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Influence of different propagation paths on the propagation of laser in atmospheric turbulence*

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The analytical expressions for the average intensity, root mean square (RMS) beam width and angular spread of Gaussian Schell-model (GSM) beams propagating under slant atmospheric turbulence are derived, and they are used to study the influence of different propagation paths on the propagation of laser beams in atmospheric turbulence. It is shown that under the same condition, the influence of atmospheric turbulence along a downlink path on the GSM beam propagation is the smallest among the three paths. Therefore, the downlink propagation is more beneficial to the beam propagation through atmospheric turbulence compared with the uplink propagation and horizontal propagation.

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The propagation of laser beams under atmospheric turbulence has attracted substantial interest for a long time because of the importance with some practical applications, such as the tracking, remote sensing, optical communications and high-power laser beaming[1-4]. The propagation properties of partially coherent beams, such as Gaussian Schell-model (GSM) beams, partially coherent Hermite-Gaussian beams and partially coherent flat-topped beam array, in horizontal atmospheric turbulence were studied extensively[5-11]. It was shown that under some circumstance partially coherent beams are less affected by turbulence than fully coherent ones, and the theoretical prediction was confirmed by experiment[12]. Recently, the propagation properties of laser beams were studied in atmospheric turbulence in a slant path and in ground-to-satellite optical links[13-22]. This paper is devoted to the study of the influence of different propagation paths on the propagation of laser beams in atmospheric turbulence based on the extended Huygens-Fresnel principle.

A GSM beam, whose cross-spectral density function is at the plane L=0, is expressed as[23]

$$W^{(0)}(\boldsymbol{s}_1,\boldsymbol{s}_2,0)=\exp(-\frac{s_1^2+s_2^2}{w_0^2})\exp[-\frac{(\boldsymbol{s}_1-\boldsymbol{s}_2)^2}{2\sigma_0^2}]\,, \tag{1}$$

where $\boldsymbol{s}_i\equiv(s_{ix},\ s_{iy})$ (i=1, 2) is the two-dimensional position vector at the source plane L = 0, and w_0 and σ_0 denote the waist width and spatial correlation length, respectively.

In accordance with the extended Huygens-Fresnel principle[2], the cross-spectral density function of GSM beams propagating through atmospheric turbulence along a slant path is given by

$$W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,L)=(\frac{k}{2\pi L})^2\iint\mathrm{d}^2s_1\iint\mathrm{d}^2s_2W^{(0)}(\boldsymbol{s}_1,\boldsymbol{s}_2,0)\times$$
$$\exp\{-\frac{\mathrm{i}k}{2L}[(\boldsymbol{\rho}_1-\boldsymbol{s}_1)^2-(\boldsymbol{\rho}_2-\boldsymbol{s}_2)^2]\}\times$$
$$\langle\exp[\psi^*(\boldsymbol{\rho}_1,\boldsymbol{s}_1)+\psi(\boldsymbol{\rho}_2,\boldsymbol{s}_2)]\rangle\,, \tag{2}$$

where * denotes the complex conjugate, k is the wave number related to the wavelength λ by k=2π/λ, $\boldsymbol{\rho}_i\equiv(\rho_{ix},\ \rho_{iy})$ is the position vector at the L plane, <·> denotes the average over the ensemble, and $\psi(\boldsymbol{\rho},\ \boldsymbol{s})$ represents the random part of the complex phase of a spherical wave due to the turbulence, and can be written as[24,25]

$$\langle\exp[\psi^*(\boldsymbol{\rho}_1,\boldsymbol{s}_1)+\psi(\boldsymbol{\rho}_2,\boldsymbol{s}_2)]\rangle=$$
$$\exp[-\frac{1}{2}D_\psi(\boldsymbol{s}_1-\boldsymbol{s}_2,\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)]\,, \tag{3}$$

where $D_\psi(\boldsymbol{s}_1-\boldsymbol{s}_2,\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)$ represents the phase structure function[2,26]

