Modulation instability in positive refractive metamaterials with higher-order dispersion and saturable nonlinearity*

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(Received 5 August 2013)

After taking the higher-order dispersion and three kinds of saturable nonlinearities into account, we investigate the characteristics of modulation instability (MI) in real units in the positive refractive region of metamaterials (MMs). The results show that the gain spectra of MI consist of two spectral regions, one of which is close to and the other is far from the zero point. In particular, the spectral region far from the zero point also has high cut-off frequency but narrow spectral width just as those revealed in the negative refractive region. Moreover, the gain spectra can change with the normalized angular frequency, the normalized optical power and the form of the saturable nonlinearity. Concretely, the spectral width increases with increase of the normalized angular frequency. But both of the spectral width and the peak gain increase and then decrease with increase of the normalized optical power. In other words, the MI characteristics and MI related applications can be controlled by adjusting the structure of the MMs, the form of the saturable nonlinearity and the normalized optical power.

Document code: A **Article ID:** 1673-1905(2013)06-0465-4
DOI 10.1007/s11801-013-3141-0

Up to now, modulation instability (MI) has been systematically investigated in various optical materials and optical effects[1-4]. Recently, a kind of special artificial composite material called metamaterial (MM), which can exhibit negative refractive effects[5,6], is referred to negative refractive MM or left-handed material. MMs are potentially utilized to effectively control the propagation of the electromagnetic wave or the movement of the photons at our own will, and thereby are applied in communications, super resolution imaging, etc. Various functional microwave devices or optoelectric devices are also expected to be designed successfully.

Studies on MI are naturally extended to the singular materials in recent years. For example, Kourakis et al[7] investigated the nonlinear instability in negative refractive MMs, and obtained the MI picture of the coupled plane wave solution. Other studies further investigate the effects of the first- and second-order nonlinear dispersions[8-12], fake quintic nonlinearity[9] and saturable nonlinearity[8,11-13]. Our recent works have considered the effects of other kinds of saturable nonlinearities[14,15] and higher-order dispersion[15]. In Ref.[15], we have investigated MI in MMs with higher-order dispersion and three kinds of saturable nonlinearities, but only in the negative refractive region. Moreover, the structures of MMs can be adjusted, and thereby MMs can also exbihit positive refractive characteristics. Thus, in this paper, we analyze the MI in terms of its gain spectra, peak gain and cut-off frequency in the positive refractive region of MMs in real units.

On the basis of our previous analyses[15], the MI investigation can be directly extended to the positive refractive region of MM readily. In order to compare with those in the negative refractive region conveniently, we set the same common parameters as those in Ref.[15]. Similarly, we also use CSN, ESN, and ASN to stand for conventional saturable nonlinearity, exponential saturable nonlinearity and another kind of saturable nonlinearity, respectively. For the reason that the relation of $g(-\Omega)=g(\Omega)$ always holds[15], only $g(\Omega)$ is displayed in the following calculations. In Fig.1, the variations of gain spectra of MI with the normalized optical power ΓP_0 are calculated by taking ASN for example. According to our detailed calculations here, it is discovered that the gain spectra consist of two spectral regions, one of which is close to and the other is far from the zero point, instead of one or two regions in ordinary materials, even if the normalized angular frequency ω_0/ω_{pe} and the electromagnetic frequency band are varied. This characteristic is also different from that in the negative refractive region where the MI gain spectra consist of only one spectral region either close to or far from the zero point[15]. To observe the two spectral regions more clearly, we show them in Figs.1(a) and (b) respectively.

