# Practical decoy-state quantum key distribution method considering dark count rate fluctuation*

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Considering fluctuant dark count rate in practical quantum key distribution (QKD) system, a new decoy-state method with one vacuum state and one weak decoy state is presented based on a heralded single photon source (HSPS). The method assumes that the dark count rate of each pulse is random and independent. The lower bound of the count rate and the upper bound of the error rate of a single photon state are estimated. The method is applied to the decoy-state QKD system with and without the fluctuation of dark count rate. Because the estimation of the upper bound of a single photon state's error rate is stricter, the method can obtain better performance than the existing methods under the same condition of implementation.

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In practice, the unconditional security of quantum key distribution (QKD)[1] has also been ensured even with imperfect technical condition[2-7]. But the cost of practical QKD's unconditional security is the limited key generation rate and secure distance. Decoy-state method[8] has been proposed for improving the perfromace of practical QKD. Following this seminal work, many researches[9-21] have been done to advance the decoy-state idea. The conclusions of theories and experiments show clearly that the decoy-state method can indeed substantially enhance the performance of practical QKD. Now, the scholars pay more attention to some questions needed to be solved when the decoy-state method is put into practical applications, such as unstable souces[22-25] and dark count rate fluctuation[26]. The existing decoy-state theory assumes that the dark count rate is a constant, whatever the circumstance changes. This assumption is invalid in practice, because the circumstances and the detector efficiency change with time. Therefore, a new problem in practice is how to carry out the decoy-state method securely and efficiently with the fluctuant dark count rate.

A decoy-state method with weak coherent states (WCSs) is presented to give the fluctuation of dark count rate in Ref. [26]. Some researches[27,28] show that the decoy-state method with a heralded single photon source (HSPS) can obtain better performance than the method with WCS. In this paper, we present a decoy-state method with one vacuum state and one weak decoy state based on HSPS to solve the fluctuation of dark count rate, and analyze the performance of the method in detail.

The state of the photons generated in two modes of T and S by HSPS can be written as

$$|\psi\rangle_{\mathrm{TS}}=\sum_{i=0}^{\infty}\sqrt{P_i}\,|i\rangle_{\mathrm{T}}|i\rangle_{\mathrm{S}}\ , \tag{1}$$

where $|i\rangle$ represents an $i$-photon state, $P_i=x^i/(1+x)^{i+1}$ is the probability to get an $i$-photon pair, and $x$ is the intensity of one mode. The photon numbers of two modes are always the same.

We define the yield $Y_i$ to be the probability of Bob getting a detection event conditioned on Alice sending an $i$-photon state, which can be named as the count rate and expressed as

$$\begin{aligned}&Y_i=Y_0+\eta_i-Y_0\eta_i\approx Y_0+\eta_i\\&\eta_i=1-(1-\eta)^i\end{aligned}\ , \tag{2}$$

where $Y_0$ is the dark count rate, $\eta_i$ is the transmittance of the $i$-photon state, and $\eta$ is the overall transmission between Alice and Bob.

The error rate of the $i$-photon state is given by

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$$e_i = \frac{e_0 Y_0 + e_{\mathrm{d}} \eta_i}{Y_i} \ , \tag{3}$$

where $e_0$ is the error rate of the background, and $e_{\mathrm{d}}$ is the probability that the survived photon hits a wrong detector.

The final key generation rate is given by

$$R \geq q\{-Q_u f(E_u) H_2(E_u) + Q_1[1 - H_2(e_1)]\} \ , \tag{4}$$

where $q$ is the basis reconciliation factor, and the $q$ for the BB84 protocol is 1/2. $f(x)$ is the bidirectional error correction efficiency. $H_2(x)$ is the binary Shannon information entropy function, which is given by $H_2(x)=-x\log_2(x)-(1-x)\log_2(1-x)$. $Q_1$ and $e_1$ are the gain and the error rate of single photon states. $Q_u$ and $E_u$ are the overall gain and quantum bit error rate (QBER) which can be observed in the experiment. In order to calculate the final key generation rate, we need to estimate the lower bound of $Y_1$ and the upper bound of $e_1$.

In our method, the mode S is coded as the signal mode sent to Bob, and the mode T going to Alice's own detector as the triggering signal to forecast the photon number and the arriving time in the mode S. Here, we are interested in the case that Alice and Bob use threshold detectors. If Alice's detector is triggered, Bob's detector needs to measure the received photon state. When the communication is over, we will estimate the lower bound of $Y_1$ and the upper bound of $e_1$ with the measured results. If the discrepancy between estimation results and theoretical results is big, Eve will be considered, this communication will be abandoned, and QKD will be restarted. Contrarily, the final key generation rate can be calculated with Eq.(4).

