Construction and performance analysis of variable-weight optical orthogonal codes for asynchronous OCDMA systems*

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A construction scheme of variable-weight optical orthogonal codes (VW-OOCs) for asynchronous optical code division multiple access (OCDMA) system is proposed. According to the actual situation, the code family can be obtained by programming in Matlab with the given code weight and corresponding capacity. The formula of bit error rate (BER) is derived by taking account of the effects of shot noise, avalanche photodiode (APD) bulk, thermal noise and surface leakage currents. The OCDMA system with the VW-OOCs is designed and improved. The study shows that the VW-OOCs have excellent performance of BER. Despite of coming from the same code family or not, the codes with larger weight have lower BER compared with the other codes in the same conditions. By taking simulation, the conclusion is consistent with the analysis of BER in theory. And the ideal eye diagrams are obtained by the optical hard limiter.

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Optical code division multiple access (OCDMA) systems have received much research interest as their advantages[1-3]. Due to the increasing requirement of a flexible support of multimedia services, the support of multirate and differentiated quality of service (QoS) transmission is becoming an essential challenge for future optical networks[4-6].

In order to meet the QoS demands of different users, variable-weight optical orthogonal codes (VW-OOCs) are always used to support different qualities. Subscribers with different code weights have the different bit error rate (BER) performance. This property can meet the requirement of QoS in multimedia networks. Therefore, codes with low weight can be assigned to the low-QoS applications, and codes with high weight can be assigned to the high-QoS requirement[7-10].

In this paper, a new arithmetic for constructing a VW-OOCs family is proposed. A smaller code length based on the actual number of weight and users' total capacity is got. The BER formula is derived in asynchronous OCDMA systems using the proposed VW-OOCs with consideration of shot noise, avalanche photodiode (APD) bulk, thermal noise and surface leakage currents. Finally, the differentiated-quality OCDMA system is simulated in OptiSys 9.0, and a method is proposed to modify the problem of the imbalance power of the mark in different users' sending-sign when they have variable code weight.

The VW-OOCs are a family of codewords with equal code length and variable code weight which can be expressed as (L, W, λ_a, λ_c, N), where L is code length, $W=\{w_1, w_2, \cdots, w_n\}$ is a set of code weights, $N=\{N_1, N_2, \cdots, N_n\}$ is a set of codeword capacities, λ_a is autocorrelation constraint, and λ_c is cross-correlation constraint, where $\lambda_a=\lambda_c=1$.

A codeword C can be denoted as ($c_1, c_2, \cdots, c_i$), where c_i ($i=1,\cdots,w_m$) is the location of '1' in the (0,1) sequences, and w_m is the weight of C. The all-interval-sets of C are composed of j interval-sets, where $j=1,2,\cdots,w_m-1$. And the j interval-set expresses the distance between two '1's where there are j '1's between them. If there is no repetitive data in all interval sets of a codeword, the autocorrelation and cross-correlation equal 1. If there is no repetitive data in all interval sets of a code family, λ_a and λ_c both equal 1. The VW-OOCs are constructed according to the important conclusion as above in this paper.

The VW-OOCs have n different weights, so n matrices are constructed to indicate the n kinds of weights. Matrix $\boldsymbol{A}_m$ ($m=1,2,\cdots,n$) denotes the 1 interval-set of the N_m codes with code weight w_m. $\boldsymbol{A}_m$ is given by

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$$\boldsymbol{A}_m = \begin{bmatrix} a_{11}^m & a_{12}^m & \cdots & a_{1w_m}^m \\ a_{21}^m & a_{22}^m & \cdots & a_{2w_m}^m \\ \vdots & \vdots & \ddots & \vdots \\ a_{N_m 1}^m & a_{N_m 2}^m & \cdots & a_{N_m w_m}^m \end{bmatrix}, \tag{1}$$

where each row is the 1 interval-set of a code, and all the N_m codes have the same weight w_m. The $\sum_{m=1}^{n} w_m N_m$ codes can be got after calculating a_{ij}^m, where m=1,2,$\cdots$,n, j=1,2,$\cdots$,w_m, and i=1,2,$\cdots$,N_m. A set of $\boldsymbol{B}$ is the part that the all-interval-sets exclude the 1 interval-set of all the codewords. In other word, the n matrices $\boldsymbol{A}$ and a set $\boldsymbol{B}$ constitute the all-interval-sets of all the VW-OOCs. The specific process to construct the codes is as follows.