* This work has been supported by the Key Project of Science Technology Research of Chinese Ministry of Education (No.210186), the Major Project of Natural Science of the Educational Department of Sichuan Province (Nos.13ZA0081 and 12ZB019), and the Scientific Research Foundation of CUIT (Nos.2010d1 and J201117).
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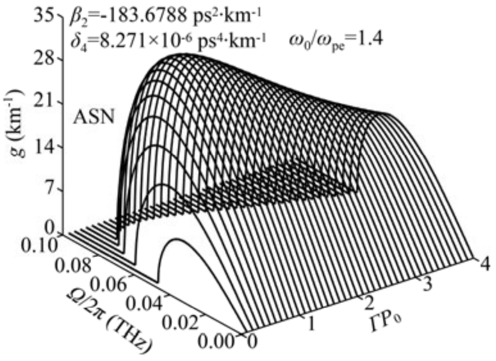
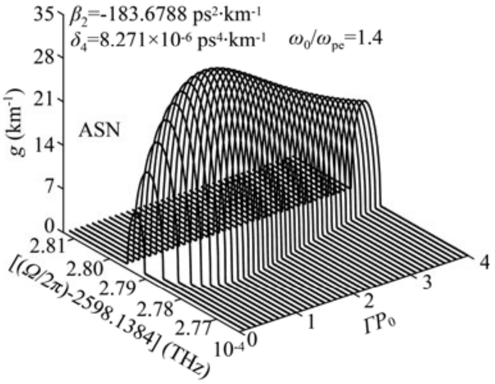


(a) The first spectral region close to the zero point

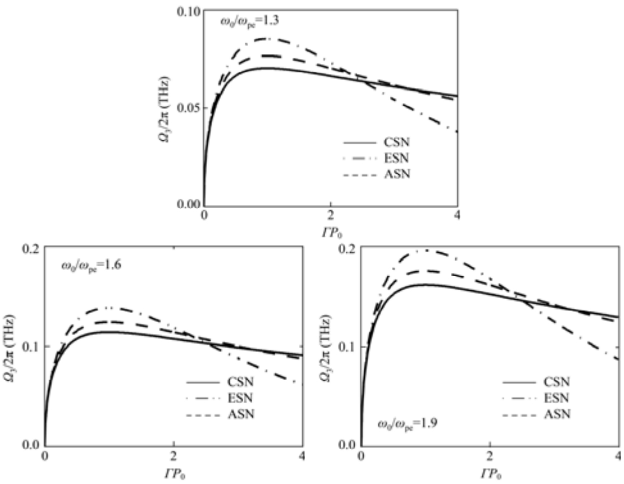


(b) The second spectral region far from the zero point

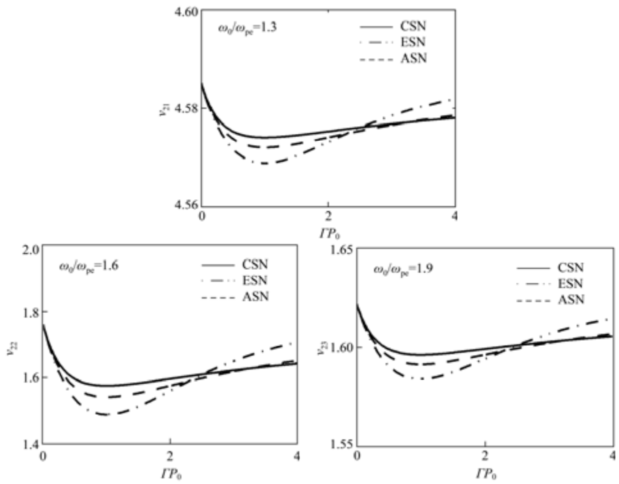
Fig.1 Variations of gain spectra of MI with ΓP_0 for ASN in the positive refractive regions of MMs when ω_0/ω_{pe} is fixed at 1.4

In fact, according to Fig.2 in Ref.[15], one can realize that in the positive refractive region, the sign combination of β_2 and δ_4 should be only $\beta_2<0$ and $\delta_4>0$. In this case, the gain spectra may theoretically consist of one spectral region of $0<\Omega^2<\Omega_1^2$ or two spectral regions of $0<\Omega^2<\Omega_3^2$ and $\Omega_2^2<\Omega^2<\Omega_1^2$. However, our detailed calculations here clearly indicate that the spectral region of $0<\Omega^2<\Omega_1^2$ does not exist in real cases. It can be seen obviously from Fig.1 that the peak gain g_m, the cut-off frequency Ω_3 and the spectral width Ω_1–Ω_2 of MI increase and then decrease with increase of the parameter ΓP_0. The definitions of the parameters β_2, δ_4, Ω_1, Ω_2 and Ω_3 mentioned above can also be found in Ref.[15]. These characteristics also can be shown for saturable nonlinearities of CSN and ESN only for different peak gains, cut-off frequencies and the spectral widths. It can be still realized that the gain spectra of MI and their variations with ΓP_0 for conventional materials and MMs with higher-order dispersion and saturable nonlinearity can be very similar to each other[3]. However, their spectral regions $\Omega_1^2<\Omega^2<\Omega_2^2$ [15] or $\Omega_2^2<\Omega^2<\Omega_1^2$ are quite different. In the MMs, the values of Ω_2 and Ω_1 are both very large, while the gain bandwidth $|\Omega_1–\Omega_2|$ is very small, which is obviously beneficial to the generation of pulse trains with high repetition rate. These characteristics can be seen more clearly in Fig.2.

