Intensity modulation of a tightly focused partially coherent and radially polarized beam*

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Because of the circular symmetry of a completely coherent radially polarized beam, the azimuthal intensity component is zero when it is focused by a high numerical aperture (NA) objective. In this paper, we show that such a conclusion is not tenable under the illumination of partially coherent beam whose coherent property depends on the azimuthal angle. Taking the Gaussian Schell-model (GSM) beam for an example, the tight focusing property of a partially coherent beam is studied, and the intensity and its radial, azimuthal and longitudinal components are particularly researched. The results show that the percentage of the components on the total intensity depends on the correlation length of the partially coherent beam. The azimuthal intensity component is produced under the illumination of partially coherent beam.

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The focusing of laser beams using a high numerical aperture (NA) system, namely, tight focusing, has attracted much attention because of its peculiar properties, such as strong longitudinal component[1-3]. The longitudinal electric component at the focal point of a radially polarized beam (RPB) is larger than that of any other focused field with the same power[1]. Other interesting phenomena are observed in the tight focusing and vectorial structure of RPB[4,5]. The size of the focal spot of RPB can be significantly smaller than that of a linearly polarized beam when focused by a high NA objective[2], and it can be further reduced by introducing a parabolic mirror and a flat diffractive lens[6]. A super-resolved quasi-spherical focal spot is produced by focusing an amplitude-modulated RPB through a high NA objective[7]. A focal beam pattern with long depth and sub-wavelength full width at half maximum (FWHM), namely, an optical needle, can be obtained by using the binary optical elements of a radially polarized Bessel-Gaussian incident beam with a high NA lens[8,9]. A similar optical needle with long focal depth can be generated by focusing radially polarized light with a high NA lens and a diffractive optical element with belts[10]. Our recent study[11] showed that a spherical dark hole, super-long optical chain and dark channel can be obtained by the tight focusing of the radially polarized vortex beam. Multi-focus with subwavelength size[12,13] can be generated by appropriately adjusting the amplitude of the radially polarized beam. With these interesting properties, the tight focusing of RPB[14-18] is applied in particle acceleration, high-density data storage, high resolution microscopy, etc.

Most of previous researches on this subject have been restricted to completely coherent fields, disregarding more general case of the partially coherent fields. Under the illumination of a completely coherent RPB, the azimuthal intensity component in the focal field is zero, which is different from the result of a linearly polarized beam[19]. All three components in the focal field emerge when a linearly polarized beam is focused by a high NA objective. The disappearance of the azimuthal intensity component of RPB can be understood by the circular symmetry of the incident beam, which means that the incident beam is independent of the azimuthal angle. An interesting problem then arises that the azimuthal intensity component in the focal field is still zero if the incident beam depends on the azimuthal coordinate. In this paper, by taking the Gaussian Schell-model (GSM) beam for an example, we focus on the tight focusing of a partially coherent RPB. Particular interest is given to the evolution of the three components on the focal plane with different correlation properties and NAs.

Under the illumination of RPB, the Cartesian components of the electric field vector near the focus can be expressed as[19]:

$$\begin{bmatrix} E_x(\rho,\psi,z) \\ E_y(\rho,\psi,z) \\ E_z(\rho,\psi,z) \end{bmatrix} = \frac{-\mathrm{i}A}{\pi}\int_0^{\alpha}\int_0^{2\pi}\sin\theta\sqrt{\cos\theta}P(\theta,\varphi)\times$$
$$\exp[\mathrm{i}k(z\cos\theta+\rho\sin\theta\cos(\varphi-\psi))]\times$$

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$$\begin{bmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ \sin\theta \end{bmatrix} \mathrm{d}\varphi\mathrm{d}\theta \ , \tag{1}$$

and the radial and azimuthal components can be obtained by

$$E_\rho(\rho,\psi,z) = E_x(\rho,\psi,z)\cos\psi + E_y(\rho,\psi,z)\sin\psi \ , \tag{2}$$

$$E_\psi(\rho,\psi,z) = E_y(\rho,\psi,z)\cos\psi - E_x(\rho,\psi,z)\sin\psi \ , \tag{3}$$

where θ is the angle of convergence, $P(\theta, \varphi)$ is the pupil apodization function at the exit pupil, λ is the wavelength of the incident beam, $k=2\pi/\lambda$ is the wavenumber, and $\alpha=\sin^{-1}(NA)$ is the maximum angle determined by the NA of the objective. Variables of ρ, ψ and z are the cylindrical coordinates of an observation point near the focus. As shown in Fig.1, an incident beam is depolarized in the focal region when it is focused by a high NA objective, so that the radial, azimuthal and longitudinal components exist in the focal region. If the illumination depends only on the radial coordinate, the azimuthal component will be zero by performing the integration over φ.

