Sensitivity analysis of light force accelerometer based on optical trapping Mie microsphere

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For achieving the greatest sensitivity, numerical simulation is proposed to optimize the parameters of light force accelerometer (LFA). Firstly, the basic principle of the LFA is analyzed, and the mathematical model is established to describe the axial and radial optical trapping forces on microspheres in the fundamental mode Gaussian laser beam. Secondly, the axial and radial optical trapping forces applied on Mie spheres are simulated to obtain the optimal parameters of open-loop LFA. Results show that the sensitivity can reach 10^3 μm/g. Finally, the LFA system is proposed based on the closed-loop scheme with dual beams. This can provide reference for the design and fabrication of LFA in future.
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Recently, micro-optical-electro-mechanical system (MOEMS) accelerometers have attracted considerable interest in the scientific community due to their advantages, such as immunity to electromagnetic interference, electrical insulating, corrosion-resistance, remote sensing, extremely high sensitivity and multiplexing ability[1]. As a novel MOEMS accelerometer, light force accelerometer (LFA) based on light trapping technology is very promising in future, because there is no cantilever in the acceleration detection system. This assures the LFA can reach high sensitivity in vacuum environment, because the errors resulting from stability of material and structural reliability are eliminated. A lot of research institutes, such as MIT and University of California, already realized the prototypes of single-beam LFA and conducted the preliminary experiment of dual-beam LFA[2-4]. Domestic research institutes, such as Zhejiang University and Shandong Normal University, have been conducting theoretical study[5-7]. In this paper, the light trapping of dual-beam mode is simulated by Matlab, and a global light path scheme of closed-loop dual-beam LFA is established on the basis of theoretical data. Low noise and high sensitivity are both achieved.

The output of the laser is basic-mode Gaussian beam. The math modeling of the light trapping of basic-mode Gaussian beam should be established firstly. Path of the light through the microsphere is shown in Fig.1.

The light absorption of the microsphere is ignored. By ray tracing method, the axial force applied on the microsphere can be calculated as

$$F_z=\left(\frac{4n_1p_0}{c_0}\right)\int_0^{\theta_m}\left(\frac{a}{\omega}\right)^2\exp\left(\frac{-2r^2}{\omega^2}\right)\cos\alpha_1\sin\theta\times \{\cos(\alpha_1-\theta)+R\cos(\alpha_1+\theta)-\frac{T^2\left[\cos(\alpha_1+\theta-2\alpha_2)+R\cos(\alpha_1+\theta)\right]}{1+R^2+2R\cos(2\alpha_2)}\}, \tag{1}$$

where p_0 is the light power, c_0 is the speed of light in vacuum, λ is the wavelength of the laser, R is the reflection coefficient, and T is the refractive coefficient. The upper limit of integral θ_m is defined as shown in Fig.2[5-9].

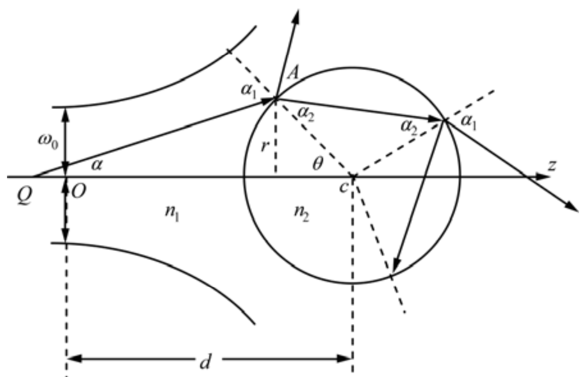

Fig.1 Path of a ray of light incident on the microsphere

When the ray of light is tangent with the microsphere, the illuminated area of the microsphere is the largest. The coordinates of the point of tangency and θ_m can be calculated as

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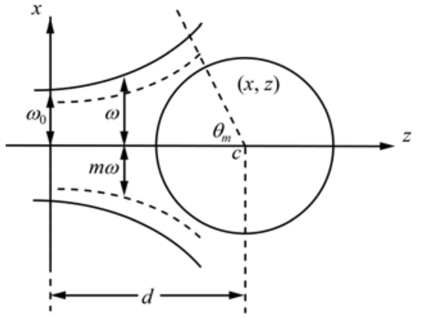
$$x = m\omega_0[1+(\lambda z/\pi\omega_0^2)^2]^{1/2}\ ,\tag{2}$$