In the protocol, Alice randomly changes the intensity of her pump light among 0, $v$ and $u$ with the probabilities of $p_1$, $p_2$ and $p_3$ ($p_1$+$p_2$+$p_3$=1), respectively. 0 and $v$ are the expected intensities of the decoy states, and $u$ is the expected intensity of the signal state, which satisfy the relations as

$$0 \leq v < u, \ \frac{v}{1+v} < \frac{u}{1+u} \ . \tag{5}$$

Then we define $Q_u$ and $Q_v$ to be the overall rates of signal states and decoy states, respectively, which can be expressed as

$$Q_u = \frac{d_{\mathrm{A}} Y_0}{1+u} + \sum_{i=1}^{\infty} Y_i [1-(1-\eta_{\mathrm{A}})^i] \frac{u^i}{(1+u)^{i+1}}$$

$$Q_v = \frac{d_{\mathrm{A}} Y_0}{1+v} + \sum_{i=1}^{\infty} Y_i [1-(1-\eta_{\mathrm{A}})^i] \frac{v^i}{(1+v)^{i+1}} \ , \tag{6}$$

where $d_{\mathrm{A}}$ and $\eta_{\mathrm{A}}$ are the dark count rate and detection efficiency of Alice's detector, respectively.

The QBERs of signal states and decoy states are given by

$$E_u = \left\{ \frac{e_0 Y_0}{1+u} + \sum_{i=1}^{\infty} e_i Y_i [1-(1-\eta_{\mathrm{A}})^i] \frac{u^i}{(1+u)^{i+1}} \right\} \Big/ Q_u$$

$$E_v = \left\{ \frac{e_0 Y_0}{1+v} + \sum_{i=1}^{\infty} e_i Y_i [1-(1-\eta_{\mathrm{A}})^i] \frac{v^i}{(1+v)^{i+1}} \right\} \Big/ Q_v \ . \tag{7}$$

We assume that the circumstances change randomly at every pulse, namely, the dark count rate of each pulse is random and independent. Suppose that Bob has two detectors. Here, $Y_{0j}$ and $Y'_{0j}$ represent the actual dark count rates of the first detector and the second detector at the $j$th pulse, respectively

$$Y_{0j} = (1+\delta_{0j}) Y_0, \ \ Y'_{0j} = (1+\delta'_{0j}) Y_0,$$

$$Y_0 = \frac{1}{J} \sum_{j=0}^{J} Y_{0j} = \frac{1}{J} \sum_{j=0}^{J} Y'_{0j} \ , \tag{8}$$

where $\delta_{0j}$ and $\delta'_{0j}$ are the dark count rate's fluctuations of the first detector and the second detector, respectively. $Y_0$ is the average dark count rate. Here we assume $J$ is infinite. We can obtain the following equation

$$\sum_{j=0}^{J} \delta_{0j} = \sum_{j=0}^{J} \delta'_{0j} = 0 \ . \tag{9}$$

Eq.(6) shows that the influence of $d_{\mathrm{A}}$ on system detection is neglectable, so we assume $d_{\mathrm{A}}$ is a constant.

Similarly, with the fluctuation of dark count rate, $Y_1$ can be given by

$$Y_{1j} = (1+\delta_{1j}) Y_1, \ \ Y_1 = \frac{1}{J} \sum_{j=0}^{J} Y_{1j}, \ \ \sum_{j=0}^{J} \delta_{1j} = 0 \ , \tag{10}$$

where $Y_1$ is the average yield of single photon states, and $\delta_{1j}$ is the dark count rate's fluctuation of single photon states.

The error rate of the dark count state of the first detector at the $j$th pulse is

$$e_{0j} = \frac{Y'_{0j}}{Y_{0j} + Y'_{0j}} = \frac{1+\delta'_{0j}}{1+\delta_{0j}+1+\delta'_{0j}} \ . \tag{11}$$

According to Eq.(3), we have

$$e_{ij} = \frac{e_{0j} Y_{0j} + e_{\mathrm{d}} \eta_i}{Y_{ij}} = \frac{1}{Y_{ij}} \left[ Y_0 \frac{(1+\delta_{0j})(1+\delta'_{0j})}{1+\delta_{0j}+1+\delta'_{0j}} + e_{\mathrm{d}} \eta_i \right] \ . \tag{12}$$

