A novel RS BTC coding scheme for optical communications*

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A novel Reed Solomon (RS) block turbo code (BTC) coding scheme of RS(63,58)×RS(63,58) for optical communications is proposed. The simulation results show that the net coding gain (NCG) of this scheme at the sixth iteration is more than that of other coding schemes at the third iteration for the bit error rate (BER) of 10^{-12}. Furthermore, the novel RS BTC has shorter component code and rapider encoding and decoding speed. Therefore, the novel RS BTC coding scheme can be better used in high-speed long-haul optical communication systems, and the novel RS BTC can be regarded as a candidate code of the super forward error correction (super-FEC) code. Moreover, the encoding/decoding design and implementation of the novel RS BTC are also presented.

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The forward error correction (FEC) becomes the first choice to improve the performance of systems because the FEC can gain a much larger transmission distance and make the system more robust under worse conditions[1-4]. In particular, the Reed-Solomon (RS) (255,239) code is now commonly used and standardized in ITU-T G.975[5] and G.709[6]. With the development of wavelength division multiplexing (WDM) optical communication technologies, more powerful super-FEC codes have become necessary to compensate for the serious degradation of transmission quality.

ITU-T has constituted the recommendation G.975.1[7] which describes that the eight code types have more powerful error-correction performance than RS (255, 239) code in ITU-T G.975. However, there is no block turbo code (BTC) in these eight code types. The analytical result shows that the BTC may apply the random encoding and decoding conditions as well as the iteration decoding idea, and it has excellent performance[8,9], so it has a very attractive application prospect in the optical communication field. As a result, the study on the novel super-FEC code based on the BTC is very important to extensively develop the ITU-T G.975.1. A novel RS BTC coding scheme of RS (63,58)×RS (63,58) code is proposed in this paper.

The two-dimensional (2D) BTC lends itself to optical communication systems, due to its easy implementation and convenient construction for the code with high code rate. RS code or Bose-Chaudhuri-Hocquenghem (BCH) code can be chosen as the component code of the 2D BTC. And RS code can obtain better error correction performance[9], so RS code is selected as the component code of the BTC in this paper.

The construction of the 2D BTC in this paper is shown in Fig.1[9]. A diagram of the encoding system is shown in Fig.2. As illustrated in Fig.2, a row/column interleaver is used between two RS component coders to assure the information flow which is transversely encoded in the first RS encoder and then is longitudinally encoded in the second BCH encoder. We can also conclude from Fig.2 that this encoding way is absolutely identical to the way of encoding RS serial concatenated code in which the row/column interleaver is used. After the above process of encoding, we get the BTC with $n=n_1 \times n_2$, $k=k_1 \times k_2$ and $d_{\min}=d_{\min 1} \times d_{\min 2}$ eventually. By this encoding system, the longer codeword with longer Hamming distance can be constructed by the shorter codeword with shorter Hamming distance. In addition, better error correction performance can also be achieved.

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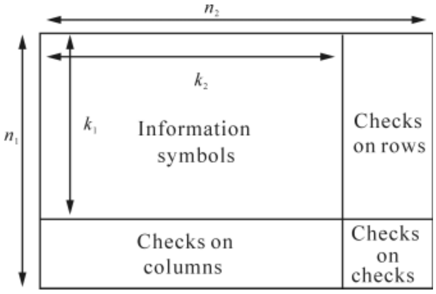

Fig.1 Encoding structural diagram of the 2D BTC

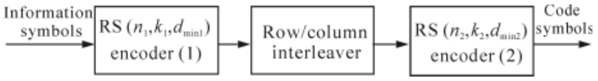

Fig.2 Encoding block diagram of the 2D BTC

The decoding of binary linear block code $c(n, k, d_{\min})$ in the condition of the additive white Gaussian noise (AWGN) channel is considered and used in this paper. In this case, the decoded binary 0, 1 information flow can be first mapped into −1, +1 information flow, and then transmitted through the AWGN channel. The mapping rule is $\{0\rightarrow-1, 1\rightarrow+1\}$. Let $\boldsymbol{x}=\{x_1, x_2,\cdots, x_n\}$ be the code vector to be sent out. After the code vector is transmitted through AWGN channel, we can obtain the monitoring vector of $\boldsymbol{r}'=\boldsymbol{x}+\boldsymbol{e}$, where $\boldsymbol{r}'=\{r_1', r_2',\cdots,r_n'\}$ and $\boldsymbol{e}=\{e_1, e_2,\cdots, e_n\}$, and $e_i(1\leqslant i\leqslant n)$ are the sampling values of white Gaussian noise whose mean is 0 and variance is σ^2.