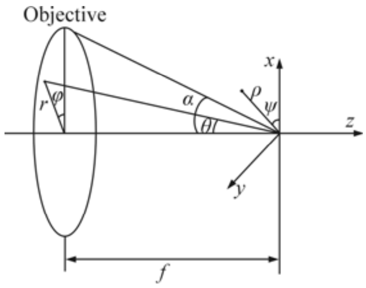


Fig.1 Schematic diagram of the tight focusing of laser beams

A typical example of a partially coherent beam whose correlation function depends on the azimuthal angle is the GSM beam. The cross-spectral function of GSM beam can be expressed as

$$W(\boldsymbol{r}_1,\boldsymbol{r}_2,z{=}0;w){=}\left\langle E^*(\boldsymbol{r}_1;w)E(\boldsymbol{r}_2;w)\right\rangle{=}$$
$$\exp\left[-\frac{\left(\boldsymbol{r}_1^2+\boldsymbol{r}_2^2\right)}{w^2}\right]\exp\left[-\frac{\left(\boldsymbol{r}_1\text{-}\boldsymbol{r}_2\right)^2}{\delta^2}\right], \tag{4}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the position vectors in the incident plane, $E(\boldsymbol{r})$ is a complex electric vector field at a point specified by the transverse position vector $\boldsymbol{r}$, the asterisk stands for the complex conjugate, the angle brackets denote ensemble averaging, w is the beam waist which is a constant, and δ is the correlation length.

As the objectives are often designed to obey the sine condition, we derive $r=f\sin\theta$, where f is the focal length of the high NA objective. Eq.(4) can be rewritten as

$$W(\theta_1,\varphi_1,\theta_2,\varphi_2,z{=}0;w) = \exp\left[-\frac{f^2\sin\theta_1^2+f^2\sin\theta_2^2}{w^2}\right]\times$$
$$\exp\left[-\frac{f^2\sin\theta_1^2+f^2\sin\theta_2^2-2f^2\sin\theta_1\sin\theta_2\cos(\varphi_1-\varphi_2)}{\delta^2}\right]. \tag{5}$$

The intensity and its components near the focus of a high NA objective can be calculated as[20]

$$I(\rho,\psi,z){=}I_\rho(\rho,\psi,z){+}I_\psi(\rho,\psi,z){+}I_z(\rho,\psi,z), \tag{6}$$

$$I_\rho(\rho,\psi,z) = \left\langle\left|E_\rho(\rho,\psi,z)\right|^2\right\rangle, \tag{7}$$

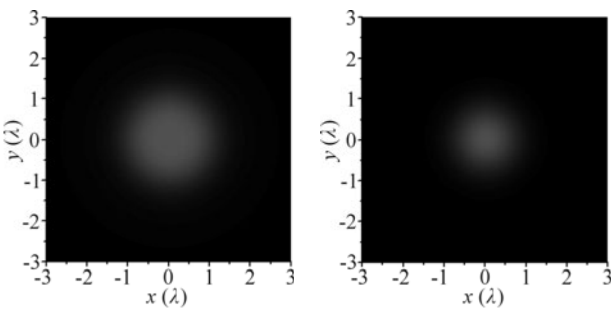
$$I_\psi(\rho,\psi,z) = \left\langle\left|E_\psi(\rho,\psi,z)\right|^2\right\rangle, \tag{8}$$

$$I_z(\rho,\psi,z) = \left\langle\left|E_z(\rho,\psi,z)\right|^2\right\rangle, \tag{9}$$