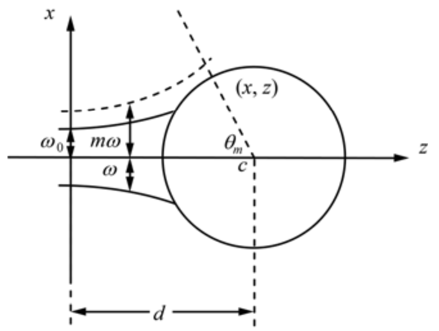
$$z = d/[1+(m\lambda/\pi\omega_0^2)^2]\ ,\tag{3}$$

$$\theta_m = \tan^{-1}[x/(d-z)]\ .\tag{4}$$

As shown in Fig.3, whether the center of microsphere is on optical axis or not, the net radial force should always point to the optical axis so that the microsphere is bounded in the radial light trap[7,10,11]. Refractive index of the microsphere n_2 must be greater than refractive index of the environment n_1, otherwise the net radial force will point to the opposite direction and the microsphere will escape from the trap.

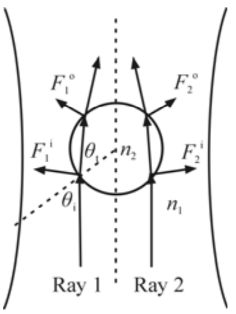


(a) The whole particle in the beam

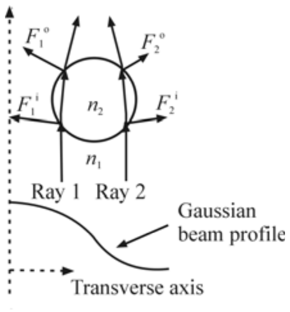


(b) Part of particle in the beam

Fig.2 Microsphere in the Gaussian beam



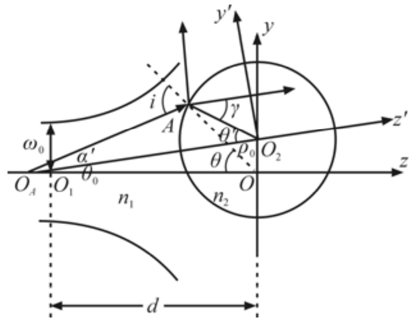
(a) Microsphere with center on optical axis



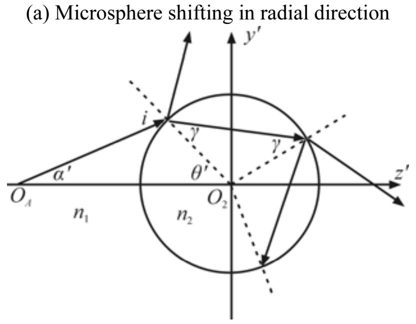
(b) Microsphere with center deviating from optical axis

Fig.3 The radial force applied on the particle

Assume the microsphere has an offset ρ_0 in radial direction as shown in Fig.4(a), and path of the light through the microsphere in rotated coordinate system $y'O_2z'$ plane is shown in Fig.4(b).



(a) Microsphere shifting in radial direction



(b) Path of the light in $y'O_2z'$ plane

Fig.4 Analysis of the radial force

The radial force applied on the microsphere can be calculated as

$$\begin{aligned} F(y) = {} & \frac{\rho^2}{2\mu_0 c^2}\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\theta_m}\mathrm{d}\theta E^2\sin\theta\cos i\times \\ & \Big\{\Big[\sin\alpha' - R\sin(i+\theta') + \\ & T^2\frac{\sin(i+\theta'-2\gamma)+R\sin(i+\theta')}{1+R^2+2R\cos(2\gamma)}\Big]\times \\ & \cos\theta_0\sin\varphi + \Big[\cos\alpha' + R\cos(i+\theta') - \\ & T^2\frac{\cos(i+\theta'-2\gamma)+R\cos(i+\theta')}{1+R^2+2R\cos(2\gamma)}\Big]\sin\theta_0\Big\}\ . \end{aligned}\tag{5}$$

As shown in Fig.5, the position signal of particle is sent to a proportional-integral-derivative (PID) controller which controls the beam power by two fast variable optical attenuators (FVOAs)[12,13] to trap the particle and pull it to the original point. Then, there is a power difference between two beams which can be multiplied by scale factor K to derive the acceleration[3]. Here, the LFA operates in the vacuum chamber. Output of PID controller is electronically divided with the same intensity and opposite sign between two FVOAs, which ensures the sum of two levitation beam power values is constant, avoiding the radial drift caused by change of gradient force. When the pressure is below 0.133 Pa, the error caused by molecular collision can be inhibited fundamentally.