For the high code-rate block code whose codeword is too long, the maximum likelihood decoding (MLD) algorithm entails exceedingly large code number. In addition, the operation quantity of the optimum decision vector is unacceptably large, and the complexity of the decoding algorithm increases at the exponent mode with k in the code number of 2^k. Therefore, a decoding algorithm with low complexity for the linear block code presented by Chase[10] is applied in this paper, which has much lower complexity compared with the above optimum MLD algorithm. In the case of greater signal to noise ratio (SNR), the optimum codeword d of MLD always falls with the very high probability in the ball, of which the centre is at $\boldsymbol{r}=\{r_1, r_2,\cdots, r_n\}$ and the radius is $(d_{\min}-1)$. Among them, $r_i=(1+\mathrm{sgn}(r_i'))/2$, and r_i' denotes the ith vector element of the channel monitoring vector. Consequently, in the inequation $|\boldsymbol{r}'-\boldsymbol{c}^i|^2\leq|\boldsymbol{r}'-\boldsymbol{c}^j|^2$, it only needs to consider the codeword in the ball whose central point is at $\boldsymbol{r}$ and radius is $(d_{\min}-1)$. The decoding process is as follows.

Use $\boldsymbol{r}'$ to determine the site of $p=d_{\min}/2$ binary elements which are the most unreliable in $\boldsymbol{r}$, and the reliability of the element r_i in $\boldsymbol{r}$ is defined as its log likelihood ratio (LLR)[11]:

$$L(r_i)=L(r'_i\,|\,x)=\ln\left[\frac{p(r'_i\,|\,x=+1)}{p(r'_i\,|\,x=-1)}\right]=\frac{2}{\sigma^2}r'_i\ . \tag{1}$$

Take the values of 0 and 1 at p, which is the most unreliable site of the n-dimensional vector, and it can obtain the number of test sequences as $q=2^p$. At the same time, take the value of 0 at the other sites. By this way, we can obtain the q testing modes of $\boldsymbol{T}^q$.

Construct the testing sequence $\boldsymbol{Z}^q=\boldsymbol{r}\oplus\boldsymbol{T}^q$, where $z_i^q=r_i\oplus t_i^q$ $(1\leqslant i\leqslant n)$, and decode $\boldsymbol{r}$ and the testing sequence $\boldsymbol{Z}^q$ by applying algebraic decoder to generate the set of Ω containing the q observation codes.

Limit the observation codes in the subset of Ω, and find out the optimum decision-making vector $\boldsymbol{d}=\boldsymbol{c}^i$ of the transmitted code vector $\boldsymbol{x}$ by applying the judging inequation of

$$|\boldsymbol{r}'-\boldsymbol{c}^i|^2\le|\boldsymbol{r}'-\boldsymbol{c}^j|^2,\boldsymbol{c}^i,\boldsymbol{c}^j\in\Omega\ . \tag{2}$$

Chase algorithm gives the decoded code of the linear block code when the channel monitoring vector is received. However, it does not offer the reliability of the decoded code. For applying the iterative soft decoding to assure the correction of decoding, the reliability of the output code of the Chase soft-input decoder must be computed. $\boldsymbol{x}$ is defined as the output code vector of the Chase decoder, and the reliability of its ith element is[12]:

$$L(\hat{x}_i)=L(x_i\,|\,\boldsymbol{r}')=\ln\left[\frac{p_{\mathrm{r}}(x_i=+1\,|\,\boldsymbol{r}')}{p_{\mathrm{r}}(x_i=-1\,|\,\boldsymbol{r}')}\right]\ . \tag{3}$$

It is very obvious in Chase decoding algorithm that the decision vector $\hat{\boldsymbol{x}}$ is one code. And the other code $\tilde{\boldsymbol{x}}$, which has the minimum Euclidean distance with $\boldsymbol{x}'$, is called as the competition code in this paper. At last we may get the soft-output value of the decoder[12]:

$$r_i''=\left(\frac{|\boldsymbol{r}'-\tilde{\boldsymbol{x}}|^2-|\boldsymbol{r}'-\hat{\boldsymbol{x}}|^2}{4}\right)\hat{x}_i\quad . \tag{4}$$

In order to search for the code $\tilde{\boldsymbol{x}}$ efficiently, we must enlarge the observation space of the Chase decoding algorithm, whereas at the same time the decoding complexity can increase at the exponent mode. So there must be a compromise between the algorithm complexity and the decoder performance. The approximate calculation practically used in the engineering is[13]:

$$r_i''=\beta\times\hat{x}_i,(\beta>0)\ . \tag{5}$$

Because Eq.(4) can offer higher accuracy than Eq.(5), we can combine the two equations in the decoding process to assure the more rapid and effective implementation of the decoding algorithm in this paper.

