A novel decoding algorithm based on the hierarchical reliable strategy for SCG-LDPC codes in optical communications*

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(Received 28 July 2013)

An effective hierarchical reliable belief propagation (HRBP) decoding algorithm is proposed according to the structural characteristics of systematically constructed Gallager low-density parity-check (SCG-LDPC) codes. The novel decoding algorithm combines the layered iteration with the reliability judgment, and can greatly reduce the number of the variable nodes involved in the subsequent iteration process and accelerate the convergence rate. The result of simulation for SCG-LDPC(3969,3720) code shows that the novel HRBP decoding algorithm can greatly reduce the computing amount at the condition of ensuring the performance compared with the traditional belief propagation (BP) algorithm. The bit error rate (BER) of the HRBP algorithm is considerable at the threshold value of 15, but in the subsequent iteration process, the number of the variable nodes for the HRBP algorithm can be reduced by about 70% at the high signal-to-noise ratio (SNR) compared with the BP algorithm. When the threshold value is further increased, the HRBP algorithm will gradually degenerate into the layered-BP algorithm, but at the BER of 10^{-7} and the maximal iteration number of 30, the net coding gain (NCG) of the HRBP algorithm is 0.2 dB more than that of the BP algorithm, and the average iteration times can be reduced by about 40% at the high SNR. Therefore, the novel HRBP decoding algorithm is more suitable for optical communication systems.

Document code: A **Article ID:** 1673-1905(2013)06-0469-4

DOI 10.1007/s11801-013-3135-y

The low-density parity-check (LDPC) code is a kind of excellent linear block code approaching the Shannon limit[1,2] and has become a hot spot in optical transmission systems[3-7]. Among many LDPC codes constructed by different methods, the systematically constructed Gallager (SCG)-LDPC code constructed by system construction method has good performance[6,7]. However, it is generally decoded by the belief propagation (BP) decoding algorithm which has higher complexity. In the aspect of reducing the decoding complexity, the applied methods are mainly divided into two categories: ①The operation process of the check nodes uses a lot of floating-point logarithm and exponential operations, which can be simplified by mathematical approximation and then compensated by correction factor[8-10]; ②Considering that all the variable nodes are involved in each iteration in the traditional decoding algorithm, the number of variable nodes can be reduced in the subsequent iterative decoding process[11,12]. In addition, the shuffled-BP decoding algorithm can accelerate the convergence rate of BP algorithm by using the updated node messages as soon as possible[13]. Through the analysis of structural characteristics of SCG-LDPC codes for optical communications, an effective hierarchical reliable belief propagation (HRBP) decoding algorithm is proposed in this paper, and the novel decoding algorithm can ensure the performance and greatly reduce the decoding complexity.

Daniel Hösli and Erik Svensson proposed a construction method for LDPC codes on the basis of Gallager, i.e., SCG(j, k) code which is shown in Fig.1[14], where $j\in\{2,3,4\}$ shows the number of submatrices of parity check matrix and the column weight of parity check matrix, and k is the row weight of parity check matrix.

In Ref.[15], it is theoretically proved that the convergence rate of shuffled schedule (SS) is faster than that of flooding schedule (FS). Although SS has better convergence rate, its parallel capability is very low and its decoding delay is higher than that of FS when the check

* This work has been supported by the National Natural Science Foundation of China (Nos.61071117, 61275077 and 61003256), and the Natural Science Foundation of Chongqing CSTC (No.2010BB2409).

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matrix is divided into many layers. Through analyses, it is shown that the check matrix of the SCG-LDPC code has the following two characteristics: ①The number of the check submatrices which are the components of the check matrix is less; ②The column weight of each check submatrix is 1. Therefore, the layering number can be equal to the number of the check submatrices, and the column weight of each check submatrix is 1. These conditions can ensure that all the variable nodes can be involved in each iteration process. In the decoding process, handle each submatrix successively (parallel iteration in layer and serial iteration in interlayer), so it can transmit the updated message of variable nodes layer by layer immediately, and thus the message of variable nodes produced in this iteration process can be better used. Since the number of the layers is less, the decoding delay brings a little effect. Combined with the criterion of reliability judgment, the number of the variable nodes involved in the same iterative process can be further reduced, and it can accelerate the convergence rate and reduce the decoding complexity. Therefore, this decoding algorithm, called as HRBP decoding algorithm based on the hierarchical reliable strategy, has a good compromise between the performance and complexity.

