Analysis of an improved Lyot depolarizer in terms of the multi-beam superposition treatment*

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In order to study the depolarization properties of an improved Lyot depolarizer with monochromatic light, the theory of multi-beam superposition is adopted. The dependences of degree of depolarization (D) on vibration azimuth angle (VAA) and total retardation (TR) are analyzed. The results show that D is ideal for any VAA when TR is $\delta=(N+1/2)\pi$ (N is an integer) on the basis of wedge angle large enough. And when $\delta=N\pi$, VAA makes the most significant impact on D. When δ is assigned to the other values, the impact on D made by VAA is between the former two. Using a 405 nm semiconductor laser, experiments for measuring D of the sample with wedge angle of 6° are conducted. The theoretical results are well verified by experiments. D is over 98.8% when TR is nearly $\delta=(N+1/2)\pi$ by changing the incident angle.

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Chi[1] presented a quasi-monochromatic (±10 nm) depolarizer named the improved Lyot depolarizer. For treating the depolarizer, most cases use the Mueller calculus[2-4]. A few related papers[5-7] take an alternative description utilizing the contemporary achievements of the theory of coherence. However, such two operations are relatively complicated and abstract. In this paper, adopting the theory of multi-beam superposition, a new discussion of the improved Lyot depolarizer is given when it works for monochromatic light.

The improved Lyot depolarizer consists of two quartz wedges whose crystal axes are crossed at 45°. Fig.1 shows the structure of the improved Lyot depolarizer and the optical path of vertical incidence, where the wedge angle is α, and d_1, d_2 and d_3 are the lengths of the three parts divided by two dotted lines parallel to y direction. The left line with double-headed arrow (parallel to y) is the direction of crystal axis, and it is oriented at 45° to the right one in the xoz plane. The depolarizer made by quartz converts the incident linearly polarized light with arbitrary vibration azimuth angle (VAA) into four kinds of polarized light. And two of them are named oo and eo whose directions of vibration are perpendicular to the other two beams named ee and oe. The propagation direction of oo and ee is the same as that of incident light for the same refractive index in depolarizer. The refractive indices of eo light are n_e in the left wedge and n_o in the right wedge, where n_e and n_o are the ordinary and extraordinary indices of refraction, respectively. So eo light is tilted down passing through the wedge plane. On the contrary, oe light is tilted up.

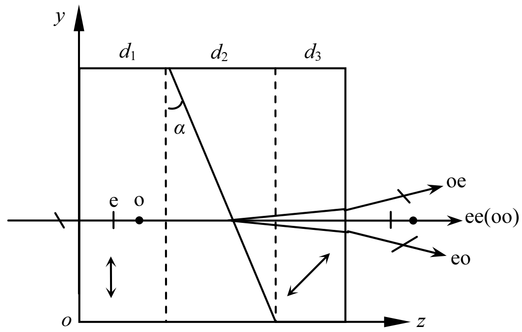

Fig.1 The improved Lyot depolarizer and optical path of vertically incident light

For mathematical convenience, we assume that the laser is expanded and collimated as uniform plane wave. Adopting the theory of multi-beam superposition, the dependences of degree of depolarization (D) on VAA and total retardation (TR) are analyzed. Firstly, suppose the initial phase of oo light is 0° at the plane of detector, and the output polarized light beams are not divided, so they are transmitted in a line. Thus, the initial phases of the output can be expressed as

$$\varphi_{oo}=0\ , \tag{1}$$

$$\varphi_{eo}=-ky+\frac{2\pi}{\lambda}(n_e-n_o)(d_1+d_2)\ , \tag{2}$$

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$$\varphi_{ee} = \frac{2\pi}{\lambda}(n_e - n_o)d = \delta, \tag{3}$$

$$\varphi_{oe} = ky + \frac{2\pi}{\lambda}(n_e - n_o)d_3, \tag{4}$$

where λ is the wavelength in vacuum, φ_{ee} is just TR, and the spatial change rate of phase (or phase difference) is

$$k = \frac{2\pi}{\lambda}(n_e - n_o)\tan\alpha. \tag{5}$$

k has little difference with its accurate value, and the difference is obtained by considering the declination of transmitted light. The suppositions of Eqs.(1)–(4) are feasible, because the factor affecting D is not the phase but the spatial change rate of phase difference.

The reflection losses of four light beams on the wedge plane are nearly the same when the wedge angle α is commonly only several degrees. Thus, neglecting the energy losses in depolarizer, the amplitudes of output light beams can be expressed as

$$A_{oo} = A\sin\theta\cos45^\circ = \frac{A}{\sqrt{2}}\sin\theta, \tag{6}$$

$$A_{eo} = A\cos\theta\sin45^\circ = \frac{A}{\sqrt{2}}\cos\theta, \tag{7}$$

$$A_{ee} = A\cos\theta\cos45^\circ = \frac{A}{\sqrt{2}}\cos\theta, \tag{8}$$

$$A_{oe} = A\sin\theta\sin45^\circ = \frac{A}{\sqrt{2}}\sin\theta, \tag{9}$$

where θ is VAA, that is the angle between the vibration direction of incident linearly polarized light and y direction.