The iterative decoding is based on the soft decoding and soft decision algorithm. Consider the decoding of the baseband transmission's 2D BTC in the AWGN channel. Let [X] be the BTC code matrix to be sent out, the corresponding received matrix is [R'] and the exterior information matrix is [$W(m)$], where m represents the times of iteration and [$W(0)$] =0. The input matrix of the decoder at the mth iteration is[13]

$$[R'(m)]=[R']+\alpha(m)[W(m)] \ , \tag{6}$$

where $\alpha(m)$ is the feedback factor at the mth iteration which reflects the difference of standard deviation between the sampling values in [R'] and [$W(m)$], and it can be adjusted according to the subcode type and the times of iteration. According to the Chase algorithm stated previously, the decoder decodes the input matrix [$R'(m)$] by applying soft decoding row after row (or column after column). Use Eqs.(4) and (5) for computing the soft output, and we can get the soft output matrix of [$R''(m)$], so we can further compute the exterior information of the next iteration to be:

$$[W(m+1)]=[R''(m)]-[R'(m)] \ . \tag{7}$$

Then use the exterior information matrix [$W(m+1)$] to implement the soft decoding column after column (or row after row) according to the above process, thus a complete iterative decoding of the 2D BTC is achieved. The decoding process is shown in Fig.3. The decoding for the different iteration times can be implemented by connecting the unit decoder in series as shown in Fig.3.

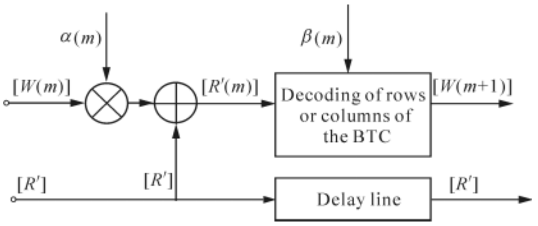

Fig.3 Block diagram of the elementary BTC decoder

The iterative soft decoding process flow of the BTC can be designed according to the relative analyses about the decoding of the BTC mentioned above. Furthermore, the iterative soft decoding software/hardware of the BTC can conveniently be implemented by the designed process flow.

Before a novel RS BTC for optical communications is constructed, the three basic guidelines to construct the FEC code are firstly proposed by analyzing the development trend of optical communication systems in this paper as follows.

Firstly, in consideration of the characteristics of the low noise in the channel of the optical communication systems, there ought to be a low error floor or even no error floor, and when the SNR of the systems is greater, the greater coding gain can still be acquired. Secondly, according to the requirement of the FEC code for optical communication systems with long distance and higher capacity, the net coding gain (NCG) should be higher, and the redundancy of the code type should be lower. In practice the NCG and the redundancy reflect the reliability and the effectiveness of transmission systems, and they construct a contradiction. Therefore, there must be a compromise in selecting the FEC code. Thirdly, the codeword cannot be too long, the time delay from encoding/decoding should not be too much, and the software/hardware implementation should be favorable.

A novel RS BTC for long-haul optical communication systems is constructed according to the above guidelines and the analyses mentioned above. The encoding of the novel RS BTC is basically identical to the encoding of two concatenated RS codes. However, its implementation is relatively complicated due to the soft input/soft output iterative decoding applied in the decoding system[8]. For the BTC, its general error correction performance and decoding complexity both depend on the selection of its component code[9]. It is impractical to use them as the component code of the BTC, because those component codes, such as the convolutional code and turbo code with the longer codeword, can result in the higher complexity for their constructed BTC. However, the RS code and BCH code have the advantage of easy implementation. Generally speaking, it is relatively complicated for implementation when the component codeword length of the BTC is bigger than 100. So the designed component codeword length is shorter than 100 in this paper, and the RS (63,58) code as the BTC component code with codeword length shorter than 100 is always thought as the good code type with better performance. In this paper, on the basis of selecting the RS(63,58) code as the component code of the BTC, we further construct the BTC of RS(63,58) × RS(63,58). The redundancy of this novel RS BTC is 17.98%, so it not only has low redundancy but also is simple to implement.

Since the component code of the novel RS BTC is the RS(63,58) code with the codeword length shorter than 100 and the decoding complexity of the BTC depends on the selection of its component code, the decoding complexity of the novel RS BTC with the RS(63,58) code as its component code is much lower than that of the BTC with the RS(255, 239) code. Furthermore, this component code of the novel RS BTC is a linear block code, and the soft-decision decoding Chase algorithm which can make the probability of the codeword error minimum is applied in the novel RS BTC. So both its operation process and its decoding complexity are greatly reduced.

The encoding block diagram of the novel RS BTC is shown in Fig.4. The encoding circuit of the BTC consists of two RS encoding circuits and a row/column interleaver. First of all, an encoding circuit of the RS code should be constructed, and then a row-write/column-read memory is connected to it in series, at last a circuit the same as the encoding circuit of the RS code stated above is connected to the memory in series. The decoding block diagram of the novel RS BTC designed in this paper is also depicted as the one in Fig.3. The time delay line unit in Fig.3 is generally implemented by applying first-in, first-out (FIFO) memory.