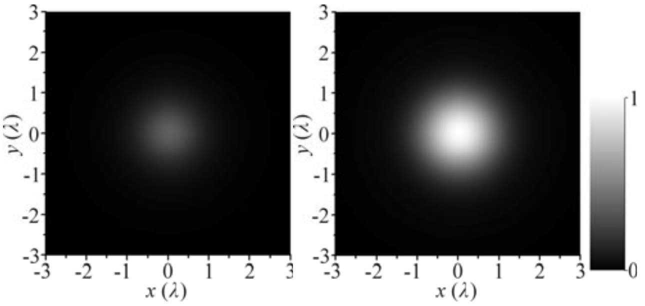
where $I_\rho(\rho,\psi,z)$, $I_\psi(\rho,\psi,z)$ and $I_z(\rho,\psi,z)$ represent the intensities of the radial, azimuthal and longitudinal components, respectively.

Unlike the tight focusing of a completely coherent RPB, the partially coherent RPB produces a focal region with an azimuthal intensity component. The magnitudes of the three components depend on the correlation property of the illumination. We use the radially polarized GSM beam with a correlation length of $\delta=0.5w$ focused by an objective with $NA=0.7$ as an example to study this effect. Fig.2 illustrates the intensity of the focal spot and its components on the focal plane. The result clearly shows the presence of the azimuthal intensity component. The numerical result also shows that the magnitude of the azimuthal intensity component at the focal point is nearly the same as that of the radial component, but the size of the beam spot is smaller. Especially, the focal pattern of the radial component in Fig.2(a) is different from the dark core pattern of a completely coherent beam in Ref.[19]. The radial intensity component of the partially coherent beam has a flap-top shape, and the size of the beam is larger. The difference is due to the influence of the correlation length of the incident beam. The size of the focal beam increases as the correlation length decreases. A bright annulus with a central dark core is produced in the radial intensity component under the illumination with high coherence, and the central dipped portion is gradually filled with diffused light as the correlation length decreases.

The correlation length of a partially coherent beam influences not only the radial and azimuthal components but also the longitudinal component. The percentages of the three components in the total intensity of the beam spot on focal plane versus the correlation length are presented in Fig.3.



(a) Radial component (b) Azimuthal component



(c) Longitudinal component (d) Total intensity

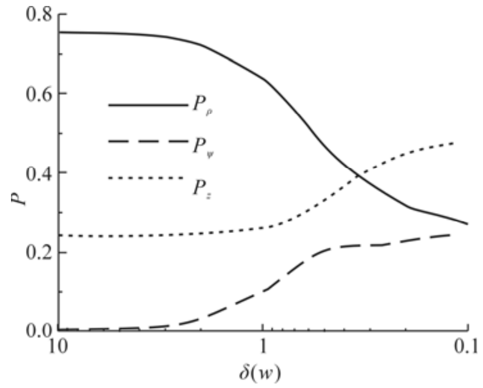
Fig.2 Intensity distributions of the GSM beam focused by an objective with NA of 0.7

The percentage of component in the total intensity (P) is defined as[10]

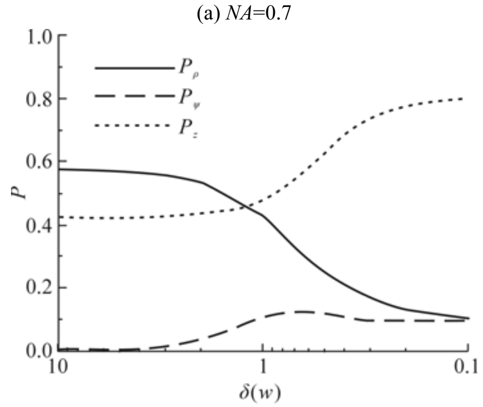
$$P_j = \frac{\phi_j}{\phi_\rho + \phi_\psi + \phi_z}, \tag{10}$$