The superpositions of oo and eo have the same frequency and vibration direction but different amplitude and initial phase. And it is the same with the superpositions of ee and oe. The amplitudes and initial phases of two superposed light beams can be expressed separately as

$$A_o^2 = \frac{A^2}{2}\left[1+\cos(ky+\delta'-\delta)\sin2\theta\right], \tag{10}$$

$$A_e^2 = \frac{A^2}{2}\left[1+\cos(ky+\delta'-\delta)\sin2\theta\right], \tag{11}$$

$$\tan\varphi_o = \frac{-\sin(ky+\delta'-\delta)}{\tan\theta+\cos(ky+\delta'-\delta)}, \tag{12}$$

$$\tan\varphi_e = \frac{\sin\delta+\tan\theta\sin(ky+\delta')}{\cos\delta+\tan\theta\cos(ky+\delta')}, \tag{13}$$

where $\delta' = \frac{2\pi}{\lambda}(n_e - n_o)d_3$. The expression of δ' changes with different coordinate systems, but it has no effect on the next analysis.

From Eqs.(10) and (11), the total intensity of transmitted light beams can be expressed by

$$I = A^2\left[1+\cos(ky+\delta'-\delta)\sin2\theta\right]. \tag{14}$$

From Eqs.(12) and (13), the initial phase difference between the two superposed light beams can be expressed by

$$\Delta\varphi = \varphi_e - \varphi_o = ky + \delta'. \tag{15}$$

As far as laser is concerned, the initial phase difference is just the phase difference.

As a pseudo-depolarizer, the improved Lyot depolarizer transforms the arbitrary incident polarization state into continuous state in the space across the exit pupil. And D is dependent on the average effect of output polarization states. The higher the spatial change rate k, the better D, meaning that a high D can be achieved by a wedge angle high enough from Eq.(5). However, depolarizer with too big wedge angle is not suitable for imaging system, because it severely degrades imaging. Moreover, experiments discover the saturability of D for α. As the important premise of the next analyses, we can select an α just high enough for ideal D.

From calculations above, the superposition of output is varying elliptically polarized light when incident linearly polarized light is perpendicular to depolarizer. From Eq.(14), the intensity of elliptically polarized light changes periodically with y. The variation range is dependent on θ. The maximum I increases from 0 to $2A^2$ as θ is 45°, I is always A^2 as θ is 0° or 90°, and it is somewhere in between as θ chooses other values. From Eq.(15), the phase difference, which determines the polarization states, varies periodically with y of the same cycle as that of Eq.(14). Comprising Eqs.(14) and (15), their different variation relations are dependent on δ. Consequently, the dependences of I and polarization state on θ and δ will be analyzed.

In Fig.2(a), the output intensity I doesn't vary with phase difference or y. And polarization states change periodically with phase difference or y, which is the same as the other three cases in Fig.2. Thus, an ideal D can be achieved when θ is 0° or 90°, because the average effect of output polarization states results in the intensity balance between every two vibration directions.

In Fig.2(b), according to a cosine curve, the output intensity I varies with phase difference or y as θ is 45°. In the interval [0, 2π] (one cycle), intensity of one circularly polarized beam is $2A^2$ as phase difference is π/2, and intensity of another is 0 as phase difference is 3π/2. Although there is an enormous difference of intensity between the two circularly polarized beams, it leads to no loss of D for the intensity balance of circularly polarized beams. There are two linearly polarized beams whose intensities are both A^2 as phase differences are 0 and π, respectively. Fortunately, the vibration directions of two linearly polarized beams are orthogonal, and A_o is equal to A_e. Thus the two linearly polarized beams have ideal D. And it is the same for the polarization states which are axisymmetric when phase difference is π/2 or 3π/2. So D is ideal as θ is 45° and δ is $(N+1/2)\pi$.

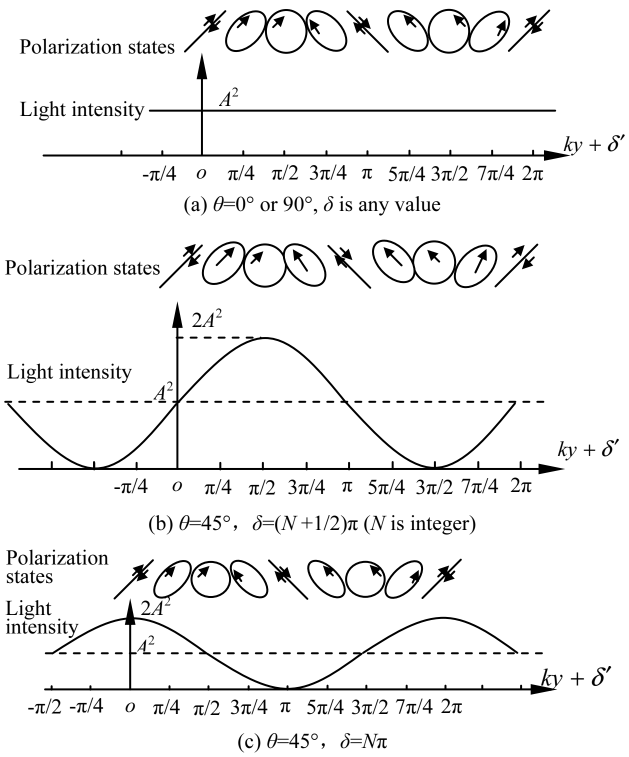


Fig.2 The output polarization states and transmitted light intensities changing with phase difference (or *y*)