From Eq.(6), we can obtain

$$\sum_{i=2}^{\infty} Y_i \left[1-(1-\eta_{\mathrm{A}})^i\right] \frac{u^i}{(1+u)^i} = (1+u) Q_u - d_{\mathrm{A}} Y_0 - Y_1 \eta_{\mathrm{A}} \frac{u}{1+u} \ . \tag{13}$$

Combining Eqs.(8), (10) and (13), we can estimate the lower bound of $Y_1$

$$\begin{aligned}
& d_{\mathrm{A}} p_1 Y_0 + p_2 Q_v (1+v) + p_3 Q_u (1+u) = \\
& \frac{1}{N}\sum_{j=0}^{N}\Bigg\{ d_{\mathrm{A}} Y_0 (1+\delta_{0j}) + Y_{1j}\eta_{\mathrm{A}}\left(\frac{p_2 v}{1+v}+\frac{p_3 u}{1+u}\right) + \\
& (1-p_1)\sum_{i=2}^{\infty} Y_{ij}[1-(1-\eta_{\mathrm{A}})^i]\frac{u^i}{(1+u)^i} - \\
& p_2\sum_{i=2}^{\infty} Y_{ij}[1-(1-\eta_{\mathrm{A}})^i]\left[\frac{u^i}{(1+u)^i}-\frac{v^i}{(1+v)^i}\right] \le \\
& \frac{1}{N}\sum_{j=0}^{N}\Bigg\{ d_{\mathrm{A}} Y_0 (1+\delta_{0j}) + Y_{1j}\eta_{\mathrm{A}}\left(\frac{p_2 v}{1+v}+\frac{p_3 u}{1+u}\right) + \\
& (1-p_1)\sum_{i=2}^{\infty} Y_{ij}[1-(1-\eta_{\mathrm{A}})^i]\frac{u^i}{(1+u)^i} - \\
& p_2\left[1-\frac{v^2(1+u)^2}{(1+v)^2u^2}\right]\sum_{i=2}^{\infty} Y_{ij}[1-(1-\eta_{\mathrm{A}})^i]\frac{u^i}{(1+u)^i}\Bigg\} = \\
& d_{\mathrm{A}} Y_0\left\{p_1 + p_2\left[1-\frac{v^2(1+u)^2}{(1+v)^2u^2}\right]\right\} + \\
& \eta_{\mathrm{A}} Y_1 p_2\left[\frac{v}{1+v}-\frac{v^2(1+u)}{(1+v)^2u}\right] + \\
& Q_u\left[p_3(1+u)+\frac{p_2 v^2(1+u)^3}{(1+v)^2u^2}\right] \quad , \qquad (14)
\end{aligned}$$

where $N$ is the number of total pulses sent by Alice. Here we assume that $N$ is infinite, and the inequality $a^i - b^i \ge a^2 - b^2$ ($a \ge 1 > b$ and $i \ge 2$) is used.

By solving Eq.(14), the lower bound of $Y_1$ is given by

$$\begin{aligned}
& Y_1 \ge \frac{1}{\eta_{\mathrm{A}}(u-v)}\Bigg\{\frac{(1+v)^3 u}{v}Q_v - \frac{(1+u)^3 v}{u}Q_u - \\
& d_{\mathrm{A}} Y_0\left[\frac{(1+v)^2 u}{v}-\frac{(1+u)^2 v}{u}\right]\Bigg\} \quad . \qquad (15)
\end{aligned}$$

It is obvious that the fluctuation of dark count rate has no influence on $Y_1$ with Eq.(9). So Eq.(15) is the same as Eq.(21) in Ref.[18] without fluctuation using a different method.

Subsequently, we estimate the upper bound of $e_1$ with Eqs.(7), (8), (9) and (11).