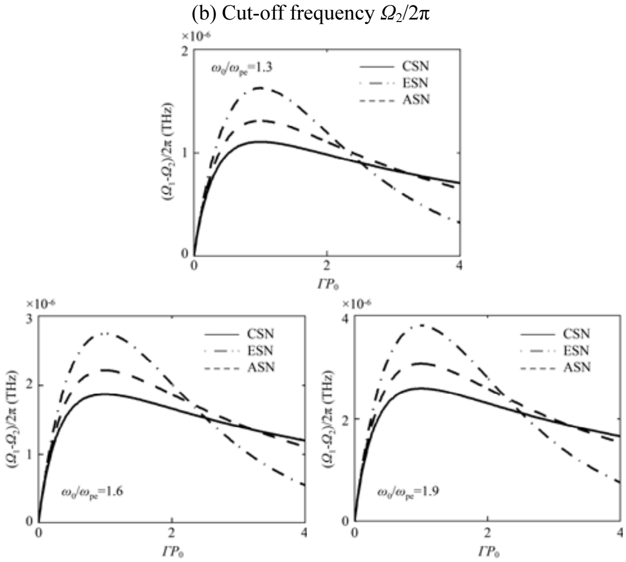
In Fig.2(b), the parameters of v_{21}, v_{22} and v_{23} stand for $(\Omega_2/2\pi–2237.8914)\times10^{-4}$ THz, $(\Omega_2/2\pi–3482.8727)\times10^{-5}$ THz and $(\Omega_2/2\pi–5094.2827)\times10^{-4}$ THz, respectively. Moreover, it can be shown from Fig.2 that when the other parameters are the same, ESN (CSN) has the largest (smallest) values of Ω_3, Ω_1–Ω_2 and g_m, but has the smallest (largest) value of Ω_2. In addition, the cut-off frequency, the spectral width and the peak gain vary more rapidly (slowly) for ESN (CSN). But the peak gains g_m for the two spectral regions are nearly the same.



