A post-coding scheme for peak-to-average power ratio reduction in intensity modulated optical OFDM systems*

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An efficient post-coding strategy is proposed in this letter to reduce the peak-to-average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signals for optical intensity modulated direct detection (IM/DD) systems. The post-coding scheme based on discrete cosine transform (DCT) is employed after the inverse fast Fourier transform (IFFT) in the transmitter to reduce the PAPR of OFDM signals. This method is different from the conventional pre-coding scheme which is employed before IFFT operation. Numerical simulations demonstrate that the new DCT post-coding strategy can significantly reduce the PAPR than the conventional pre-coding scheme. Meantime, the bit error rate (BER) performance of the proposed post-coding system can be improved compared with the conventional pre-coding scheme.

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In recent years, the orthogonal frequency division multiplexing (OFDM) technique has gained remarkable interest in the optical communications, mainly due to its high spectral efficiency and strong resistance to dispersive channels[1,2]. The performance of OFDM technique has been extensively researched in the intensity-modulated direct-detection (IM/DD) systems[3,4]. However, the high peak-to-average power ratio (PAPR) of OFDM signals is the main problem for OFDM systems. A large PAPR will cause strong nonlinear impairments, thus degrade the transmission performance. In wireless communications, many methods are proposed to reduce the PAPR of OFDM signals. These techniques have been summarized in Ref.[5] and can be categorized into two types, signal distortion techniques and signal scrambling techniques. Distortion techniques, such as companding and clipping, can reduce the PAPR but will cause the signal distortion at the same time. Scrambling techniques, such as selective level mapping (SLM) and partial transmit sequences (PTS), can reduce the PAPR by improving the distribution characteristics of OFDM signals with no signal distortion. A pre-coding approach was discussed in Refs.[6,7], which can not only reduce the PAPR but also improve the BER of the OFDM systems. This technique involves a signal pre-coding prior to the IFFT. Other pre-coding schemes, such as Hadamard transform, discrete cosine transform (DCT) and discrete Fourier transform (DFT), are employed to reduce the PAPR of OFDM signals in wireless OFDM systems[8-10].

However, for an IM/DD optical OFDM system, the OFDM signals must be real values. When these pre-coding techniques are employed into IM/DD optical system, the Hermitian symmetry of the input symbols of IFFT has to be satisfied. DFT pre-coding scheme and Hadamard pre-coding scheme are applied to optical OFDM systems, and both of the PAPR and BER performance are improved[11,12].

In these conventional pre-coding schemes, only half of the input symbols of IFFT are transformed by pre-coding matrix, while the other half is transformed by using Hermitian symmetry. So the capability of reducing PAPR based on pre-coding scheme is restricted. When the post-coding matrix is applied after the IFFT operation, all the symbols of OFDM data frame are transformed by the post-coding matrix. Thus the effect of PAPR reduction can be improved.

In this paper, a DCT post-coding algorithm for optical OFDM signals is proposed. When DCT post-coding is applied to real optical OFDM signal, the transformed signal is also real. Thus the DCT matrix can be used as a post-coding matrix to reduce the PAPR of the real optical OFDM signal.

In an OFDM system with N orthogonal subcarriers, a high rate data stream is split into N low rate streams

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which are transmitted simultaneously. The information bit stream is first mapped to quadratic-amplitude modulation (QAM) symbols or phase-shift keying (PSK) symbols. The frequency-domain and the time-domain OFDM symbols can be denoted as $\boldsymbol{X}=\left[X(0) \quad X(1) \quad \cdots \quad X(N-1)\right]^{\mathrm{T}}$ and $\boldsymbol{x}=\left[x(0) \quad x(1) \quad \cdots \quad x(N-1)\right]^{\mathrm{T}}$, respectively. According to the property of the inverse Fourier transform, a real-valued time signal $x(n)$ corresponds to a frequency-domain signal $X(k)$ which is Hermitian symmetric, i.e.,

$$X(k)=X^{*}(N-k-1), 0 \le k \le N-1 , \tag{1}$$

where $*$ denotes complex conjugate.