$$\begin{aligned}
& E_v Q_v (1+v) = \frac{1}{N'}\sum_{j=0}^{N'}\Bigg\{ e_{0j} Y_{0j} d_{\mathrm{A}} + e_{1j} Y_{1j}\eta_{\mathrm{A}}\frac{v}{1+v} + \\
& \sum_{i=2}^{\infty} e_{ij} Y_{ij}\frac{v^i}{(1+v)^i}[1-(1-\eta_{\mathrm{A}})^i]\Bigg\} \ge \\
& \frac{1}{N'}\sum_{j=0}^{N'}\Bigg\{ Y_0 d_{\mathrm{A}}\frac{(1+\delta_{0j})(1+\delta'_{0j})}{1+\delta_{0j}+1+\delta'_{0j}} + e_{1j} Y_{1j}\eta_{\mathrm{A}}\frac{v}{1+v} + \\
& Y_0\frac{(1+\delta_{0j})(1+\delta'_{0j})}{1+\delta_{0j}+1+\delta'_{0j}}\sum_{i=2}^{\infty}\frac{v^i}{(1+v)^i}[1-(1-\eta_{\mathrm{A}})^i]\Bigg\} = \\
& \frac{1}{N'}\sum_{j=0}^{N'}\Bigg\{\frac{Y_0 d_{\mathrm{A}}}{1/(1+\delta_{0j})+1/(1+\delta'_{0j})} + e_{1j} Y_{1j}\eta_{\mathrm{A}}\frac{v}{1+v}(1+\delta_{1j}) + \\
& \frac{Y_0}{1/(1+\delta_{0j})+1/(1+\delta'_{0j})}\sum_{i=2}^{\infty}\frac{v^i}{(1+v)^i}\Big[1-(1-\eta_{\mathrm{A}})^i\Big]\Bigg\} \ge \\
& e_0 Y_0 d_{\mathrm{A}}(1+\delta) + e_1 Y_1 \eta_{\mathrm{A}}\frac{v}{(1+v)}(1+\delta) + \\
& e_0 Y_0 (1+\delta)(1+v)[1-(1-\eta_{\mathrm{A}})^2]\sum_{i=2}^{\infty}\frac{v^i}{(1+v)^{i+1}} = \\
& e_0 Y_0 d_{\mathrm{A}}(1+\delta) + e_1 Y_1 \eta_{\mathrm{A}}\frac{v}{1+v}(1+\delta) + \\
& e_0 Y_0 (1+\delta)[1-(1-\eta_{\mathrm{A}})^2]\frac{v^2}{1+v} \quad , \qquad (16)
\end{aligned}$$

where $N'$ is the number of decoy pulses sent by Alice, and it is infinite. The first inequality in Eq.(16) uses the fact that the error rate of the $i$-photon state is greater than that of the dark count state, namely, $e_{ij}Y_{ij} \ge e_{0j}Y_{0j}$. The second inequality in Eq.(16) uses the condition of $\delta$=min$\{\delta_{0j}\}$=min$\{\delta'_{0j}\}$=min$\{\delta_{1j}\}$.

Consequently, the upper bound of $e_1$ is

$$e_1 \le \frac{E_v Q_v (1+v)^2 - e_0 Y_0 (1+\delta)[d_{\mathrm{A}}(1+v) + v^2(2\eta_{\mathrm{A}} - \eta_{\mathrm{A}}^2)]}{Y_1 \eta_{\mathrm{A}} v (1+\delta)}. \qquad (17)$$

Combining Eqs.(12) and (16), $Q_v E_v$ can be expressed by

$$\begin{aligned}
& Q_v E_v = \frac{e_0 Y_0}{(1+v)} + \sum_{i=1}^{\infty}\frac{e_i Y_i [1-(1-\eta_{\mathrm{A}})^i] v^i}{(1+v)^{i+1}} = \\
& \frac{1}{N'}\sum_{j=0}^{N'}\Bigg\{ Y_0\frac{(1+\delta_{0j})(1+\delta'_{0j})}{1+\delta_{0j}+1+\delta'_{0j}}\frac{1}{1+v} + \\
& Y_0\eta_{\mathrm{A}}\frac{(1+\delta_{0j})(1+\delta'_{0j})}{1+\delta_{0j}+1+\delta'_{0j}}\frac{v}{(1+v)^2} + e_{\mathrm{d}}\eta_{\mathrm{A}}\eta_1\frac{v}{(1+v)^2} + \\
& \sum_{i=2}^{\infty}\left[Y_0\frac{(1+\delta_{0j})(1+\delta'_{0j})}{1+\delta_{0j}+1+\delta'_{0j}} + e_{\mathrm{d}}\eta_i\right]\frac{[1-(1-\eta_{\mathrm{A}})^i]v^i}{(1+v)^{i+1}}\Bigg\} = \\
& \frac{1}{N'}\sum_{j=0}^{N'}\Bigg\{ Y_0\frac{(1+\delta_{0j})(1+\delta'_{0j})}{1+\delta_{0j}+1+\delta'_{0j}}\Bigg[\frac{1}{1+v} + \\
& \sum_{i=1}^{\infty}\frac{[1-(1-\eta_{\mathrm{A}})^i]v^i}{(1+v)^{i+1}}\Bigg] + \sum_{i=1}^{\infty} e_{\mathrm{d}}\eta_i[1-(1-\eta_{\mathrm{A}})^i]\frac{v^i}{(1+v)^{i+1}}\Bigg\} = \\
& e_0 Y_0\frac{\eta_{\mathrm{A}} v}{1+\eta_{\mathrm{A}} v} + e_{\mathrm{d}}\Bigg[1 - \frac{1}{1+\eta_{\mathrm{A}} v} - \frac{1-Y_0}{1+\eta v} + \\
& \frac{1-Y_0}{1+(\eta_{\mathrm{A}}+\eta-\eta_{\mathrm{A}} v)v} - \frac{Y_0\eta_{\mathrm{A}} v}{1+\eta_{\mathrm{A}} v}\Bigg] \quad . \qquad (18)
\end{aligned}$$

