 8 and 9, respectively. With increasing the number of simultaneous users, the BER becomes larger, but BERs of all the three codes are stabilized. Obviously, when the number of simultaneous users is larger than 50, the BER can not increase quickly. The BER with code weight of 6 versus the number of simultaneous users is shown in Fig.3, where the five curves represent the BERs of the 2nd, 3rd, 4th, 5th and 6th users, respectively. When the code weight is the same as that shown in Fig.3, the longer the length, the lower the BER.

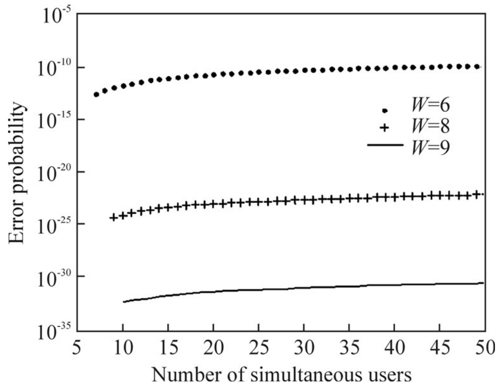


Fig.2 Error probabilities of the fourth user with different code weights versus the number of simultaneous users

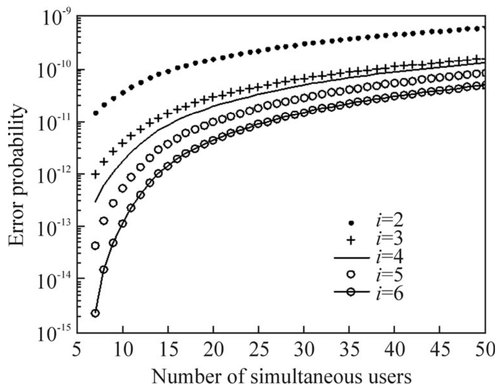


Fig.3 Error probabilities of different users versus the number of simultaneous users

Fig.4 shows the BER of multilength OOC devised by Kwong[13] who analyzed the BERs of the codes with weight of 5 and lengths of 241 and 1 205 respectively. As shown in Fig.4, when the total number of users is 25, the BER for the length 241 is about 10^{-5}, and that for the length 1 205 is about 9×10^{-4}. The BER of NRI code with the weight of 5 and the length of 1 132 is 1.78×10^{-8}, and that with length of 1 278 is 1.35×10^{-8}, while the BER of NRI code is 3.82×10^{-7} for the same weight of 5 and the length of 241. Obviously, the BER of NRI code is lower than that of the multilength OOCs in the same situation. The lengths of the multilength OOCs are integer relation, which can't meet the more requirement of the multirate, and the code lengths are generally too large. On the contrary, each user can get a different-length code with NRI codes, which meets the requirement of the multirate better.

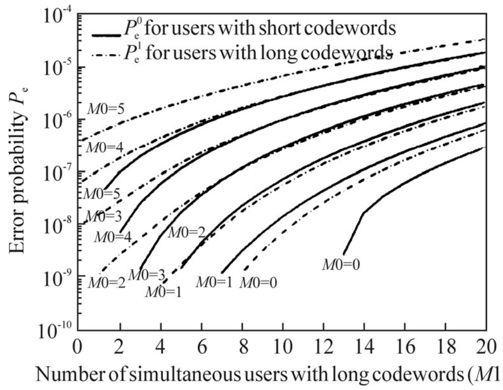
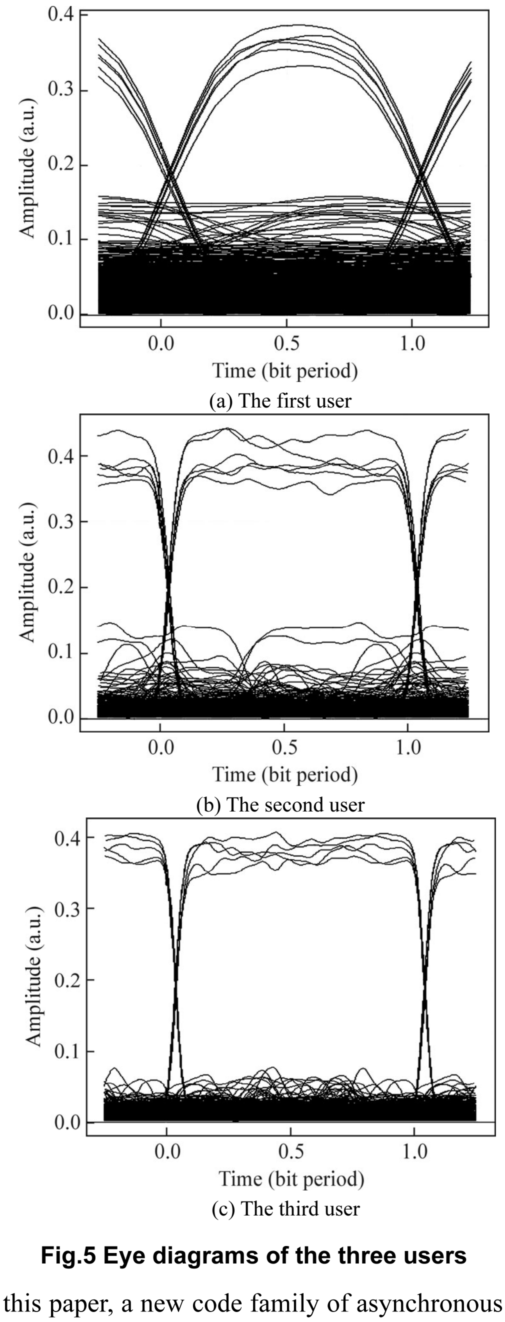