In IM/DD OFDM system, the first $N/2$ subcarriers are set zero to ensure that the output signal only consists of real value. Suppose $X_k=a_k-\mathrm{j}b_k$ is the transmitted data for the k-th subcarrier. The n-th element of OFDM symbol $x(n)$ is given by using the IDFT of $\boldsymbol{X}$, i.e.,

$$x(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X(k)\exp\left(\frac{\mathrm{j}2\pi kn}{N}\right), 0 \le n \le N-1 . \tag{2}$$

Then, the power of OFDM signal $x(n)$ can be expressed as

$$|x(n)|^{2}=\frac{1}{N}\sum_{m=0}^{N-1}\sum_{k=0}^{N-1} X_m X_k \exp\left(\frac{\mathrm{j}2\pi(m-k)n}{N}\right). \tag{3}$$

The PAPR of OFDM signals $x(n)$ is defined as

$$PAPR=\frac{\max\limits_{0\le n\le N-1}\left[|x(n)|^{2}\right]}{\mathrm{E}\left\{|x(n)|^{2}\right\}} , \tag{4}$$

where $\mathrm{E}\{\bullet\}$ denotes the expectation and $|x(n)|$ denotes the magnitude of $x(n)$. The peak power occurs when the N modulated symbols are added with the same phase.

Usually, the complementary cumulative distribution function (CCDF) is used to measure the efficiency of PAPR reduction technique. The CCDF is defined as the probability that the PAPR of a certain OFDM symbol exceeds the given threshold $PAPR_0$. The CCDF can be written as

$$CCDF(PAPR(\boldsymbol{x}))=\Pr(PAPR(\boldsymbol{x})>PAPR_0) . \tag{5}$$

Pre-coding scheme can reduce PAPR of the OFDM signal both in wireless communication and optical OFDM system. Fig.1 shows the conventional IM/DD optical OFDM system based on DCT pre-coding scheme. As shown in Fig.1, the input bit stream at the transmitter is mapped to complex symbols according to the chosen modulation scheme, e.g., M-QAM. We can write the complex vector with size M as $\boldsymbol{S}=\left[S_0 \quad S_1 \quad \cdots \quad S_{M-1}\right]^{\mathrm{T}}$, where $[\cdot]^{\mathrm{T}}$ denotes the matrix transpose. The DCT pre-coding matrix transforms this vector $\boldsymbol{S}$ into a new vector $\boldsymbol{X}$ with size M as $\boldsymbol{X}=\boldsymbol{AS}=\left[X_0 \quad X_1 \quad \cdots \quad X_{M-1}\right]^{\mathrm{T}}$, where $\boldsymbol{A}$ is the DCT pre-coding matrix. Let $\boldsymbol{A}_M$ be the $M\times M$ DCT matrix with entry given by[13]

$$[\boldsymbol{A}]_{i,j}=\begin{cases}\dfrac{1}{\sqrt{M}}, & i=1\\ \sqrt{2/M}\cos\left[\dfrac{(2j-1)(i-1)\pi}{2M}\right], & i\ne 1, 1\le i,j\le M.\end{cases} \tag{6}$$

It can be readily checked that $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}=\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}=\boldsymbol{I}_M$, where $\boldsymbol{I}_M$ denotes an identity matrix with size M.

Then, the symmetric conjugate data vector $\bar{\boldsymbol{X}}$ is formed as $\bar{\boldsymbol{X}}=\left[X_0^{*} \quad X_1^{*} \quad \cdots \quad X_{M-1}^{*}\right]^{\mathrm{T}}$. The input data symbol vector of the IFFT unit is formed as $\boldsymbol{X}=\left[\boldsymbol{X}^{\mathrm{T}} \quad \bar{\boldsymbol{X}}^{\mathrm{T}}\right]^{\mathrm{T}}$ with $X_k=X_{N-k-1}^{*}$ in order to generate a real-valued OFDM signal. The precoded symbol vector $\boldsymbol{X}$ is fed into the IFFT block. At the receiver, the inverse DCT pre-coding matrix is employed after the FFT unit.

Fig.2 shows the block diagram of an IM/ID optical OFDM system based on post-coding scheme. We can see that the difference between the pre-coding scheme and the post-coding scheme is that the size of post-coding matrix is $N\times N$ and the size of pre-coding is $M\times M$ with $M=N/2$. So only half of the symbols are protected by pre-coding scheme, while all of the symbols are protected by post-coding scheme. But the complexity of post-coding scheme is higher than the pre-coding scheme. DCT is a real transform in which the data are multiplied by a cosine function. The output of IFFT is real-valued signal, then the output signal of the DCT is also real. That's why DCT post-coding scheme can be used here.

