Effect of SGC on transient evolution of GWI in a Doppler broadened quasi Λ-type four-level system*

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In this paper, we study the effect of spontaneously generated coherence (SGC) on transient evolution of gain without inversion (GWI) in a Doppler broadened quasi Λ-type four-level atomic system. It is shown that transient evolution of GWI is very sensitive to the variation of SGC strength, and the transient maximum value and steady value of GWI both increase with SGC strength increasing. The transient and steady values of GWI with SGC are much larger than those without SGC. When Doppler broadening is present, the transient maximum value and steady value of GWI first increase and then decrease with Doppler broadening width (D) increasing, and the value of D which corresponds to the maximum transient GWI is different from that corresponding to the maximum steady GWI. The time needed for reaching the steady GWI increases with D increasing. The steady GWI, which is larger than that without Doppler broadening (D = 0), can be obtained by choosing appropriate D and SGC strength.
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The generation of lasing without inversion (LWI) is the result of quantum coherence and interference in an atomic system. The study of LWI has attracted tremendous attention[1-4], because this study has an important scientific sense and potential wide applications. There are many ways to generate quantum coherence and interference. Generally, they can be realized by coherent driving fields or by initial coherence injections, and the interference between different spontaneous emission paths also can lead to coherence, which is usually called as spontaneously generated coherence (SGC). At present, the effects of SGC on optical properties and phenomena (including LWI) in an atomic system have been widely investigated[5-9]. However, due to the complexity of mathematical treatment, the studies about effects of SGC are most related to the static three-level atomic systems without Doppler broadening, and there are only a few papers considering more complex atomic system with four levels without Doppler broadening. In fact, many experimental researches on optical phenomena, such as LWI, are made in an atomic vapor, in which there is very significant Doppler broadening. The results show that Doppler broadening has an obvious effect on LWI gain (also called as gain without inversion (GWI)) in an atomic system[10-12]. It is an important project in the LWI study that how to enhance GWI. It is necessary to study the transient evolution of atomic response in order to understand the dynamic process of producing LWI and find the way of increasing GWI. So far, all the investigations about transient evolution of GWI[13-15] were made for the atomic system without Doppler broadening. In this paper, we explore the effects of SGC and Doppler broadening on transient evolution of GWI in a quasi Λ-type four-level atomic system, and find the way to get larger GWI.

The quasi Λ-type four-level atomic system considered here is shown in Fig.1. The transition of $|2\rangle \rightarrow |3\rangle$ with frequency ω_{23} is coupled by a strong driving field with frequency of ω_c and Rabi frequency of $\Omega_c = \boldsymbol{\mu}_{32} \cdot \boldsymbol{E}_c/(2\hbar)$, while the transition of $|1\rangle \rightarrow |3\rangle$ with frequency ω_{13} is coupled by a weak probe field with frequency of ω_p and Rabi frequency of $\Omega_p = \boldsymbol{\mu}_{31} \cdot \boldsymbol{E}_p/(2\hbar)$. A coherent pump field with frequency of ω_s and Rabi frequency of $\Omega_s = \boldsymbol{\mu}_{41} \cdot \boldsymbol{E}_s/(2\hbar)$ is applied between levels $|1\rangle$ and $|4\rangle$.

Using the rotating-wave and electric dipole approximations, the density matrix equations of motion of this system with SGC can be written as[12]

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$$\dot{\rho}_{11}=\gamma_{31}\rho_{33}+\gamma_{41}\rho_{44}+\mathrm{i}\Omega_{\mathrm{p}}^{*}\rho_{31}-\mathrm{i}\Omega_{\mathrm{p}}\rho_{13}+\mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{41}-\mathrm{i}\Omega_{\mathrm{s}}\rho_{14}\ , \tag{1}$$

$$\dot{\rho}_{22}=\gamma_{32}\rho_{33}+\gamma_{42}\rho_{44}+\mathrm{i}\Omega_{\mathrm{c}}^{*}\rho_{32}-\mathrm{i}\Omega_{\mathrm{c}}\rho_{23}\ , \tag{2}$$

$$\dot{\rho}_{33}=-(\gamma_{31}+\gamma_{32})\rho_{33}+\mathrm{i}\Omega_{\mathrm{p}}\rho_{13}-\mathrm{i}\Omega_{\mathrm{p}}^{*}\rho_{31}+\mathrm{i}\Omega_{\mathrm{c}}\rho_{23}-\mathrm{i}\Omega_{\mathrm{c}}^{*}\rho_{32}\ , \tag{3}$$