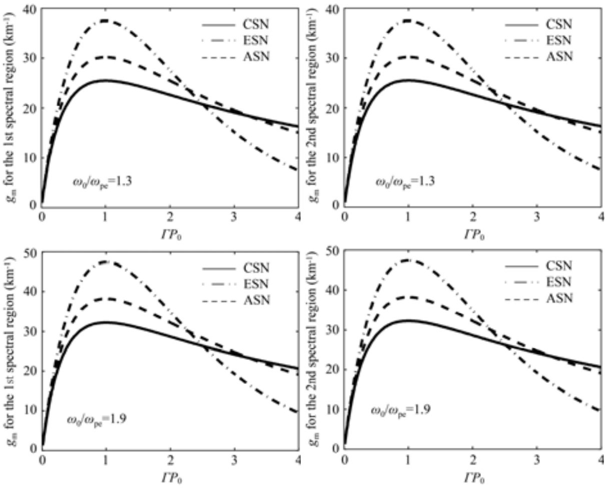
(a) Cut-off frequency $\Omega_3/2\pi$



(b) Cut-off frequency $\Omega_2/2\pi$



(c) Gain bandwidth $(\Omega_1–\Omega_2)/2\pi$

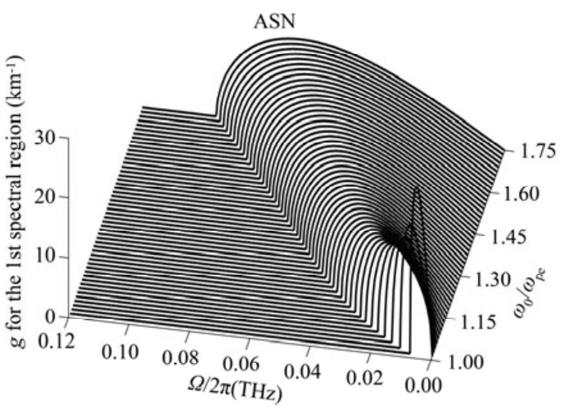


(d) Peak gain g_m

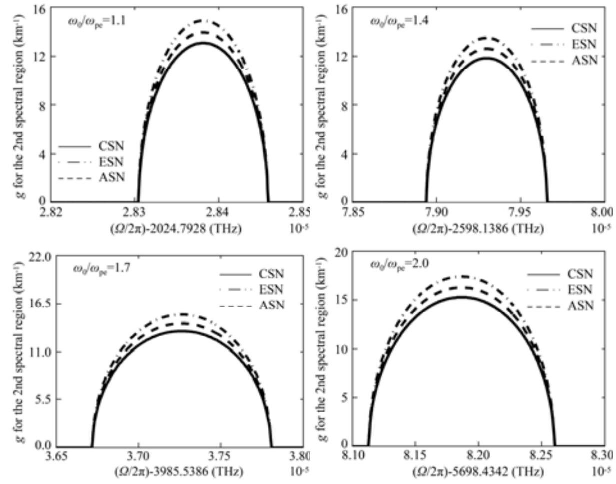
Fig.2 Variations of cut-off frequencies $\Omega_3/2\pi$ and $\Omega_2/2\pi$, gain bandwidth $(\Omega_1–\Omega_2)/2\pi$, and peak gain g_m with ΓP_0 for different normalized frequencies ω_0/ω_p for ESN, CSN and ASN

It is worth mentioning that compared with that of conventional materials, there is still another important novel feature for MI in MMs, that is the MI characteristics and their related applications can be modified by adjusting the structures and the normalized angular frequency of MMs. Thus, the variations of the first and the second gain spectral regions with the normalized frequencies ω_0/ω_{pe} are shown in Fig.3(a) and (b) for the fixed ΓP_0, respectively.

One can see from Fig.3 that both of the cut-off frequency Ω_3 for the first spectral region and the frequency of the second one increase with ω_0/ω_{pe}, while it is different from that for the peak gain. Thus, to observe it more clearly, we provide the variations of cut-off frequencies of $\Omega_3/2\pi$ and $\Omega_2/2\pi$, gain bandwidth $(\Omega_1–\Omega_2)/2\pi$ and peak gain g_m with ω_0/ω_{pe} for different ΓP_0 and three kinds of saturable nonlinearities, as shown in Fig.4. It is indicated that with the increase of ω_0/ω_{pe}, Ω_3 and $\Omega_1–\Omega_2$ increase, while Ω_2 and g_m decrease and then increase, which is a little different from that in the negative refractive region[15].



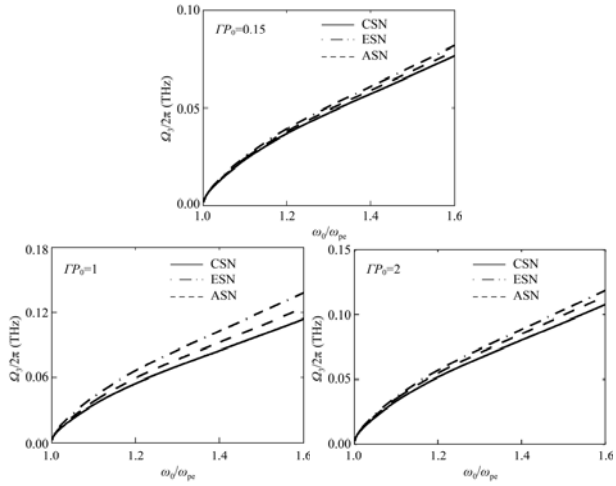
(a)