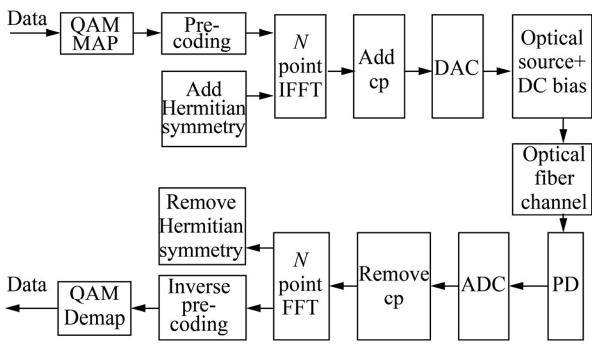

Fig.1 IM/DD optical OFDM system with pre-coding

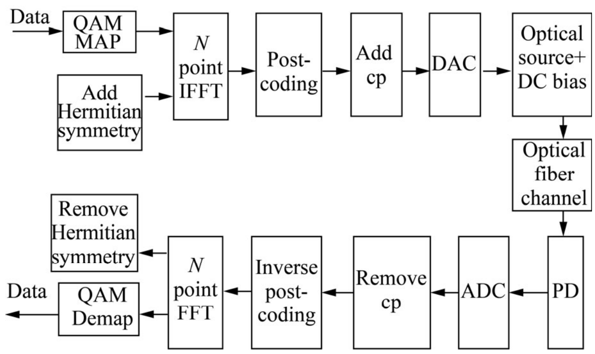

Fig.2 IM/DD optical OFDM system with post-coding

Let $\boldsymbol{F}$ be the $N \times N$ FFT matrix with entry given by

$$[\boldsymbol{F}]_{n,k} = 1/\sqrt{N} \exp(-\mathrm{j}2\pi(n-1)(k-1)/N), \quad (7)$$

where n=1, $\cdots$, N and k=1, $\cdots$, N.

Let $\boldsymbol{X}$ be a modulated symbol with size $N \times 1$, thus the output of IFFT is an OFDM symbol in the form of an $N \times 1$ vector and can be expressed as

$$\boldsymbol{x} = \boldsymbol{F}^{\mathrm{H}} \boldsymbol{X}. \quad (8)$$

The superscript H represents the complex conjugate transpose.

Let $\boldsymbol{A}$ be the DCT post-coding matrix with size $N \times N$. The post-encoded OFDM symbol can be written as

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} = \boldsymbol{A}\boldsymbol{F}^{\mathrm{H}} \boldsymbol{X}. \quad (9)$$

Let r be the received signal. The inverse post-coded transform is applied to the signal r, and the new transformed signal is given as

$$\boldsymbol{Y} = \boldsymbol{A}^{\mathrm{H}} r. \quad (10)$$

Finally, $\boldsymbol{Y}$ is processed by the FFT unit. The output of the FFT can be given as

$$\hat{\boldsymbol{y}} = \boldsymbol{F}\boldsymbol{Y}. \quad (11)$$

In our simulation, each OFDM frame has 256 subcarriers, among which 192 (96×2) subcarriers are used for the data, 8 pilot subcarriers, and 56 subcarriers are set to zero as the guard interval. Cyclic prefixing with 32 sample symbols is used to avoid the inter-block interference. For evaluating the capability of PAPR reduction based on DCT post-coding schemes, the real-valued QPSK or 16QAM OFDM signals are employed.

In Fig.3, we compare the PAPR performance of proposed DCT post-coding scheme and conventional pre-coding scheme for QPSK signals. It can be found that the proposed DCT post-coding scheme outperforms the original OFDM signal without coding and the conventional DCT pre-coding scheme. At $CCDF = 10^{-3}$, the PAPR of the proposed DCT post-coding scheme is 3 dB less than the original OFDM signal and 1 dB less than the conventional DCT pre-coding scheme.

Fig.4 shows the PAPR performance of different coding scheme for 16QAM signals. The results are similar to the results in Fig.3. We can see at $CCDF = 10^{-3}$, the PAPR of the proposed DCT post-coding scheme is 2 dB less than the original OFDM signal without coding and 1 dB less than the conventional DCT pre-coding scheme. When comparing Fig.4 with Fig.3, we can see the 16QAM OFDM system shows higher PAPR than the QPSK OFDM system.

Fig.5 shows the BER performance in AWGN channel. We can find that BER performance of the proposed DCT post-coding scheme outperforms the other two schemes. At BER of 10^{-3}, the system with proposed DCT post-coding scheme may obtain 1.5 dB SNR gain compared with the conventional DCT pre-coding scheme. It is observed that post-coding takes advantage of the frequency selectivity of the communication channel and improves the system performance considerably, which is consistent with previously reported results[11,12].