$$\dot{\rho}_{44}=-(\gamma_{41}+\gamma_{42})\rho_{44}+\mathrm{i}\Omega_{\mathrm{s}}\rho_{14}-\mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{41}\ , \tag{4}$$

$$\dot{\rho}_{12}=\mathrm{i}(\Delta_{\mathrm{p}}-\Delta_{\mathrm{c}})\rho_{12}+\mathrm{i}\Omega_{\mathrm{p}}^{*}\rho_{32}+\mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{42}-\mathrm{i}\Omega_{\mathrm{c}}\rho_{13}+\sqrt{\gamma_{31}\gamma_{32}}\,p_{1}\rho_{33}+\sqrt{\gamma_{41}\gamma_{42}}\,p_{2}\rho_{44}\ , \tag{5}$$

$$\dot{\rho}_{13}=-[(\gamma_{31}+\gamma_{32})/2-\mathrm{i}\Delta_{\mathrm{p}}]\rho_{13}+\mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{43}-\mathrm{i}\Omega_{\mathrm{c}}^{*}\rho_{12}+\mathrm{i}\Omega_{\mathrm{p}}^{*}(\rho_{33}-\rho_{11})\ , \tag{6}$$

$$\dot{\rho}_{14}=-[(\gamma_{41}+\gamma_{42})/2-\mathrm{i}\Delta_{\mathrm{s}}]\rho_{14}+\mathrm{i}\Omega_{\mathrm{p}}^{*}\rho_{34}+\mathrm{i}\Omega_{\mathrm{s}}^{*}(\rho_{44}-\rho_{11})\ , \tag{7}$$

$$\dot{\rho}_{23}=-[(\gamma_{31}+\gamma_{32})/2-\mathrm{i}\Delta_{\mathrm{c}}]\rho_{23}-\mathrm{i}\Omega_{\mathrm{p}}^{*}\rho_{21}+\mathrm{i}\Omega_{\mathrm{c}}^{*}(\rho_{33}-\rho_{22})\ , \tag{8}$$

$$\dot{\rho}_{24}=-[(\gamma_{41}+\gamma_{42})/2-\mathrm{i}(\Delta_{\mathrm{s}}-\Delta_{\mathrm{p}}+\Delta_{\mathrm{c}})]\rho_{24}+\mathrm{i}\Omega_{\mathrm{c}}^{*}\rho_{34}-\mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{21}\ , \tag{9}$$

$$\dot{\rho}_{34}=-[(\gamma_{31}+\gamma_{32}+\gamma_{41}+\gamma_{42})/2-\mathrm{i}(\Delta_{\mathrm{s}}-\Delta_{\mathrm{p}})]\rho_{34}+\mathrm{i}\Omega_{\mathrm{p}}\rho_{14}+\mathrm{i}\Omega_{\mathrm{c}}\rho_{24}-\mathrm{i}\Omega_{\mathrm{s}}^{*}\rho_{31}\ . \tag{10}$$