$$D_\psi(\boldsymbol{s}_1-\boldsymbol{s}_2,\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)=2.914k^2\sec(\xi)\times$$
$$\int_{h_0}^{H}C_n^2(h)\,|(1-\eta)(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)+\eta(\boldsymbol{s}_1-\boldsymbol{s}_2)|^{5/3}\,\mathrm{d}h=$$
$$T_1(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2+T_2(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)\cdot(\boldsymbol{s}_1-\boldsymbol{s}_2)+T_3(\boldsymbol{s}_1-\boldsymbol{s}_2)^2\,, \tag{4}$$

where

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$$T_1 = 2.914k^2 \sec(\xi) \int_{h_0}^{H} C_n^2(h)(1-\eta)^2 \mathrm{d}h , \tag{5}$$

$$T_2 = 2.914k^2 \sec(\xi) \int_{h_0}^{H} 2C_n^2(h)\eta(1-\eta) \mathrm{d}h , \tag{6}$$

$$T_3 = 2.914k^2 \sec(\xi) \int_{h_0}^{H} C_n^2(h)\eta^2 \mathrm{d}h . \tag{7}$$

The difference between uplink propagation and downlink propagation is the different values of η in Eqs.(5)–(7). $\eta=1-(h-h_0)/(H-h_0)$ and $\eta=(h-h_0)/(H-h_0)$ are for the uplink propagation and downlink propagation, respectively, where h_0 is the height above ground level of the uplink transmitter and/or downlink receiver, $H= h_0+L\cos(\xi)$ is the receiver altitude for the uplink propagation and/or transmitter altitude for the downlink propagation, L is propagation distance in a slant path, and ξ is the zenith angle[2]. $C_n^2(h)$ describes the variation of the structure constant versus the altitude h. One of the most widely used models is the Hufnagel-Valley (H-V) model which is given by

$$C_n^2(h) = 0.00594(\upsilon/27)^2(10^{-5}h)^{10}\exp(-h/1000) +$$
$$2.7\times10^{-16}\exp(-h/1500) + A\exp(-h/100) , \tag{8}$$

where υ is the wind speed, and A is a nominal value at ground level. In our calculations, we use the H-$\mathrm{V}_{5/7}$ model with υ=21 m/s and $A=1.7\times10^{-14}$ $\mathrm{m}^{-2/3}$[2].

Let $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\boldsymbol{\rho}$ in Eq.(2), so the average intensity of GSM beams in slant atmospheric turbulence obtained from the straightforward integration of Eq.(2) is written as

$$I(\boldsymbol{\rho},L) = \frac{k^2 w_0^2}{8L^2\varepsilon}\exp[-\frac{k^2\rho^2}{4L^2\varepsilon}] , \tag{9}$$

where

$$\varepsilon = \frac{1}{2w_0^2} + \frac{1}{2\sigma_0^2} + \frac{T_3}{2} . \tag{10}$$

The on-axis intensity of GSM beams propagating through slant atmospheric turbulence can be written as

$$I(\boldsymbol{\rho}=0,L) = \frac{k^2 w_0^2}{8L^2\varepsilon} , \tag{11}$$

which provides the analytical propagation expression for the on-axis average intensity of GSM beams through slant atmospheric turbulence. For the case of altitude h=0, $C_n^2(0)=1.727\times10^{-14}$ $\mathrm{m}^{-2/3}$ in Eq.(8) and $T_1=T_2=T_3=0.9713k^2LC_n^2(0)$ in Eqs.(5)–(7), Eq.(11) reduces to the on-axis average intensity of GSM beams through horizontal atmospheric turbulence.