From the constellation graphs of the demodulated signals we can also compare the performance of different systems. Fig.6 and Fig.7 show the constellations of the demodulated OFDM signal without post-coding and with post-coding at SNR of 11 dB. We can see that the constellations of Fig.7 are more concentrated than Fig.6. In other words, the quality of the modulated symbols with post-coding is better than that of conventional symbols.

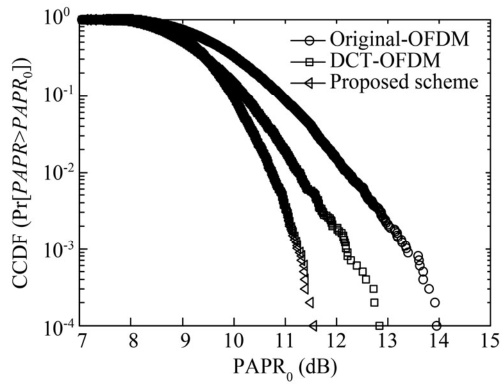

Fig.3 PAPR performance of QPSK OFDM signals

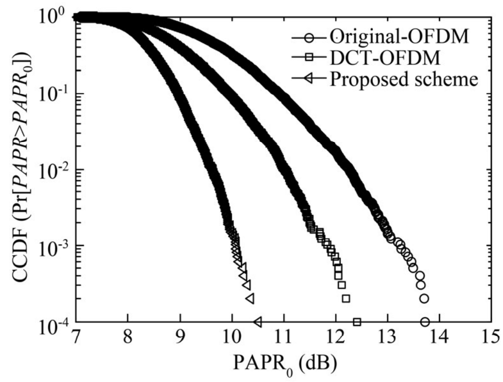

Fig.4 PAPR performance of 16QAM OFDM signals

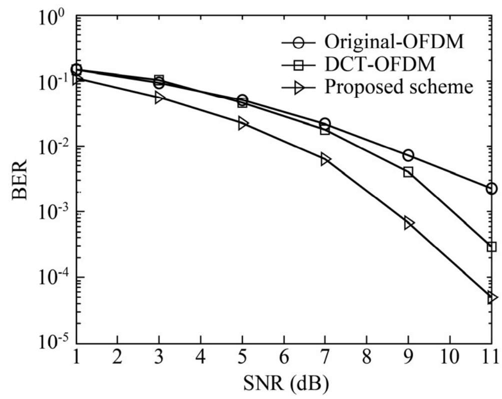

Fig.5 BER performance in AWGN channel

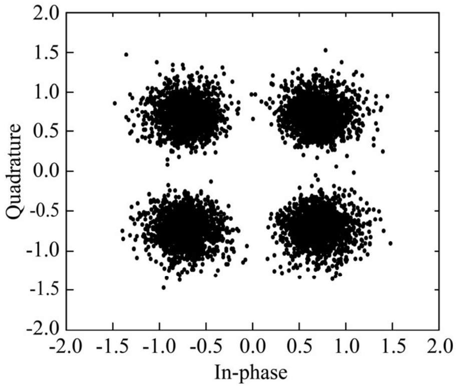

Fig.6 Constellation of demodulated signals without post-coding at SNR of 11 dB

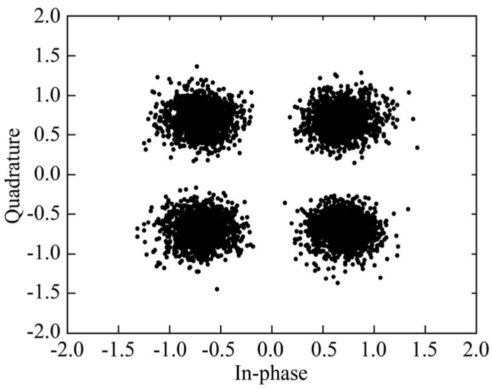

Fig.7 Constellation of demodulated signals with post-coding at SNR of 11 dB

In this paper, a new DCT post-coding scheme for an IM/DD OFDM communication systems is proposed. Since only half of the symbols are protected by pre-coding scheme and all of the symbols are protected by post-coding scheme, the performance of post-coding scheme is better than pre-coding scheme. Simulation results demonstrate the DCT post-coding scheme not only reduces the PAPR, but also improves the BER of the optical system compared with the conventional DCT pre-coding scheme. Future works may include considering the complexity of the proposed scheme and finding more effective schemes for OFDM optical system to reduce the PAPR.

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