According to the users' need, make sure the values of w_1,w_2,$\cdots$,w_n and the values of N_1,N_2,$\cdots$,N_n respectively. Then, calculate the value of a_{ij}^m.

First, when j=1, the method to calculate a_{i1}^m is divided into the following three cases. When m=i=1, a_{i1}^m=1; when $m \neq 1$ and i=1, $a_{i1}^m = a_{N_{m-1}(w_{m-1}-1)}^{m-1}$+1; and when $m \neq 1$ and $i \neq 1$, $a_{i1}^m = a_{(i-1)(w_m-1)}^m$+1. Then, check to see if there is a number equal to a_{i1}^m in set $\boldsymbol{B}$. Let a_{i1}^m be added by 1 until there is no one equal to a_{i1}^m in set $\boldsymbol{B}$.

Second, when $j \neq 1$ and $j \neq w_m$, $a_{ij}^m = a_{i(j-1)}^m + 1$. Check to see if there is a number equal to a_{ij}^m in $\boldsymbol{B}$, and let a_{ij}^m be added by 1 until there is no one equal to a_{ij}^m in $\boldsymbol{B}$. Put j–1 numbers of $a_{ij}^m + a_{i(j-1)}^m$, $a_{ij}^m + a_{i(j-1)}^m + a_{i(j-2)}^m$, $\cdots$, $a_{ij}^m + a_{i(j-1)}^m + \cdots + a_{i1}^m$ into $\boldsymbol{B}$, and check to see if there are repetitive numbers in $\boldsymbol{B}$. Let a_{ij}^m be added by 1 until a_{ij}^m is not equal to the number in $\boldsymbol{B}$ and there is no repetitive data in $\boldsymbol{B}$.

With m from 1 to n, j from 1 to w_m–1 and i from 1 to N_m, repeat the two steps above to calculate the values of $\sum_{m=1}^{n}(w_m-1)N_m$. Next, the algorithm for obtaining the last number in 1 interval-sets is different from others. First, calculate the last number of the last code's 1 interval-set, so the code length and other numbers can be got as

$$a_{N_n w_n}^n = a_{N_n(w_n-1)}^n + 1, \tag{2}$$

$$L = \sum_{j=1}^{w_n} a_{N_n j}^n, \tag{3}$$

$$a_{iw_m}^m = L - \sum_{j=1}^{w_m-1} a_{ij}^m. \tag{4}$$

With m from 1 to n and i from 1 to N_m, use Eqs.(2)–(4) to calculate the last values of $\sum_{m=1}^{n} N_m$. Check to see if there is any number in $\boldsymbol{B}$ equal to $\sum_{m=1}^{n} N_m$, and let $a_{N_n w_n}^n$ be added by 1 until there is no equal number. The values of $\frac{(w_m-2)(w_m+1)}{2}$ shown in Tab.1 should be added into $\boldsymbol{B}$. Check to see if there are repetitive numbers in $\boldsymbol{B}$, let $a_{N_n w_n}^n$ be added by 1, and recalculate and recheck as above until there is no repetitive data in $\boldsymbol{B}$. Finally, all the values of a_{ij}^m can be got after the calculation as above, and the class of VW-OOCs can be obtained by using

$$(0, a_{i1}^m, a_{i1}^m + a_{i2}^m, \cdots, \sum_{j=1}^{w_m-1} a_{ij}^m),\ m=1,2,\ \cdots,w_n;\ i=1,2,\cdots,N_n. \tag{5}$$