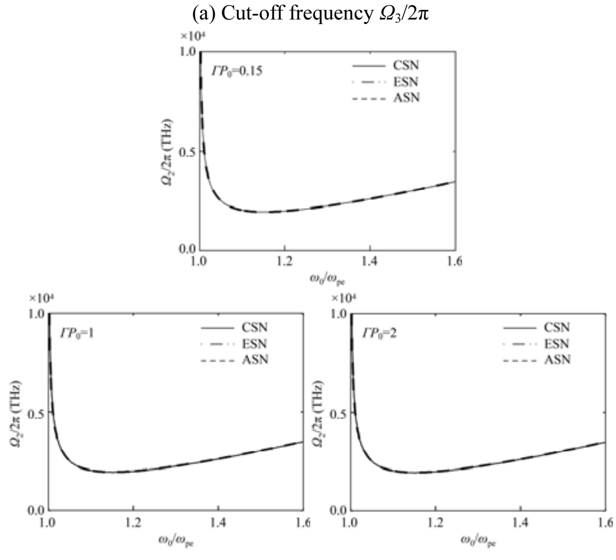
(b)

Fig.3 Variations of (a) the first and (b) the second gain spectral regions with the normalized frequency ω_0/ω_{pe} with the fixed ΓP_0 of 0.15

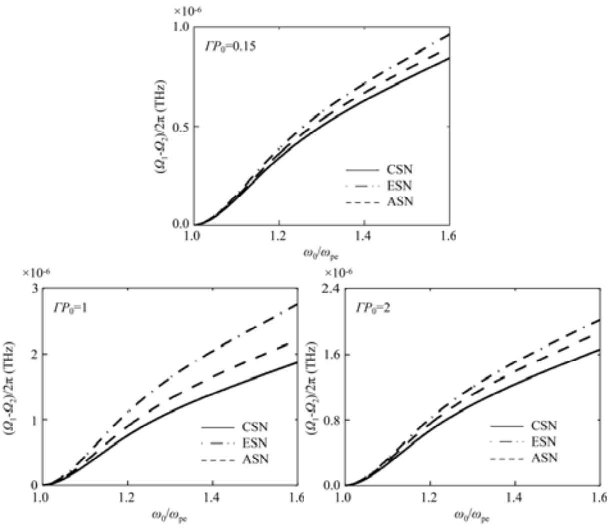
In summary, MI is investigated in the positive refractive regions of MMs with higher-order dispersion and three kinds of saturable nonlinearities. The results show



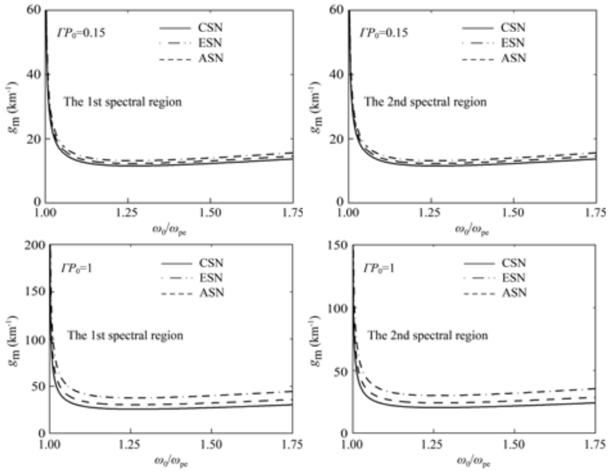
(a) Cut-off frequency $\Omega_3/2\pi$



(b) Cut-off frequency $\Omega_2/2\pi$



(c) Gain bandwidth $(\Omega_1-\Omega_2)/2\pi$



(d) Peak gain g_m

Fig.4 Variations of cut-off frequencies of $\Omega_3/2\pi$ and $\Omega_2/2\pi$, gain bandwidth $(\Omega_1-\Omega_2)/2\pi$ and peak gain g_m with ω_0/ω_{pe} for different ΓP_0 for ESN, CSN and ASN

that the gain spectra consist of two spectral regions instead of one or two regions in ordinary materials, one of which is close to and the other is far from the zero point. Especially, the spectral region far from the zero point also has high cut-off frequency but narrow spectral width, which is similar to the case in the negative refractive region, which is obviously beneficial to the generation of pulse trains with high repetition rate. Moreover, the gain spectra can vary with the normalized angular frequency, the normalized optical power and the form of the saturable nonlinearity. That is to say, the MI characteristics and MI related applications can be adjusted by adjusting the structures of the MMs, the form of the saturable nonlinearity and the normalized optical power.

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