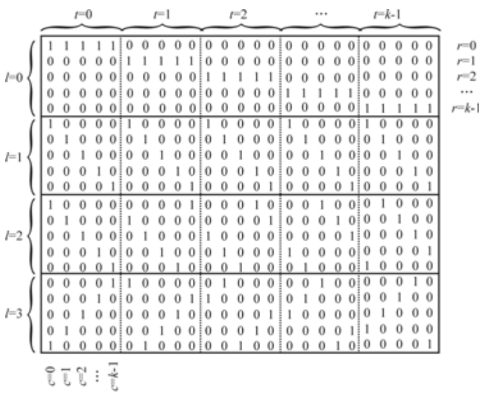

Fig.1 The check matrix of the SCG(4,5) code constructed by Daniel Hösli and Erik Svensson

After encoding, the code-word c_i (i=1,2,3⋯,n) is mapped to $x_i=(-1)^{c_i}$ by binary phase-shift keying (BPSK) digital modulation. And then through the additive white Gaussian noise (AWGN) channel, the code-word $y_i=x_i+n_i$ is received, where n_i is Gaussian stochastic sequence. The initial log-likelihood ratio (LLR) message from the variable node i is expressed as $L(v_i)$. In the lth iterative process, the external probability information of the variable node i to the check node j in the nth layer is expressed as $L(q_{ijn}^{(l)})$, the external probability information of the check node j in the nth layer to the variable node i is expressed as $L(r_{ijn}^{(l)})$, and the posterior probability of the variable node i is expressed as $L(Q_i^{(l)})$. The set of variable nodes connected with check node j is expressed as $N(j)$, the set of check nodes connected with variable node i is expressed as $M(i)$, and the set of θ includes variable nodes with high reliability before each iterative process. The concrete steps of the HRBP decoding algorithm are as follows in detail:

①Initialization

Set θ as NULL, for every variable node, i=1,2,⋯,M and j=1,2,⋯,N,

$$L(r_{ijn}^{(0)})=0\,, \tag{1}$$

$$L(Q_i^{(0)})=L(v_i)=2y_i/\sigma^2\,, \tag{2}$$

where σ^2 is the noise variance of the channel.

②Layer by layer decoding

For $i\notin\theta$, the variable node is updated. In the lth iterative process, calculate the information of the variable node i to the check node j in the nth layer:

$$L(q_{ijn}^{(l)})=L(Q_{i(n-1)}^{(l)})-L(r_{ijn}^{(l-1)})\,. \tag{3}$$

Then update the check node. Calculate the information of the check node j to the variable node i:

$$L(r_{ijn}^{(l)})=2\tanh^{-1}\left\{\prod_{i'\in N_j\backslash i}\tanh\left[\frac{1}{2}L\left(q_{i'jn}^{(l)}\right)\right]\right\}. \tag{4}$$

For the posterior probability of updating the variable node, calculate the information of the variable node i after receiving the check node message:

$$L(Q_{in}^{(l)})=L(r_{ijn}^{(l)})+L(q_{ijn}^{(l)})\,. \tag{5}$$

③Decoding decision

$$c_i=\begin{cases}0, & L(Q_i^{(l)})>0\\ 1, & L(Q_i^{(l)})\le 0\end{cases}. \tag{6}$$

Stop the decoding, if $\boldsymbol{H}c^{\mathrm{T}}=0$ or the maximum iterating time is reached. Otherwise, turn into the next step.

④Reliability judgment

For every variable node, calculate the number of the illegal check equations and the posteriori probability. If this number equals 0 and the posteriori probability meets $|L(Q_i^{(l)})|>L_{\mathrm{th}}$, it can be considered to be a high reliability variable node and added into the set θ, and then return to the second step and keep on the iteration. For the variable node judged as the high reliability, only the probability information is passed out while the information of the connected node can't be received in the next iteration process. Once a node is judged as the high reliability node, it will always be considered to be the high reliability node. This updating strategy can reduce the effect of the external error information of the high reliability variable node and be helpful for ensuring the high reliability of the output information so that the errors of other updating variable nodes are corrected.