Tab.1 *j*-1 numbers which should be added into *B*

$a_{i(w_m-1)}^m + a_{iw_m}^m$	$a_{i(w_m-2)}^m + a_{i(w_m-1)}^m + a_{iw_m}^m$	$\cdots$	$a_{i2}^m + a_{i3}^m + \cdots + a_{i(w_m-1)}^m + a_{iw_m}^m$
$a_{iw_m}^m + a_{i1}^m$	$a_{i(w_m-1)}^m + a_{iw_m}^m + a_{i1}^m$	$\cdots$	$a_{i3}^m + a_{i4}^m + \cdots + a_{iw_m}^m + a_{i1}^m$
	$a_{iw_m}^m + a_{i1}^m + a_{i2}^m$	$\cdots$	$\vdots$
			$a_{iw}^m + a_{i1}^m + \cdots + a_{i(w_m-3)}^m + a_{i(w_m-2)}^m$

Using the algorithm mentioned above and the program in Matlab, such as inputting w_1=3, w_2=5, N_1=10 and N_2=10, we can get the VW-OOCs with L=376 as shown in Tab.2.

Tab.2 VW-OOCs of (376, {3,5}, 1, 1, {10/20,10/20})

Codes with w_1=3	Codes with w_2=5
(0,68,138)	(0,1,3,7,12)
(0,74,150)	(0,8,8,31,45)
(0,77,156)	(0,15,32,51,71)
(0,80,161)	(0,21,43,67,93)
(0,82,166)	(0,28,57,87,120)
(0,85,173)	(0,34,69,107,147)
(0,89,179)	(0,41,83,127,174)
(0,94,189)	(0,48,97,149,202)
(0,96,194)	(0,54,10,169,230)
(0,99,199)	(0,62,126,191,257)

We analyze the performance of the asynchronous OCDMA system using the proposed VW-OOCs with consideration of thermal noise, shot noise, APD bulk and surface leakage currents.

In the APD, the total photon arrival rate influenced by the sign, body leakage current and background light is given by

$$\lambda = \begin{cases} \lambda_s + \lambda_b + \dfrac{I_b}{e}, b=1 \\ \dfrac{\lambda_s}{M_e} + \lambda_b + \dfrac{I_b}{e}, b=0 \end{cases}, \tag{6}$$

where b is the sending bit, λ_s and λ_b are the photon arrival rates influenced by sign and background light, respectively, e=1.6×10^{-19} C is the electron charge, $\frac{I_b}{e}$ is the contribution of the APD bulk leakage current to the APD output, and M_e is the extinction ratio. $\lambda_s=\frac{\eta P_{T_c}}{hf}$, where P_{T_c} is the received signal power, η is the APD quantum efficiency, h=6.63×10^{-34} J·s is the Planck's constant, and f is the optical frequency.

w_i chips can get the optical signal which is carried by a code in VW-OOCs with weight w_i after the operation in optical correlator, and the other $L-w_i$ chips have no optical signal. When the number of total users is K, and K_i user is with weight w_i (i=1,2,$\cdots$,n), the number of sending signals including all '1' and '0' is $\sum_{i=1}^{n} w_i K_i$. When the sending bit is 1, there are w_i+I marks at the photon arrival rate of λ_s, where I indicates the interference by other users. The other null signal arrives at the photon arrival rate of $\frac{\lambda_s}{M_e}$. The impacts of APD body, surface leakage current and thermal noise exist in each chip. Therefore, the probability density function of the output of the photodetector can be expressed as a Gaussian function by

$$P(y\mid I,b=1)=\frac{1}{\sqrt{2\pi\sigma_{b1}^2}}\mathrm{e}^{-\frac{(y-u_{b1})^2}{2\sigma_{b1}^2}}, \quad (7)$$

where

$$u_{b1}=GT_{\mathrm{C}}[(w_i+I)\lambda_s+(\sum_{i=1}^{n} w_i K_i-(w_i+I))\frac{\lambda_s}{M_e}+L(\lambda_b+\frac{I_b}{e})]+LT_{\mathrm{C}}\frac{I_S}{M_e}, \quad (8)$$

$$\sigma_{b1}^2=G^2F_eT_c[(w_i+I)\lambda_s+(\sum_{i=1}^{n} w_i K_i-(w_i+I))\frac{\lambda_s}{M_e}+L(\lambda_b+\frac{I_b}{e})]+L(T_{\mathrm{C}}\frac{I_S}{e}+\sigma_{\mathrm{th}}^2), \quad (9)$$

