Construction of a new regular LDPC code for optical transmission systems*

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A novel construction method of the check matrix for the regular low density parity check (LDPC) code is proposed. The novel regular systematically constructed Gallager (SCG)-LDPC(3969,3720) code with the code rate of 93.7% and the redundancy of 6.69% is constructed. The simulation results show that the net coding gain (NCG) and the distance from the Shannon limit of the novel SCG-LDPC(3969,3720) code can respectively be improved by about 1.93 dB and 0.98 dB at the bit error rate (BER) of 10^{-8}, compared with those of the classic RS(255,239) code in ITU-T G.975 recommendation and the LDPC(32640,30592) code in ITU-T G.975.1 recommendation with the same code rate of 93.7% and the same redundancy of 6.69%. Therefore, the proposed novel regular SCG-LDPC(3969,3720) code has excellent performance, and is more suitable for high-speed long-haul optical transmission systems.
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The development of the super forward error correction (super-FEC) code with more powerful error correction performance has become a research hot in optical transmission systems[1-4]. The low-density parity-check (LDPC) code is a kind of linear block codes, which can approach the Shannon limit, and has the advantage of reducing the decoding complexity for hardware implementation[5-9].

According to the check matrix $\boldsymbol{H}$, the LDPC code is classified into the regular LDPC code and the irregular LDPC code[10]. For optical transmission systems, the optical channel is a high signal-to-noise ratio (SNR) channel and requires low output bit error rate (BER), so it is not convenient to directly use the irregular LDPC code as the super-FEC code[10]. It can be considered to reform the irregular LDPC code generally by concatenating with other codes to eliminate or reduce the error floor. However, it can increase the encoding/decoding complexity and bring about more time delay arisen from the encoding/decoding[10]. As an alternative, the regular LDPC code can attain very low or even no error floor through appropriate design[5-8], and is easier for the implementation of the encoding/decoding[6-10] compared with the irregular LDPC code. As a result, the regular LDPC code is more preferred in the applications of optical transmission systems.

In view of the above reasons, a novel regular LDPC code is constructed and studied in this paper. In addition, for further reducing the implementation complexity, the regular LDPC code is constructed in the GF(2) field. The novel regular SCG-LDPC(3969,3720) code is constructed and compared with other two codes, and the error-correction performance is simulated and analyzed.

The encoder/decoder of the regular LDPC code are based on the sparse check matrix $\boldsymbol{H}_{m\times n}$ as

$$\boldsymbol{H}=\begin{bmatrix}1&0&0&1&0&1&1&0&0&0\\0&1&0&0&0&1&0&0&1&1\\0&1&1&0&1&0&0&0&0&1\\0&0&1&1&0&0&1&1&0&0\\1&0&0&0&1&0&0&1&1&0\end{bmatrix}. \quad (1)$$

The random arrangement of non-zero elements and the sparseness of the matrix have direct impact on the encoding/decoding complexity and error correction performance[6,10]. Fig.1 shows the corresponding bipartite graph of the check matrix $\boldsymbol{H}$ for a regular LDPC code in the GF(2) field.

For the above construction requirements of the regular LDPC code's sparse check matrix, the further explanation is made as follows: ①The distribution of "1" should have randomness and be as sparse as possible under the conditions of construction; ②The number of "1" in each row and column should satisfy certain conditions. For the regular code, the number of "1" is fixed; ③The girth-4 phenomenon cannot exist. That is to say the number of

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"1" in the same position of any two rows or columns can not be more than one. If the girth-4 phenomenon exists, in terms of the sparse check matrix, it may not be full rank; ④Because the inverse matrix is used in the process of encoding, if the full rank cannot be insured in a part of the sparse check matrix, it would have a great effect on the encoding process, even the unable encoding would occur. When the code length is shorter, this step is easy to guarantee; but when the code length is grown by several magnitudes, it's very difficult to achieve this goal. In summary, this is a crucial step for the construction of the sparse check matrix.

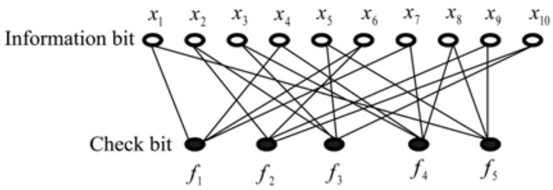

Fig.1 The corresponding bipartite graph of the check matrix *H* for a regular LDPC code in the GF(2) field