The root mean square (RMS) beam width reflects the spreading of beams propagating through atmospheric turbulence, and the smaller the RMS beam width is, the less the beam is affected by turbulence. The RMS beam width is defined as[8]

$$w(L) = \sqrt{\frac{\int I(\boldsymbol{\rho},L)\rho^2\mathrm{d}^2\rho}{\int I(\boldsymbol{\rho},L)\mathrm{d}^2\rho}} . \tag{12}$$

Substituting Eq.(9) into Eq.(12), tedious but straightforward integral calculations lead to the RMS beam width of GSM beams in slant atmospheric turbulence as

$$w(L) = [\frac{w_0^2}{2} + (\frac{2}{k^2w_0^2} + \frac{2}{k^2\sigma_0^2})L^2 + \frac{2T_3}{k^2}L^2]^{1/2} , \tag{13}$$

which indicates that the beam width of GSM beams in turbulence consists of three terms. The first two terms under the square root represent the beam-width spreading in free space due to diffraction, where the first term $w_0^2/2$ is independent of the propagation distance L and the second term increases with L^2. The third term $2T_3L^2/k^2$ describes the effect of turbulence on the beam-width spreading. Therefore, the beam width spreads more rapidly in turbulence than in free space. For h=0, Eq.(13) reduces to the RMS beam width of GSM beams through horizontal atmospheric turbulence.

From Eq.(13), the angular spread[7] of GSM beams in slant atmospheric turbulence is expressed as

$$\theta_{\mathrm{sp}} = \frac{w(L)}{L}\Big|_{L\to\infty} = \sqrt{(\frac{2}{k^2w_0^2} + \frac{2}{k^2\sigma_0^2}) + \frac{2T_3}{k^2}} . \tag{14}$$

The angular spread θ_{sp} can be generally used to describe the laser beam quality in the far field. The smaller the angular spread θ_{sp} is, the less the beam is affected by turbulence.

Fig.1 gives the on-axis intensity distributions $I(0, L)$ of GSM beams through atmospheric turbulence along horizontal path, uplink path and downlink path versus the propagation distance L, and the calculation parameters are λ=1.06 μm, w_0=3 cm, σ_0=3 cm, ξ=60° and h_0=0. It is seen from Fig.1 that the on-axis intensity distributions of GSM beams are different along different paths. The intensity of slant propagation of the GSM beam is bigger than that of horizontal propagation, and the result can be interpreted reasonably from Eq.(8). Because the structure constant $C_n^2(h)$ decreases with the increase of altitude h, the effect of turbulence along the slant path is less than that along the horizontal path. Fig.1 indicates that the intensity of downlink propagation is bigger than that of uplink propagation. It can be explained from Eqs.(10) and (11) that the on-axis intensity varies inversely with T_3, the value of T_3 in downlink propagation is smaller than that in uplink propagation, so the intensity of downlink propagation is bigger than that of uplink propagation of GSM beams through atmospheric turbulence. It means that the effect of turbulence along the downlink path is less than that along the uplink path.

The RMS beam widths $w(L)$ of GSM beams in atmospheric turbulence versus the propagation distance L for different propagation paths are depicted in Fig.2, where the other calculation parameters are the same as those in Fig.1. From Fig.2 we see that the RMS beam width of horizontal propagation is the biggest among the three paths, while the RMS beam width of uplink propagation is bigger than that of downlink propagation. Therefore, the beam-width spreading of GSM beams of downlink propagation is the least affected by atmospheric turbulence among the three paths, and the beam-width spreading of uplink propagation is less affected by turbulence than that of horizontal propagation. The physical explanations can be derived from Eqs.(7) and (13), Eq. (11) implies that the RMS beam width $w(L)$ varies directly with T_3, T_3 in Eq.(7) of downlink propagation is smaller than that of uplink propagation or horizontal propagation, and horizontal propagation is the biggest. From Figs.1 and 2, it is shown that the influence of atmospheric turbulence along downlink path on the GSM beam propagation is the smallest among the three paths, while the influence of atmospheric turbulence of uplink propagation is smaller than that of horizontal propagation.