Fig.4 Error probability versus the number of simultaneous users of the multilength OOCs

The simulation of incoherent multirate OCDMA system is conducted using OptiSystem Version 9.0. Here, a simple system with three users is simulated, using NRI codes with w=5. The tests are carried out by continuous wave (CW) laser at 1 550 nm for three active users at bit rates of 10 Gbit/s, 1.782 Gbit/s and 1.286 Gbit/s, respectively with fiber optic time delay. The attenuation is 0.2 dB/km, the dispersion is 16.75 s/nm·km, the dark current value is set as 10 nA, and the thermal noise coefficient is 10^{-22} W/Hz for each photo-diode at the detection part. Nonlinear effects are activated and specified according to the typical industry values to simulate the environment as real as possible[14].

The three users have codes of (0,1,3,7,15) with L=31, (0,32,65,99,135) with L=174, and (0,40,81,123,167) with L=241. The fiber optic time delays in parallel construction for codecs are (0.1,0.3,0.7,1.5,3.1) ns, (3.2,6.5,9.9,13.5) ns and (4,8.1,12.3,16.7,24.1) ns for the three encoders and the corresponding negative time delays for the decoders. It is the most important that the maximum delay time must be less than the minimum time interval of the (0,1) sequence which is sent. The three users have the rate ratio of $\frac{1}{L_1}:\frac{1}{L_2}:\frac{1}{L_3}$, so the unit length of (0,1) sequence must have the ratio of $\frac{1}{L_1}:\frac{1}{L_2}:\frac{1}{L_3}$. The method makes sure the orthogonality of each user.

Fig.5 shows the eye diagrams of the three users. As shown in Fig.5, the third user has the best BER, and the first user has the worst one. It clearly depicts that the users with higher speed have worse BER, that is, the higher the rate, the worse the BER performance as the theory. In actual situation, we can use the optical hard limiter to reduce the BER and get the clearer eye diagram. The NRI code system gives better performance, and the system based on time delay line as codecs can reduce the complexity, especially when the value of w increases.



(a) The first user
(b) The second user
(c) The third user

Fig.5 Eye diagrams of the three users

In this paper, a new code family of asynchronous multirate OCDMA multimedia network is proposed. The NRI code is proved that a large number of lengths are applicable for users who require the higher QoS. The BER performance of the NRI code system is compared with that of an existing code system, such as multilength OOCs, considering the influence of MAI, and the BER is closely related with the code weight. The theoretical and simulation results show that the eye diagrams of NRI code have superior performance. The decoding scheme uses the time delay to reduce the complexity for the system with large w and L. Furthermore, the simulation can get the result that the users with high speed have worse BER performance.

References

[1] Shrikant S. Tangade and Shivaleela E. S., Better Quality of Servies in Multimedia Transmission by Using Wavelength/Time Single-Pulse-per-Row Encoding Technique in OCDMA Networks, IEEE India Conference, 525 (2012).

[2] S. Sahuguede, M. Morelle, A. Julien-Vergonjanne and J.-P. Cances, Multi-Rate and QoS Differentiation in Optical Code Division Multiple Access Networks, Consumer Communications and Networking Conference, 581 (2008).

[3] Shaar A. A. and Davis P. A., Electronics Letter **19**, 888 (1983).

[4] R. C. S. Chauhan, Ratenesh Kumar, M. Shukla, Ashutosh Kumar Sigh and G. P. Bagaria, Proposal for One Dimensional Optical Orthogonal Codes: Design, Analysis, & Algorithm, Communication Systems and Network Technologies, 514 (2011).

[5] J. O. Anaman and Shanthi Prince, Correlation Properties and Performance Evaluation of 1-Dimensional OOC's for OCDMA, International Conference on Devices, Circuits and Systems, 167 (2012).

[6] Xiaobin Li and Yongjie Shi, Construction of Variable-Weight Optical Orthogonal Codes for OCDMA Passive Optical Network, IEEE 11th International Conference on Signal Processing, 1 (2012).

[7] Li Chuan-qi, Xia Qing, Zhang Yuan and Zhou Yuan-yuan, Optoelectronics Letters **5**, 372 (2009).

[8] John S. Vardakas and Ioannis D. Moscholios, IEEE Transactions on Communications **61**, 3374 (2013).

[9] Li chuanqi, Hu jinlin, He dongdong, Chen meijuan, Wang dachi and Chen yan, Optoelectronics Letters **9**, 473 (2013).

[10] Yih-Fuh Wang, A Novel OVSF-based Multi-rate OCDMA-PON and Its Optimal Configuration, Information Security and Intelligence Control, 133 (2012).

[11] John S. Vardakas, Ioannis D. Moscholiosy, Michael D. Logothetis and Vassilios G. Stylianakis, On Code Reservation in Multi-Rate OCDMA Passive Optical Networks, 8th IEEE IET International Symposium on Communication Systems, Networks and Digital Signal Processing, 1 (2012).

[12] Liu Xiao-lei, En De and Wang Li-guo, Journal of Optoelectronics·Laser **25**, 876 (2014). (in Chinese)

[13] Kwong W. C. and Yang G. C., IEEE Transactions on Communications **50**, 1258 (2002).

[14] Ryosuke Matsumoto, Takahiro Kodama, Satoshi Shimizu, Ryujiro Nomura, Koji Omichi, Naoya Wada and Ken-ichi Kitayama, Apodized SSFBG Encoder/Decoder for 40G-OCDMA-PON System, OptoElectronics and Communications Conference, 1 (2013).