where

$$\phi_j = 2\pi \int_0^\infty I_j(\rho, z)\rho \mathrm{d}\rho. \tag{11}$$



(a) NA=0.7

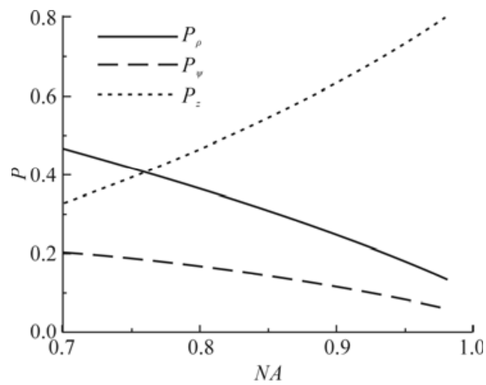


(b) NA=0.9

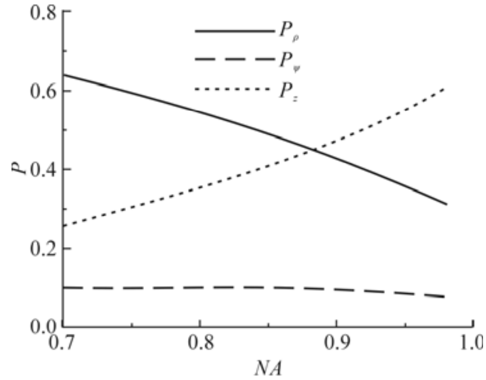
Fig.3 Percentages of the components in the total intensity of the beam on the focal plane with different NAs

The azimuthal intensity component is zero under the illumination with high coherence. It increases gradually and reaches the maximum, and then decreases when the correlation length decreases. The percentage can be larger than 20% when the beam is focused by an objective with NA of 0.7, indicating that the azimuthal component plays an important role in shaping the total intensity. The variation in the percentage of the longitudinal intensity component is contrary to that of the azimuthal intensity component. The longitudinal component decreases first and reaches the minimum, and then increases as the correlation length decreases. A stronger longitudinal component is useful in some applications, such as particle acceleration. Previous studies showed that the radially polarized beam has a stronger longitudinal component than a linearly polarized beam[1]. Our investigation shows that the radially polarized beam with lower coherence may have a stronger longitudinal component than the beam with higher coherence. However, it should be noticed that a lower coherence can lead to a larger focal spot.

The NA of the objective also influences the beam pattern on the focal plane. Generally, the larger the NA, the smaller the focal spot. The effect of NA on the percentages of the three components is shown in Fig.4. Under the illumination of the partially coherent beams with the correlation lengths of δ=0.5w and 1w, the radial and azimuthal components decrease, and the longitudinal component increases with the increase of NA. This result accords with the notion that the incident beam in the outer region of the pupil becomes more depolarized than that in the inner region. Therefore, the contribution of the longitudinal component to the total intensity increases significantly.



(a) δ=0.5w



(b) δ=1w

Fig.4 Percentages of the components in the total intensity of the beam on the focal plane with different correlation lengths

Based on Eq.(3), the calculation result of the azimuthal component is zero by performing the integration over the azimuthal angle φ when the incident beam is expressed as $P(\theta)$, i.e., the incident beam is independent of the azimuthal angle. The azimuthal component is generated only when the incident beam depends on the azimuthal angle. Under the illumination of the GSM beam with a sufficiently long correlation length, the magnitude of the coherence function is calculated as 1, indicating that the beam is independent of the azimuthal angle. Under the illumination of a partially coherent GSM beam, the correlation property depends on the azimuthal angle. The results shown in Fig.3 clearly show that the azimuthal intensity component is zero in the former case but non-zero in the latter case. Moreover, the azimuthal component is also zero under the illumination of a partially coherent beam whose correlation property is independent of the azimuthal angle.

In conclusion, the intensity and the radial, azimuthal and longitudinal components of a partially coherent GSM beam focused by a high NA objective are studied. Different from the completely coherent RPB whose azimuthal component is zero, the azimuthal component is produced under a radially polarized illumination depending on the azimuthal angle. The correlation length of the partially coherent GSM beam and the NA of the objective significantly influence the depolarization effect. Compared with the zero azimuthal component of a completely coherent beam, the azimuthal intensity component formed by a partially coherent radially polarized beam is zero with a sufficiently long correlation length. The value increases and reaches the maximum and then decreases with the reduction of correlation length. When the NA increases, the longitudinal component also increases, while the radial and azimuthal components decrease.

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