Eq.(18) accords with Eq.(10) in Ref.[29].

We can now calculate the final key rate with Eqs.(4), (15) and (17). We use the parameters mainly from GYS experiment[30] and Ref.[18]. At Alice's side, $d_{\mathrm{A}}$=5 × 10$^{-6}$, and $\eta_{\mathrm{A}}$= 0.6. At Bob's side, $d_{\mathrm{B}}$=1.7 × 10$^{-6}$, $\eta_{\mathrm{B}}$=0.045, $e_{\mathrm{d}}$=0.033, $f$=

1.16. In simulation, the optimal $u$ is chosen at each distance.

We calculate the final key rates based on our scheme with $\delta = 0, -0.1, -0.2, -0.3$, respectively. Fig.1 shows that the key generation rate and maximal secure distance decrease with the decrease of $\delta$. The maximal secure distance is about 172 km with $\delta$=0, and the maximal secure distance decreases to 169 km, 162 km and 148 km with $\delta = -0.1, -0.2, -0.3$, respectively. The dark count rate plays a more important role, and the effect of the error rate of the dark count rate becomes more important as the distance increases. For example, when the transmission distance is less than 100 km, the difference between the key generation rate with $\delta$=−0.3 and that with$\delta$=0 is not large. However, when the transmission distance is larger than 100 km, the key generation rate with $\delta$=−0.3 begins to decrease rapidly.

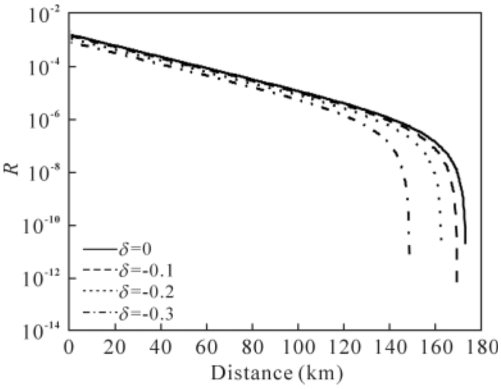

**Fig.1 Performance comparison of the decoy-state method with various values of $\delta$**

The decoy-state method in Ref.[18] taking no account of the fluctuant dark count rate also has one vacuum state and one weak decoy state, namely, the method and our method are under the same condition of implementation. Fig.2 shows that the two curves seem to be one curve when the transmission distance is less than 140 km. The performance of our method is better when the transmission distance is over 140 km, because our estimation of $e_1$ is stricter. We not only consider the error rate of the dark count rate, but also consider the error rate of the multi-photon state. However, Ref.[18] makes a pessimistic assumption that all error rates are induced by single states and dark count.

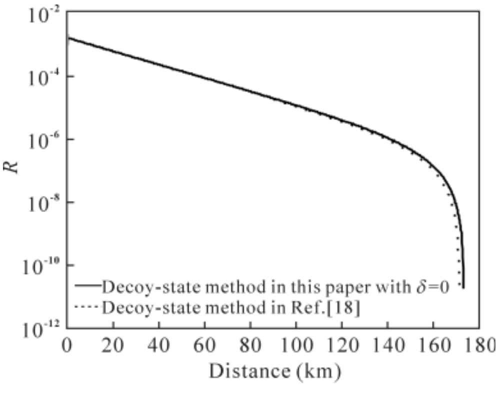

**Fig.2 Performance comparison between two decoy-state methods**

In summary, considering the fluctuant dark count rate in practical QKD system, we have presented a new decoy-state method with one vacuum state and one weak decoy state based on an HSPS. Our method is applied to the decoy-state QKD system with and without the fluctuation of dark count rate. We estimate $e_1$ considering the error rates of the dark count rate and the multi-photon state, and get a tighter bound. So our method obtains better performation than the existing methods under the same condition of implementation.

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