From the construction principles of the bipartite graph for the regular LDPC code, it can be known that the shortest girth number in the bipartite graph is the girth-4. The girth number of the LDPC code has great negative effects on its decoding, and especially the decoding performance can be seriously degraded under the condition of the short girth[11]. Through analyzing the noise characteristics of optical transmission systems, the construction requirements of the check matrix for the regular LDPC code and the development trend of optical transmission systems, as well as considering the theoretical analysis for the high bit-rate regular LDPC code and the special characteristics of the regular LDPC code itself, it can be concluded that the five construction methods of the regular LDPC code can be summarized as follows[4-7,12]: ①There should be a low error floor or no error floor; ② The net coding gain (NCG) should be high, and the redundancy of the regular LDPC code should be low; ③ The codeword length cannot be too long, the time delay arisen from encoding/decoding should not be too much, and the software/hardware implementation should be favorable; ④ The constructed regular LDPC codes should have no girth-4 to meet the requirements of the Steiner limit, so the decoding of the regular LDPC code has better decoding constringency; ⑤The constructed regular LDPC code should have lower density, that is to say, in the check matrix ***H*** of the regular LDPC code, the number of "1" should be absolutely smaller than that of "0". Thus, there will be less calculation each time when iteratively decoding, and the decoding complexity will be reduced.

According to the requirements of the above construction methods for the regular LDPC code, a novel check matrix of the SCG(4,5) code for optical transmission system is constructed as shown in Fig.2, which is based on the construction method of the check matrix in Ref.[9] and the systematically constructed Gallager (SCG) random construction method proposed by Daniel Hösli and Erik Svensson[13]. This novel check matrix is a check matrix of the regular LDPC code with the column number of N=30 and the row number of M=23. It can directly be seen that there is no girth-4 phenomenon in Fig.2, and the conclusion of no girth-4 phenomenon can also be proved by the girth-4 check program written with MATLAB. From Fig.2, it's not difficult to find the conclusion that the fourth submatrix of the check matrix for the above constructed SCG(4,5) code consists of all the phalanxes of the third submatrix to be re-adjusted in the arrangement order. While in the check matrix of the SCG(4,5) code proposed by Daniel Hösli and Erik Svensson, the phalanxes of the third and fourth submatrices are not identical. Therefore, the construction method of the SCG(4,5) code can save storage space of hardware and reduce the calculation complexity in the future hardware implementation.

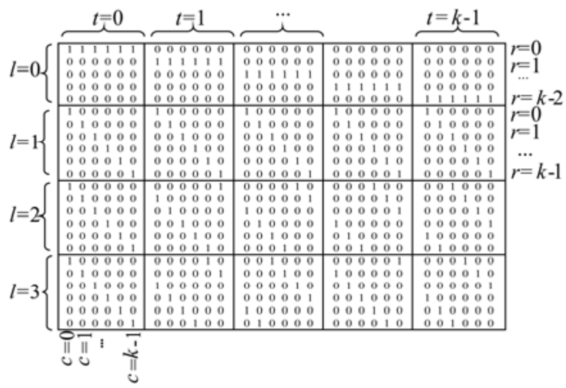

Fig.2 The novel check matrix of the SCG(4,5) regular LDPC code with *N*=30 and *M*=23 constructed in this paper

The regular LDPC code constructed by the SCG construction method is called as SCG(j,k) code[13], and it is also named as the regular SCG-LDPC code in order to be better combined with the relative characteristics of the LDPC code in this paper.

As known from the theoretical analysis of the LDPC code: the LDPC code is more easily constructed in GF(2) field, and has the advantage of reducing encoding/decoding complexity compared with those not constructed in GF(2) field. The corresponding modulation in GF(2) field is the binary phase shift keying (BPSK), and the LDPC code is applied in optical transmission system whose channel is approximately taken as additive white Gaussian noise (AWGN). Therefore, the basic simulation environment including GF(2) field, BPSK modulation and AWGN channel is used in this paper. The source code of the check matrix for SCG(4,k) code using the construction algorithm proposed in this paper is compiled by MATLAB. Considering the characteristics of the optical transmission system and the analysis of the LDPC code pattern, the parameters of j=4 and k=63 are emphatically selected to construct the check matrix of SCG-LDPC code. The reason for this consideration is that the LDPC code constructed by the above parameters has the same code rate and redundancy with the classic RS(255,239) code in ITU-T G.975[14] recommendation and the LDPC(32640,30592) code in ITU-T G.975.1[15] recommendation, so the comparative analyses are even more meaningful. Since the check matrix constructed by this method is not full rank, the indirect encoding method through the check matrix is used. In the process, the key consideration is that the parameters after check matrix ***H*** are converted into generator matrix ***G***. These parameters are the actual parameters of the final LDPC code, and the numbers of columns and rows for the generator matrix correspond to the code length and information bit of the LDPC code. The column weight and row weight of the check matrix constructed by the above parameters are the fixed values, namely j=4 and k=63. So the constructed LDPC code is the regular SCG-LDPC (3969,3720) code with the code rate of 93.7% and the redundancy of 6.69%, which meet the high speed and low redundancy requirements of the LDPC code for optical communication and don't have girth-4 phenomenon by the girth-4 check test. In addition, this regular LDPC code can save the storage space of hardware, and relatively reduce the calculation complexity in the future hardware implementations. As a result, it is more suitable for the regular LDPC code in the optical transmission systems.