The SCG-LDPC(3969,3720) code with the code rate of 0.937 for optical communication systems is constructed by the SCG construction method when setting j=4, k=63 and simulated by MATLAB. Fig.2 shows the comparison of the error correction performance among the three decoding algorithms which are BP algorithm, min-sum algorithm and HRBP algorithm respectively. When the maximum iteration times are 30, it shows that the bit error rate (BER) performance of the HRBP algorithm with the threshold value L_{th} of 15 is better than that of the BP algorithm. When the threshold value L_{th} is further increased, the HRBP algorithm gradually degenerates into the layered-BP algorithm, but the net coding gain (NCG) of the HRBP algorithm is 0.2 dB more than that of the BP algorithm at the BER of 10^{-7}. Fig.3 shows the average iteration times of BP algorithm and HRBP algorithm. In the low SNR region, the reliability of the node is relatively low, and more average iteration times are needed. And it is converse in the high SNR region. At the same time, the average iteration times of the HRBP algorithm, compared with the BP algorithm, can be reduced by about 40% at the high SNR. Fig.4 shows the normalized variable nodes in subsequent iterative process. As SNR increases, the number of variable nodes which meet the conditions of the reliability judgment is increased. That is the number of variable nodes decreasing in subsequent iterative process. It is proved that the number of variable nodes of HRBP algorithm can be reduced by about 70% at the high SNR.

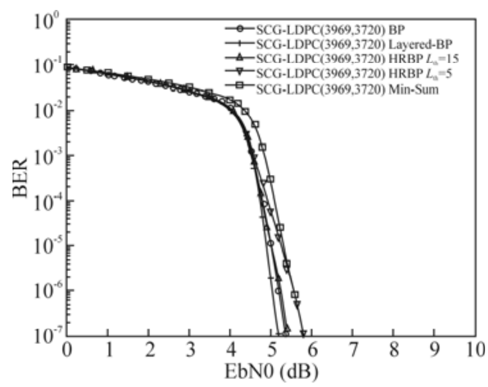

Fig.2 The error performance of the three decoding algorithms

The computation in subsequent iterative decoding process is determined by the average number of variable nodes and the average iteration times in subsequent iteration. Fig.5 shows the normalized computation amounts of different decoding algorithms, from which it can be proved that the computation amount of HRBP algorithm is far less than that of the traditional BP algorithm in higher SNR.

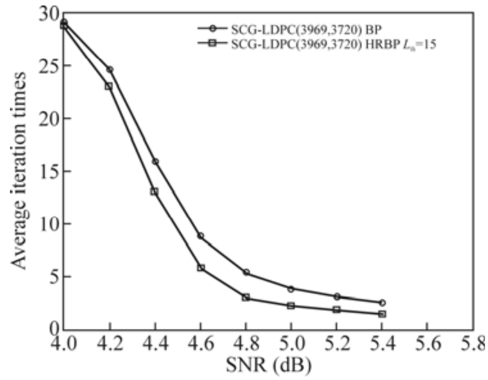

Fig.3 The comparison of the average iteration times between BP algorithm and HRBP algorithm

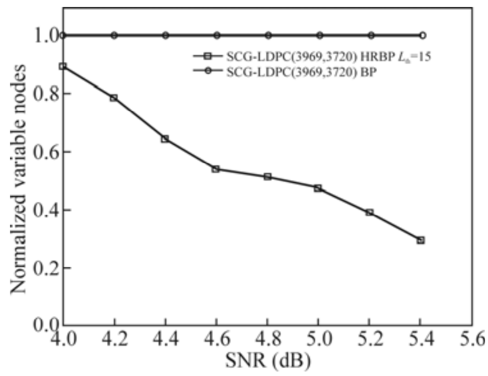

Fig.4 The comparison of normalized variable nodes between BP algorithm and HRBP algorithm

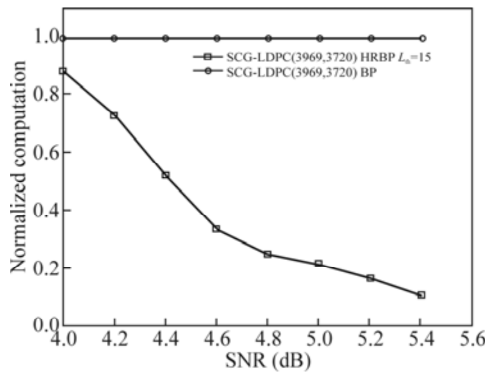

Fig.5 The comparision of normalized computation between BP algorithm and HRBP algorithm

A novel effective decoding algorithm according to the SCG-LDPC codes for optical communication system is proposed in this paper. It can accelerate the convergence rate and reduce the number of the decoding nodes involved in the iterative process. Furthermore, the decoding complexity can be greatly reduced in the condition of ensuring the performance by combining the layered decoding with the reliability judgment. Since the average iteration times and the number of the decoding nodes involved in the iterative process are reduced, the decoding time can be greatly reduced. Therefore, the novel

HRBP decoding algorithm is more effective and suitable for optical communication systems.

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