where G is the average APD gain, I_s is the APD surface leakage current, and F_e is the excess noise factor. $F_e=k_{\mathrm{eff}}G+(2-\frac{1}{G})(1-k_{\mathrm{eff}})$, where k_{eff} is the APD effective ionization ratio, and σ_{th}^2 is the variance of thermal noise which is expressed as $\sigma_{\mathrm{th}}^2=\frac{2k_B T_r T_c}{e^2 R_L}$, where k_B=1.38×10^{-23} J/K is the Boltzmann's constant, T_r is the receiver noise temperature, and R_L is the receiver load resistance.

When the sending bit is 0, I marks arrive at the photon arrival rate of λ_s, other null signals arrive at $\frac{\lambda_s}{M_e}$, and noise exists at each chip like the condition of sending bit 1. Therefore, the probability density function of the output of the photodetector is expressed as

$$P(y|I,b=0)=\frac{1}{\sqrt{2\pi\sigma_{b0}^2}}\mathrm{e}^{-\frac{(y-u_{b0})^2}{2\sigma_{b0}^2}}, \quad (10)$$

where

$$u_{b0}=GT_{\mathrm{C}}[I\lambda_s+(\sum_{i=1}^{n} w_i K_i-I)\frac{\lambda_s}{M_e}+L(\lambda_b+\frac{I_b}{e})]+LT_{\mathrm{C}}\frac{I_S}{e}, \quad (11)$$

$$\sigma_{b0}^2=G^2F_eT_c[I\lambda_s+(\sum_{i=1}^{n} w_i K_i-I)\frac{\lambda_s}{M_e}+L(\lambda_b+\frac{I_b}{e})]+L(T_{\mathrm{C}}\frac{I_S}{e}+\sigma_{\mathrm{th}}^2). \quad (12)$$

In the OCDMA system, the influence due to multiuser has a big part in BER. The probability density function of the multiuser influence (MUI) with code weight w_i is given by

$$P(I)=\sum_{m=I}^{K-1}\sum_{m_1=0}^{m}\binom{K_1-1}{m_1}p_1^{m_1}(1-p_1)^{K_1-1-m_1}\times\prod_{j=1}^{n}\binom{K_j}{m_j}p_j^{m_j}(1-p_j)^{K_j-m_j}, \quad (13)$$

where $p_1=\frac{w_i^2}{L}$ and $p_i=\frac{w_i w_j}{L}$ are the probabilities that one specified mark position of user with weight w_i is hit by the interfering user with weight w_i and w_j, respectively. m_1 and m_j are the numbers of users hitting the target users with the weights of w_i and w_j, respectively, and they should meet the conditions of $m_1\le K_1-1$ and $m_j\le K_j$. K_j is the number of synchronous users with code weight w_j.

Summing up the above, the BER caused by MUI and all kinds of noise is expressed by

$$BER=\frac{1}{2}\sum_{I=th}^{K}\left[P(I)+Q(\frac{th-u_{b0}(I)}{\sigma_{b0}(I)})+Q(\frac{th-u_{b1}(I)}{\sigma_{b1}(I)})\right], \quad (14)$$

where $Q(x)$ is the Gaussian integral function which is given as $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}\mathrm{e}^{-\frac{u^2}{2}}\mathrm{d}u$, and th is the threshold which generally equals w_i. The parameters used to calculate BER are as follows: light wavelength λ=1 550 nm, APD quantum efficiency η=0.6, APD gain G=100, APD effective ionization ratio k_{eff}=0.02, the received signal power P_{T_c}=−10 dBm, the photon arrival rate caused by background light λ_b=10^9 s^{-1}, body leakage current I_b=0.1 nA, the extinction ratio M_e=100, APD surface leakage current I_s=10 nA, receiver noise temperature T_r=1 100 K, receiver load resistor R_L=1 030 Ω and chip duration T_c=0.1 ns. Putting Eqs.(8), (9), (11)–(13) into Eq.(14) and using the parameters shown above, we get the BER finally.