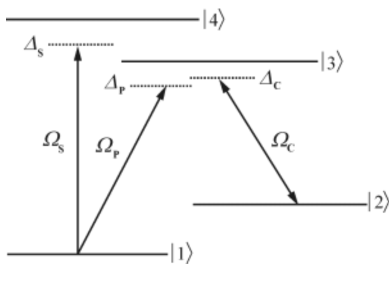

Fig.1 Quasi Λ-type four-level system

The above equations are constrained by $\rho_{mn}^{*}=\rho_{nm}$ and $\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44}=1$, where ρ_{mm} is the atomic population of state $|m\rangle$, and ρ_{mn} ($m\neq n$) is the atomic polarization between states $|m\rangle$ and $|n\rangle$. γ_{mn} represents the spontaneous decay rate from level $|m\rangle$ to level $|n\rangle$, and in Eqs.(1)-(10), we neglected the spontaneous decay rate of γ_{21} which is from level $|2\rangle$ to level $|1\rangle$. $\Delta_s=\omega_{41}-\omega_s$, $\Delta_p=\omega_{31}-\omega_p$ and $\Delta_c=\omega_{32}-\omega_p$ denote the frequency detunings of the pump, probe and driving fields from their relevant atomic transitions, respectively. $\sqrt{\gamma_{31}\gamma_{32}}\,p_1\rho_{33}$ and $\sqrt{\gamma_{41}\gamma_{42}}\,p_2\rho_{44}$ represent the quantum interference effects from the cross coupling, i.e., SGC effect. The parameter p_1 is defined as $p_1\equiv\mu_{31}\cdot\mu_{32}/|\mu_{31}\|\mu_{32}|=\cos\theta_1$, and θ_1 represents the angle between the two dipole moments of $\boldsymbol{\mu}_{31}$ and $\boldsymbol{\mu}_{32}$. Likewise, $p_2\equiv\mu_{41}\cdot\mu_{42}/|\mu_{41}\|\mu_{42}|=\cos\theta_2$, and θ_2 represents the angle between the two dipole moments of $\boldsymbol{\mu}_{41}$ and $\boldsymbol{\mu}_{42}$. If $\boldsymbol{\mu}_{31}$ and $\boldsymbol{\mu}_{32}$ ($\boldsymbol{\mu}_{41}$ and $\boldsymbol{\mu}_{42}$) are orthogonal to each other, $\theta_1=\pi/2$($\theta_2=\pi/2$), then $p_1(p_2)=0$, and SGC is absent. Otherwise, $\theta_1\neq\pi/2$ ($\theta_2\neq\pi/2$), then $p_1(p_2)\neq 0$, and the SGC is present. The strength of the SGC effect changes with the values of θ_1 and θ_2. If level $|4\rangle$ is absent, Eqs.(1)-(10) reduce to the motion equations for a Λ-type three-level atomic system with SGC[11], so the system which we are considering is called as a quasi Λ-type four-level atomic system.

Up to now, the above discussion has referred to the static atomic system without Doppler broadening. In order to account for the Doppler broadening, the velocity-dependent detunings should be considered in Eqs.(1)-(10), which are given by

$$\delta_{\mathrm{p}}(\upsilon)=\Delta_{\mathrm{p}}\mp\omega_{\mathrm{p}}\upsilon/c, \tag{11}$$

$$\delta_{\mathrm{c}}(\upsilon)=\Delta_{\mathrm{c}}\pm\omega_{\mathrm{c}}[\delta_{\mathrm{p}}(\upsilon)-\Delta_{\mathrm{p}}]/\omega_{\mathrm{p}}, \tag{12}$$

$$\delta_{\mathrm{s}}(\upsilon)=\Delta_{\mathrm{s}}\pm\omega_{\mathrm{s}}[\delta_{\mathrm{p}}(\upsilon)-\Delta_{\mathrm{p}}]/\omega_{\mathrm{p}}, \tag{13}$$

where υ is the velocity of a single atom which moves along z axis, c is the light speed, $\delta_p(\upsilon)$, $\delta_c(\upsilon)$ and $\delta_s(\upsilon)$ are the detunings of the probe, driving and pump fields, respectively, and the negative (positive) sign corresponds to co-propagating (counter-propagating) atom and field. For convenience, in the following calculations, we assume that the driving and pump fields both propagate in the direction which the atoms move along, so co-propagating (counter-propagating) probe and driving fields mean co-propagating (counter-propagating) probe field and atom. Under this condition, we only need to take the negative signs in Eqs.(12) and (13).

We assume that atomic velocity obeys Maxwell distribution. When Doppler broadening is present, the transient evolutions of the density matrix elements can be represented by

$$\overline{\rho}_{ij}(t)=\int_{-\infty}^{\infty}\rho_{ij}(\delta_{\mathrm{p}},t)\rho(\delta_{\mathrm{p}})\mathrm{d}\,\delta_{\mathrm{p}}\ , \tag{14}$$

where

$$\rho(\delta_{\mathrm{p}})=\exp[-(\delta_{\mathrm{p}}-\Delta_{\mathrm{p}})^{2}/2D^{2}]/\sqrt{2\pi D^{2}}\ , \tag{15}$$

where D represents Doppler broadening width. The gain or absorption coefficients of the probe, driving and pump fileds correspond to the imaginary parts of $\overline{\rho}_{13}$, $\overline{\rho}_{23}$ and $\overline{\rho}_{14}$, respectively. If $\mathrm{Im}\,\overline{\rho}_{13}(\mathrm{Im}\,\overline{\rho}_{23},\mathrm{Im}\,\overline{\rho}_{14})>0$, the system exhibits the gain for the probe (driving, pump) filed; if $\mathrm{Im}\,\overline{\rho}_{13}(\mathrm{Im}\,\overline{\rho}_{23}$, $\mathrm{Im}\,\overline{\rho}_{14})<0$, the probe (driving, pump) filed is attenuated. When $\overline{\rho}_{11}>\overline{\rho}_{33}$ (without inversion) and $\mathrm{Im}\,\overline{\rho}_{13}>0$ are simultaneously satisfied, GWI can be obtained, and then LWI can be realized.