The relevant curve between the bit error rate (BER) and SNR can be achieved through the MATLAB programming simulation of the regular SCG-LDPC(3969, 3720) code in the AWGN channel. The simulation result applying the sum-product-algorithm decoding at 18 iterations is shown in Fig.3, and the comparative analyses of the error-correction performance of the novel constructed regular SCG-LDPC(3969,3720) code, the RS(255,239) code and the LDPC(32640,30592) code are performed.

From Fig.3, it can be seen that the NCGs of the regular SCG-LDPC(3969,3720) code, RS(255,239) code and LDPC (32640,30592) code are about 5.38 dB, 3.44 dB and 4.15 dB, and their distances from the Shannon limit are about 1.32 dB, 3.26 dB and 2.55 dB at BER of 10^{-6}, respectively. The NCGs of the regular SCG-LDPC(3969,3720) code, RS(255,239) code and LDPC(32640,30592) code are about 6.30 dB, 4.38 dB and 5.33 dB, and their distances from the Shannon limit are about 1.80 dB, 3.72 dB and 2.77 dB at BER of 10^{-8}, respectively. The comparative data among the regular SCG-LDPC(3969,3720) code, RS(255,239) code and LDPC(32640,30592) code are listed in Tab.1 in detail.

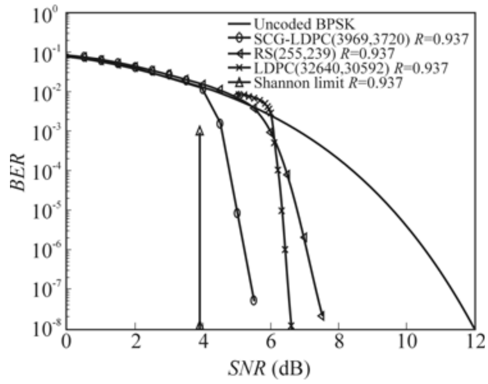

Fig.3 BER curves of the regular SCG-LDPC(3969,3720) code, RS(255,239) code and LDPC(32640,30592) code

Tab.1 Error-correction performance of the regular SCG-LDPC(3969,3720) code, RS(255,239) code and LDPC (32640,30592) code

Code type	Code rate	Redundancy	BER =10^{-6}		BER =10^{-8}	
			NCG (dB)	Distance from the Shannon limit (dB)	NCG (dB)	Distance from the Shannon limit (dB)
SCG-LDPC (3969,3720)	93.7%	6.69%	5.38	1.33	6.30	1.80
RS(255,239)	93.7%	6.69%	3.44	3.26	4.38	3.72
LDPC (32640,30592)	93.7%	6.69%	4.15	2.55	5.33	2.77

The analyses from Tab.1 show that both the NCG and the distance from the Shannon limit of the novel SCG-LDPC (3969,3720) code can be respectively improved by about 1.95 dB and 1.24 dB at the BER of 10^{-6}, compared with those of the classic RS(255,239) code and the LDPC(32640,30592) code. Meanwhile, the NCG and the distance from the Shannon limit of the novel SCG-LDPC (3969,3720) code can be respectively improved by about 1.93 dB and 0.98 dB at the BER of 10^{-8}, compared with those of the classic RS (255,239) code and the LDPC(32640,30592) code. Therefore, the error-correction performance of the novel regular SCG-LDPC(3969,3720) code is better than that of the classic RS(255,239) code in ITU-T G.975 recommendation and the LDPC(32640,30592) code in ITU-TG.975.1 recommendation with the same code rate of 93.7% and redundancy of 6.69%.

The construction methods of the regular LDPC code for optical transmission systems are analyzed and compared in this paper. A novel regular SCG-LDPC(3969, 3720) code with the code rate of 93.7% and the redundancy of 6.69% is constructed based on the construction methods. The novel regular SCG-LDPC(3969,3720) code can save the storage space and reduce the calculation complexity in the future hardware implementations. The simulation results show that the error-correction performance of the novel SCG-LDPC(3969,3720) code is excellent, and it is more suitable for high-speed long-haul optical transmission systems.

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