In order to obtain the BER conveniently, let the capacity N equal 100, and there are two kinds of weights w_1 and w_2 in the code family. The BER versus the number of simultaneous users with different w_1 and the same w_2 is shown in Fig.1, when the numbers of users with the different weights are equal, i.e., K_1=K_2. Obviously, BER is increased with the quantity of users increasing, and the users with the bigger weight have the lower BER. The BER of the users with different weights in the same system is shown in Fig.2, when the numbers of users with different weights are equal too. In the same system, the users with the bigger weight have the lower BER, which is consistent with the conclusion shown in Fig.1. The BER is shown in Fig.3 when the quantities of users with various weight are different, i.e., $K_1 \neq K_2$. The difference of BER is not very significant when the proportions of K_1 and K_2 change. So if the number of simultaneous users is random, the characteristics in Fig.3 will be more beneficial to estimate the actual BER.

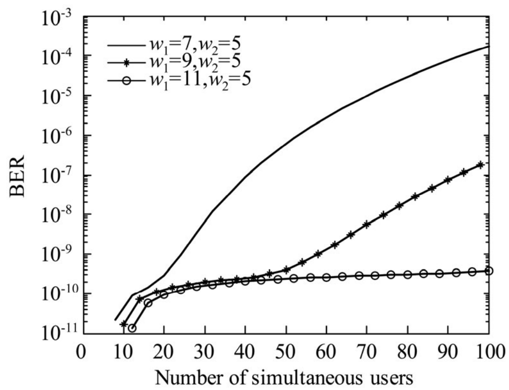


Fig.1 BER versus the number of simultaneous users with different w_1 and the same w_2

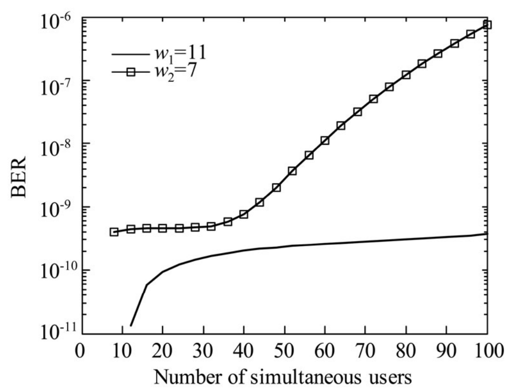


Fig.2 BER of users in the same system with different weights

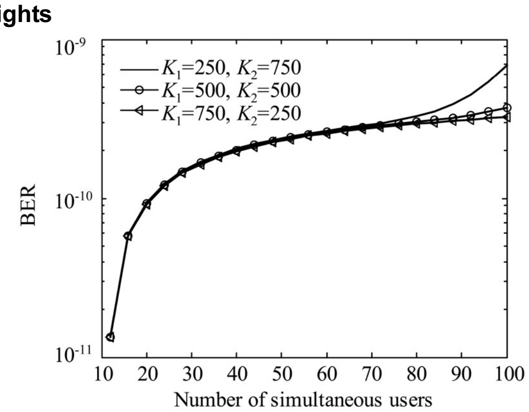
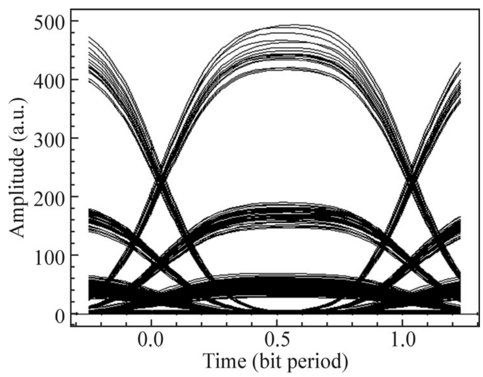