We set the initial conditions as $\bar{\rho}_{22}=\bar{\rho}_{11}=0.5$, $\bar{\rho}_{33}=\bar{\rho}_{44}=0$, and other $\bar{\rho}_{ij}=0$ ($i\neq j$, i, j=1, 2, 3, 4). Except for special statement, the system parameters take following values that the spontaneous decay rates are $\gamma_{32}=\gamma_{31}$ and $\gamma_{41}=\gamma_{42}=1.2\gamma_{31}$, the frequency detunings of the pump, probe and driving fields are $\Delta_p=4.9\gamma_{31}$, $\Delta_c=0$ and $\Delta_s=0$, and the Rabi frequencies of the probe, driving and pump fields are $\Omega_p=0.02\gamma_{31}$, $\Omega_c=3.3\gamma_{31}$ and $\Omega_s=2.0\gamma_{31}$, respectively. In addition, we always let $p_1\equiv p_2\equiv p$. Because all parameters except for $\bar{\rho}_{ij}$, p and t are scaled by γ_{31}, from now on, for simplicity, we can omit γ_{31} when we give the parameters in the following.

It should be pointed out that the inversionless condition of $\bar{\rho}_{33}-\bar{\rho}_{11}<0$ is always satisfied in the following discussion, and if $\mathrm{Im}\bar{\rho}_{13}>0$ is also satisfied, we can obtain the GWI (simply called as gain below).

Fig.2 gives the transient evolutions of $\mathrm{Im}\bar{\rho}_{13}$ for different values of p with D=0.5 when the probe and driving fields are co-propagating. We can see that when t is smaller, all evolution curves are oscillating, and with t increasing, each parameter tends to a steady value. Under the condition of $p=0$, i.e., SGC is absent, value of gain $\mathrm{Im}\bar{\rho}_{13}$ is very small, and the transient maximum value $(\mathrm{Im}\bar{\rho}_{13})_{\max}$ and the steady value $(\mathrm{Im}\bar{\rho}_{13})_{s}$ are also very small. Under the condition of $p\neq 0$, i.e., SGC is present, the transient evolution rules of $\mathrm{Im}\bar{\rho}_{13}$ for different values of p are similar. When t is small, the probe field is absorbed ($\mathrm{Im}\bar{\rho}_{13}<0$), and when t is large enough, the probe field is amplified ($\mathrm{Im}\bar{\rho}_{13}>0$). For different values of p, the gain begins to appear at the nearly same time of $t\approx 0.7$, and gets its transient maximum value $(\mathrm{Im}\bar{\rho}_{13})_{\max}$ at the nearly same time of $t\approx 1.1$. Both values of $(\mathrm{Im}\bar{\rho}_{13})_{\max}$ and $(\mathrm{Im}\bar{\rho}_{13})_{s}$ increase with p increasing. When p =1, the $(\mathrm{Im}\bar{\rho}_{13})_{\max}$ and $(\mathrm{Im}\bar{\rho}_{13})_{s}$ are maximum, so in the following discussion, we let p =1 when SGC is present. The gain with SGC is much larger than that without SGC.

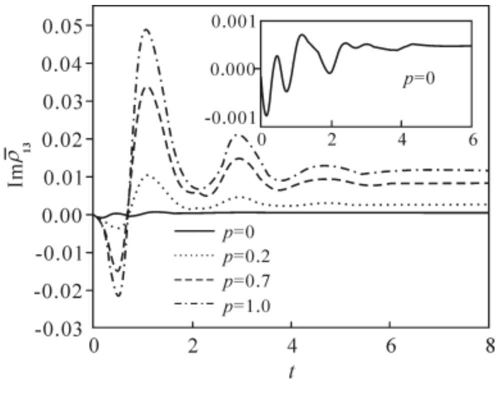

Fig.2 $\mathrm{Im}\bar{\rho}_{13}$ as a function of *t* for different values of *p* with *D*=0.5 when the probe and driving fields are co-propagating under Ω_p = 0.02, Ω_c = 3.3, Ω_s = 2.0, $\gamma_{31}=\gamma_{32}$=1, $\gamma_{41}=\gamma_{42}$=1.2, Δ_p= 4.9, Δ_c= 0 and Δ_s= 0

Fig.3 denotes the transient evolutions of $\mathrm{Im}\bar{\rho}_{13}$ for different values of D when the probe and driving fields are co-propagating. Note that the vertical ordinate scale in Fig.3(a) corresponding to p=0 is much smaller than that in Fig.3(b) corresponding to p=1.