Different from Fig.2(b), δ is $N\pi$ in Fig.2(c), which leads to the opposite result. The intensities of two circularly polarized beams are both A^2. The intensity of one linearly polarized beam is $2A^2$ as phase difference is 0 or 2π, and that of another linearly polarized beam is 0 as phase difference is π. The enormous difference of intensities of the two linearly polarized beams leads to degradation of D. And it is the same for the other symmetric polarization states. So D is very poor as θ is 45° and δ is $N\pi$.

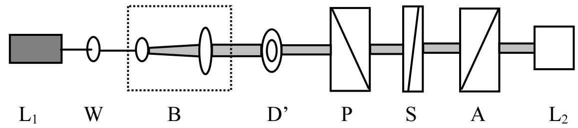
According to the analyses above, D is always ideal independent of δ as θ is 0° or 90°, and D generally increases with θ varing from 45° to 0° or 90°. However, D is always ideal independent of θ as δ is $(N+1/2)\pi$.

The sample with the thickness of 8 mm and the area of 12 mm×12 mm is made of quartz with wedge angle of 6°. The light path for measuring D[8-11] is shown in Fig.3, in which laser is a 405 nm semiconductor laser, $\lambda/4$ wave plate transforms the polarization state of light source from linearly polarized light to circularly polarized light, battery of lens can accomplish the alignment and expansion of incident light, circular diaphragm is to control the radius of pupil, rotatable polarizer outputs linearly polarized light with varying θ, and analyzer achieves the minimum and maximum intensities by revolving. As D is affected significantly by phase difference, light must be vertically incident, accomplished by self-alignment of light path. The degree of polarization can be expressed by

$$P=\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}, \tag{16}$$

where $I_{\min}$ and $I_{\max}$ are the minimum and maximum intensities of transmitted light, respectively. And D can be expressed as

$$D=1-P=\frac{2I_{\min}}{I_{\max}+I_{\min}} . \tag{17}$$



L1: Laser; W: $\lambda/4$ wave plate; B: Battery of lens; D': Diaphragm; P: Polarizer; S: Sample; A: Analyzer; L2: Light power meter

Fig.3 Test system for *D* of the improved Lyot depolarizer

We select the reliable values to calculate the average $I_{\min}$ and $I_{\max}$ which are measured repeatedly for the diminution of error caused by the fluctuation of light source. In Fig.4, the curve is in full accord with the analyses above. The lowest D is 84.27% as θ is 45°, and it is about 98.4% as θ is 0° or 90°. The wedge angle is not big enough for more ideal D, so the highest values are a bit smaller than the ideal value of 100%.

In Fig.4, it is obvious that the TR of the sample (δ) is not equal to $(N+1/2)\pi$. It is feasible to adjust δ to be $(N+1/2)\pi$ by oblique incidence. As shown in Tab.1, D is over 98.8% as incident angle is 5.2°. Corresponding to the theoretical analysis, the sample is very close to an ideal depolarizer as δ is equal to $(N+1/2)\pi$.

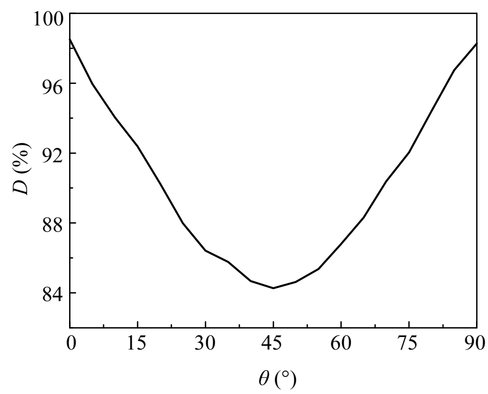


Fig.4 *D* as a function of VAA under vertical incidence

Tab.1 Test results for *D* as incident angle is 5.2°

VAA (θ)	0°	10°	30°	45°	60°	70°	80°	90°
D	98.82%	98.92%	99.63%	99.19%	99.14%	99.05%	99.03%	98.88%

In conclusion, an improved Lyot depolarizer is analyzed for D of linearly polarized monochromatic light, adopting the theory of multi-beam superposition. It is theoretically and experimentally shown that the dependence of D on VAA is affected by the TR. D changes significantly with VAA when TR is close to $N\pi$. And D is ideal value and independent of VAA when TR is $(N+1/2)\pi$. Though the TR of sample is not $(N+1/2)\pi$, we adjust the incident angle on the sample to be 5.2°, and then anticipated values are achieved.

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