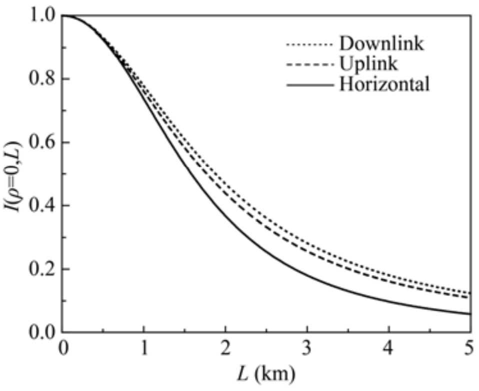

Fig.1 The on-axis intensity distributions *I*(0, *L*) of GSM beams through atmospheric turbulence along horizontal path, uplink path and downlink path versus the propagation distance *L*

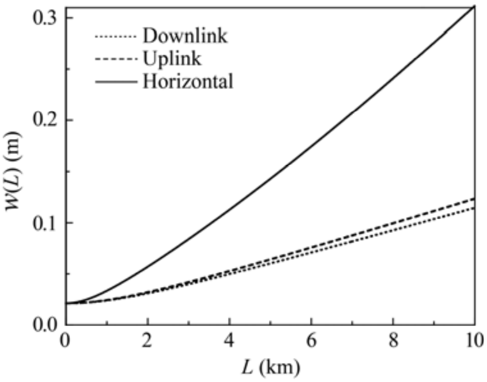

Fig.2 The RMS beam widths *w*(*L*) of GSM beams in atmospheric turbulence for different propagation paths versus the propagation distance *L*

The angular spread $\theta_{\rm sp}$ of GSM beams in atmospheric turbulence versus the spatial correlation length σ_0 for different propagation paths at the plane L=10 km is plotted in Fig.3, where the other calculation parameters are the same as those in Fig.1. From Fig.3, it is seen that the angular spread of horizontal propagation is the biggest among the three paths, while the angular spread of uplink propagation is bigger than that of downlink propagation. The physical explanation of angular spread in Fig.3 is similar to that of the beam width in Fig.2. From Figs.1–3, it is shown that the slant path is more beneficial to the beam propagation through atmospheric turbulence compared with the horizontal propagation, and the downlink path is more beneficial to the beam propagation compared with the uplink propagation.

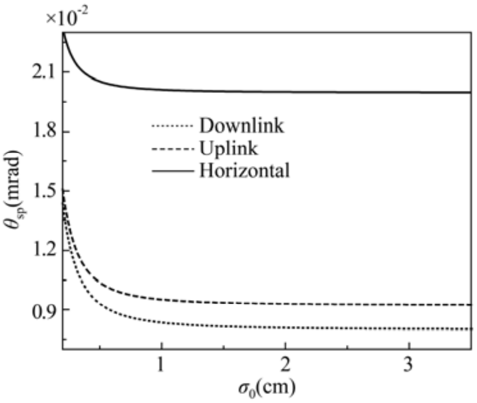

Fig.3 The angular spread $\theta_{\rm sp}$ at the plane *L*=10 km of GSM beams in atmospheric turbulence for different propagation paths versus the spatial correlation length σ_0

In this paper, a comparative study of the influence of different propagation paths on the propagation of laser beams in atmospheric turbulence is made by using the average intensity, RMS beam width and angular spread. It is found that the intensity of slant propagation of the GSM beam is bigger than that of horizontal propagation, and the intensity of downlink propagation is bigger than that of uplink propagation. The RMS beam width of horizontal propagation is the biggest among the three paths, while the RMS beam width of uplink propagation is bigger than that of downlink propagation. The angular spread of horizontal propagation is the biggest among the three paths, while the angular spread of uplink propagation is bigger than that of downlink propagation. Therefore, the influence of atmospheric turbulence along downlink path on the GSM beam propagation is the smallest among the three paths, while the influence of atmospheric turbulence of uplink propagation is smaller than that of horizontal propagation. Consequently, the slant path is more beneficial to the beam propagation through atmospheric turbulence in comparison with the horizontal propagation, and the downlink path is more beneficial to the beam propagation in comparison with the uplink propagation. Results obtained in this paper may provide potential applications in free-space optical communications.

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