It can be seen from Fig.3 that whether SGC is present or not, the transient gain maximum value $(\mathrm{Im}\bar{\rho}_{13})_{\max}$ and the gain steady value $(\mathrm{Im}\bar{\rho}_{13})_{s}$ always first increase and then decrease with D increasing, but the value of D which corresponds to the largest value of $(\mathrm{Im}\bar{\rho}_{13})_{\max}$, i.e., the largest transient gain $(\mathrm{Im}\bar{\rho}_{13})_{L}$, is different from that corresponding to the largest value of $(\mathrm{Im}\bar{\rho}_{13})_{s}$, i.e., the largest steady gain $(\mathrm{Im}\bar{\rho}_{13})_{sL}$. When D=1.0, we obtain the largest transient gain $(\mathrm{Im}\bar{\rho}_{13})_{L}$ and when D=0.5, we obtain the largest steady gain $(\mathrm{Im}\bar{\rho}_{13})_{sL}$. When SGC is present, $(\mathrm{Im}\bar{\rho}_{13})_{L}$= 0.053 and $(\mathrm{Im}\bar{\rho}_{13})_{sL}$=0.012 can be obtained, which are much larger than those without SGC as $(\mathrm{Im}\bar{\rho}_{13})_{L}$= 0.00075 and $(\mathrm{Im}\bar{\rho}_{13})_{sL}$= 0.00048. Numerical calculation result shows that when $p=0$, $D>2.0$ and when $p=1$, $D>10.0$, the steady value of $\mathrm{Im}\bar{\rho}_{13}$ is negative, which means that we can not obtain steady gain again but only obtain steady absorption. The value of t which corresponds to the transient gain maximum value of $(\mathrm{Im}\bar{\rho}_{13})_{\max}$ in the case with SGC is smaller than that in the case without SGC.

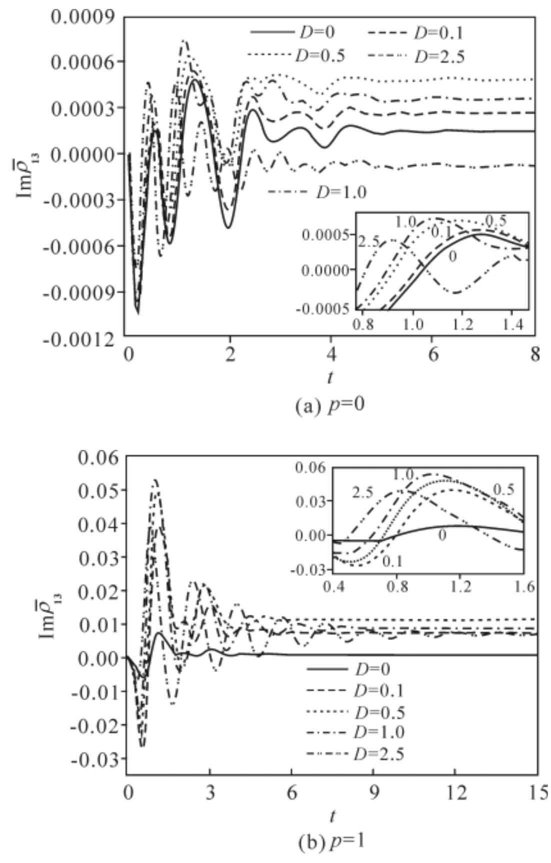

Fig.3 $\mathrm{Im}\bar{\rho}_{13}$ as a function of *t* for different values of *D* when the probe and driving fields are co-propagating

Our numerical calculation results show that in the counter-propagating case of the probe and driving fields, the rule of transient evolution of GWI is almost similar to that in the co-propagating case. But there are some differences when SGC is absent: the largest transient GWI in the counter-propagating case is larger than that in the co-propagating case, but the largest steady GWI in the counter-propagating case is smaller than that in the co-propagating case, and when SGC is present, the largest transient and steady GWI in the co-propagating case are much larger than those in the counter-propagating case.

In conclusion, we study the influence of SGC on transient evolution of GWI in a Doppler broadened quasi Λ-type four-level atomic system. The transient evolution of GWI is very sensitive to the variation of SGC strength, and the transient maximum and steady values of GWI increase with SGC strength increasing. The transient and steady values of GWI with SGC are much larger than those without SGC. When Doppler broadening is present, Doppler broadening width and propagation directions of the probe and driving fields have considerable influence on the transient evolution of SGC-dependent GWI. By choosing the appropriate SGC strength, *D* and propagation directions of the probe and driving fields, we can get much larger transient and steady GWI.

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