The dual-beam trap is realized by using the closed-loop mode. Then the open-loop sensitivity is analyzed, and the preliminary scheme of closed-loop mode LFA is established. The scale factor K should be calibrated in real time. When each of the beams from two FVOSs is incorporated by a sinusoidal modulating signal with the same amplitude δP but opposite signs[4,14,15], the power difference is $2\delta P$. According to the scale factor K changing actually with time, the force applied on the microsphere is $F=mK2\delta P$ in vacuum. Assuming there is little air in high vacuum environment, the oscillation of microsphere position is described by the dynamics as $F=mx$. So K satisfies $K2\delta P=\omega^2\delta x/2$. The power dither is applied at a frequency ω which is outside the bandwidth of the control loop but within that of the detector[4,16,17], so the scale factor K can be determined without affecting the acceleration measurement.

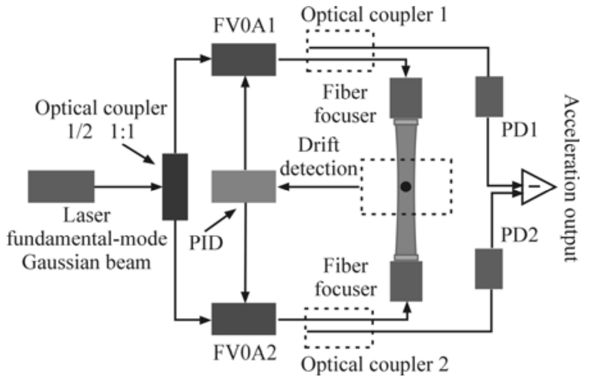


Fig.5 Schematic diagram for integral structure of the LFA

The simulation parameters are set as follows. The distance between two waists is 400 μm, refractive index of environment is 1, refractive index of microsphere is 2.66, and laser power is 80 mW. The simulations of the axial force on the transverse displacement are shown in Figs.6–8 when wavelength, microsphere diameter and beam waist change, respectively.

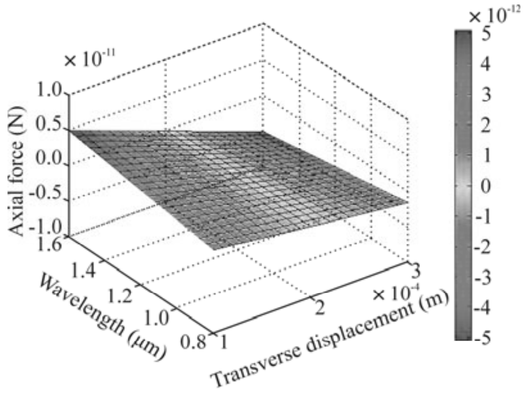


Fig.6 Simulation results for different wavelengths

The variables in the simulations above are continuous. But in practice, the wavelengths used in optical communication are 980 nm, 1064 nm and 1550 nm, and the microsphere diameters and the beam waists are both processed with interval of 5 μm, such as the microsphere diameters of 5 μm, 10 μm and 15 μm and the beam waists of 10 μm, 15 μm and 20 μm. The comparison is mainly focused on the situations corresponding to the discrete points.