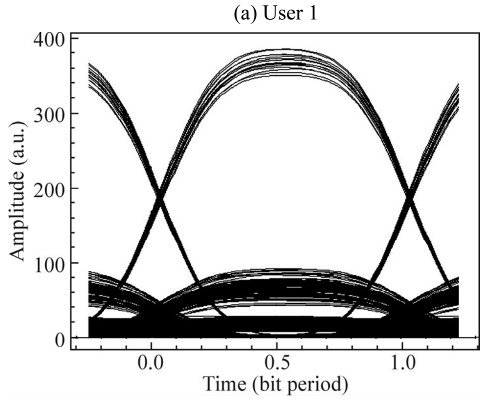
Fig.3 BER versus the number of simultaneous users with different proportions of K_1 and K_2

The performance of VW-OOC is simulated using OptiSys 9.0[11,12]. We use a simple system with two users for simulation. A continuous wave (CW) laser with wavelength of 1 550 nm is used for light source, and the time delay is used for coding. The codes of user 1 and use 2 are (0,8,18) and (0,1,3,7,12), respectively, and the code length is 31. The encoded part of user 2 is with the unit time delay of 0.2 ns, and the tests are carried out at a rate of 10 Gbit/s. A codec is made up of a splitter, a combiner and w time delay, where w is the weight of the user's code. The power of signal is reduced by w times after a splitter and w times after a combiner, so the users with various weights have different signal powers. Further, the signal with small weight has high power, and that with big weight has low power. To avoid this result, an erbium-doped fiber amplifier (EDFA) with different magnifications is added after encoding. For example, if the weights of user 1 and user 2 are w_1 and w_2, respectively, the signal power can be reduced by w_1^2 and w_2^2 times, so the EDFA magnifications of user 1 and user 2 are w_1^2 and w_2^2, respectively.

The performance of the system characterized by BER is compared with others. In the simulation system, the BER is expressed in the clarity of the eye diagram. The clearer the eye diagram, the smaller the BER. The eye diagrams of user 1 and user 2 are shown in Fig.4. Because the time delay is used for coding, the side lobe exits unavoidably. Therefore, the optical hard limiters are used in simulation, and the eye diagram with optical hard limiter is shown in Fig.5. Obviously, the optical hard



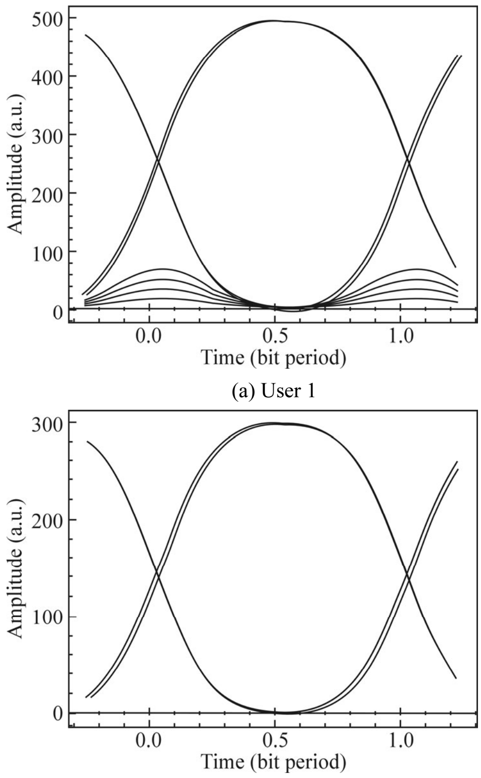
(a) User 1



(b) User 2

Fig.4 Eye diagrams without optical hard limiters

limiter can well reduce the side lobe and get the lower BER. Based on the eye diagrams in Figs.4 and 5, we get the conclusion that the users with the smaller code have the higher BER, which is consistent with the conclusion in theoretical analysis.



(a) User 1

(b) User 2

Fig.5 Eye diagrams with optical hard limiter

A construction scheme of VW-OOCs is investigated in this paper. By using the program in Matlab, an all-interval-set with no repetitive number is produced. Therefore, the VW-OOCs have ideal correlation, and we can easily get the code family only using the code weights and corresponding capacity. The BER performance of the proposed codes is also evaluated analytically with consideration of all kinds of noise. The conclusion shows that the codes with the bigger weight have the lower BER, and changing the proportion of the users with different weights only affects the BER a little. The improved system also shows the conclusion that users with the bigger weight have the lower BER.

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