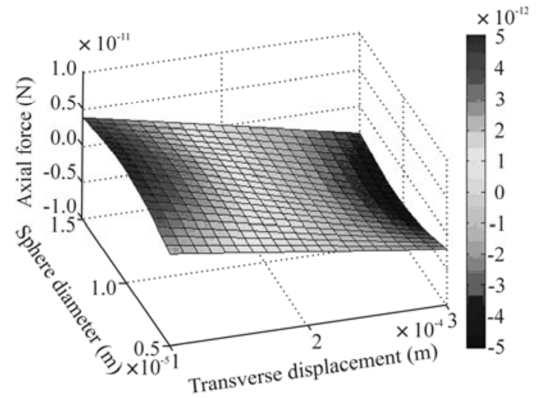


Fig.7 Simulation results for different microsphere diameters

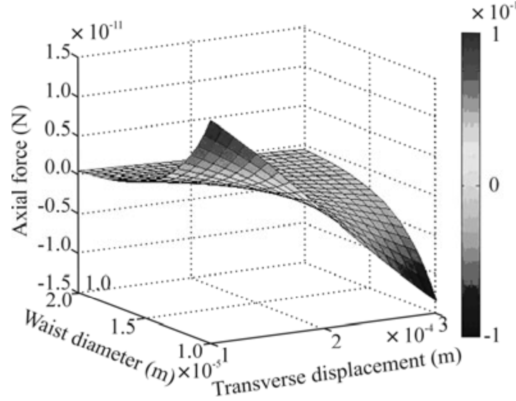


Fig.8 Simulation results for different beam waists

The variation of radial force as a function of radial displacement for different beam waists of 15 μm and 20 μm is shown in Fig.9. The gravity of the microsphere is 5.24×10^{-12} N. As shown in Fig.9, the maximum radial force for beam waist of 20 μm is hardly greater than the gravity of the microsphere compared with that for beam waist of 15 μm. So we choose wavelength of 980 nm, beam waist of 15 μm and microsphere diameter of 5 μm to design the open-loop mode system. In this case, the sensitivity of the open-loop mode light trapping system can reach $S=10^3$ μm/g. The net axial force of open-loop mode system points to the direction opposite to the drift of particle constantly.

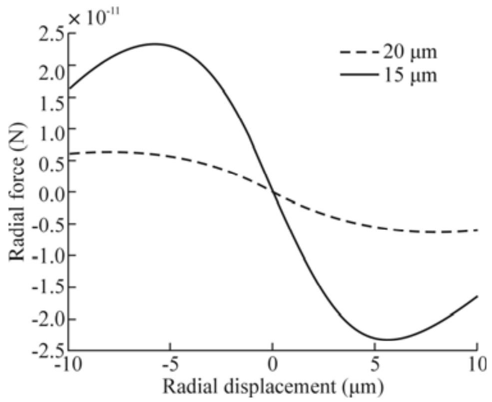


Fig.9 Variation of radial force as a function of radial displacement for different beam waists

Open-loop mode establishes a stable trap, but the sensitivity in this case is lower than that in closed-loop mode. As shown in Fig.10, in closed-loop mode, the particle is completely free in the axial direction when there is no inertial input. Since the beams are confocal and of identical shape and power, their scattering forces disappear over the entire beam axis. In practice, this configuration is approximated using beams with Rayleigh ranges that are many particle diameters[4]. Such beams are nearly collimated in the trapping region, so that the net force does not change drastically for small displacements in the beam axis, which makes the real-time scale factor calibration available.

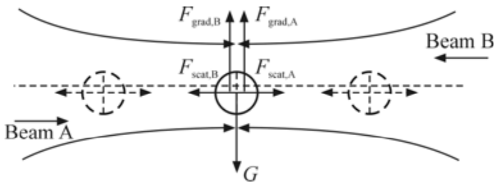


Fig.10 Optical path of the closed-loop dual-beam trap

When there is no inertial input in closed-loop mode, the directions of the two beams are opposite and the other characteristics of two beams are identical, so shot noise of the laser is the main factor to limit the sensitivity. When external forces are applied on the LFA, i.e., there is inertial input, the two beams are no longer equivalent. The relative intensity noise in the laser becomes the important noise source. There is a positive correlation between this noise and $1/\gamma$, where γ is the time for calculating the average value in acceleration measurement.

As shown in Fig.11, the illumination beam at 1550 nm passes through the levitation beam with microspheres, and the small-angle refraction light is obtained[4] to be used as detection source for reducing the error caused by the asphericity of microsphere. Lens L_1 combined with diaphragm masks the uncalled-for light. The small-angle refraction light beam should be expanded and collimated firstly to illuminate the whole area, in which the intensity of light is constant and the particle exists possibly.

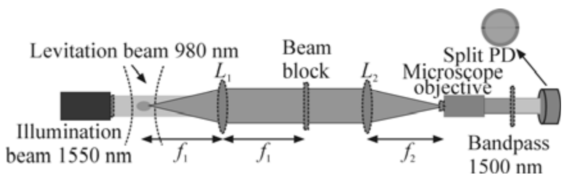


Fig.11 Detection principle of the small-angle refraction light from an illumination beam

The optimized parameters of open-loop LFA are established as wavelength of 980 nm, beam waist of 15 μm and microsphere diameter of 5 μm. With the optimized parameters, the sensitivity can achieve 10^3 μm/g, and the closed-loop dual-beam LFA scheme is put forward. Experiments of LFA will be conducted and the sensitivity calibration of open-loop will be measured in the future work. This article provides the fundamental academic consultations for the further improvement and commercialization of LFA.

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