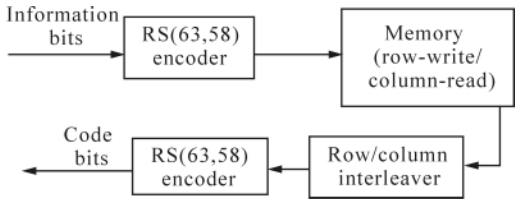

Fig.4 Encoding block diagram of the novel RS BTC

The simulation of the novel super-FEC code is achieved in this paper. Because the inherent thermal noise in optical communication systems is an AWGN that plays a vital role in producing the bit-error, the AWGN channel must be considered when simulating it. The simulation result is shown in Fig.5.

The NCG at the BER_{ref} of 10^{-6} and 10^{-12} can be calculated by applying the definition of the NCG according to Fig.5. The applied definition of the NCG in the calculation is as follows[14,15]:

$$NCG\,(\text{dB})=20\,\text{Log}_{10}[\text{erfc}^{-1}(2BER_{\text{ref}})]-20\,\text{Log}_{10}[\text{erfc}^{-1}(2BER_{\text{in}})]+10\text{Log}_{10}(R)\quad , \tag{8}$$

where BER_{ref} represents the reference output BER, BER_{in} is the input BER, R is the code rate, and erfc^{-1} is the reciprocal form of the assistant error function, where erfc(x)=1-erf(x). The NCG of the novel super-FEC code is calculated and analyzed in Tab.1 by comparing with that of RS(255,239) code and the other two super-FEC codes in ITU-T G.975.1.

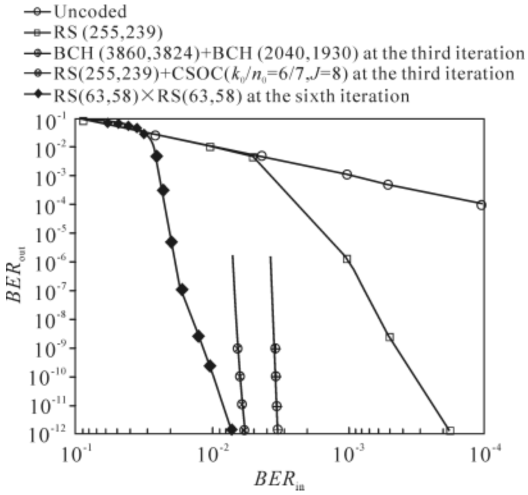

Fig.5 Simulation result of the novel RS BTC

It can be seen from Tab.1 that the NCG of the novel super-FEC code at the sixth iteration is respectively 0.23 dB and 0.26 dB more than that of the BCH(3860,3824) + BCH (2040,1930) code and the RS(255,239) + CSOC(k_0/n_0=6/7, J=8) code in ITU-T G.975.1 at the third iteration for the BER of 10^{-12}. Moreover, the component code of the BTC is shorter, so its encoding/decoding speed is very high. These advantages reduce not only the complexity for implementing the software/hardware but also the time delay for the encoding/decoding. Of course, the novel RS BTC also pays cost to some extent, namely, its redundancy is greater than that of the BCH(3860, 3824) + BCH(2040, 1930) code and the RS (255,239) + CSOC(k_0/n_0= 6/7, J=8) code in ITU-T G.975.1.

Tab.1 Comparison of error-correction performance among the novel RS BTC and other FEC codes

FEC code-type	Redundancy	BER_{in} for BER_{out} $=BER_{\text{ref}}=10^{-6}$	BER_{in} for BER_{out} $=BER_{\text{ref}}=10^{-12}$	NCG for BER_{ref} $=10^{-6}$ (dB)	NCG for BER_{ref} $=10^{-12}$ (dB)
RS (255,239)	6.69%	9.70×10^{-4}	1.8×10^{-4}	3.43	5.62
BCH(3860, 3824)+BCH(2040, 1930) at the third iteration	6.69%	3.68×10^{-3}	3.3×10^{-3}	4.70	7.98
RS (255,239)+CSOC(k_0/n_0 =6/7, J=8) at the third iteration	24.48%	7.14×10^{-3}	5.8×10^{-3}	4.82	7.95
RS(63,58) × RS(63,58) at the sixth iteration	17.98%	1.863×10^{-2}	7.18×10^{-3}	6.19	8.21

The novel RS BTC of the RS(63,58)× RS(63,58) code type is constructed, simulated and analyzed in this paper. The simulation analysis shows that the NCG of the novel RS BTC at the sixth iteration is respectively 0.23 dB and 0.26 dB more than that of the BCH(3860,3824)+BCH(2040,1930) code and the RS(255,239)+ CSOC(k_0/n_0 =6/7, J=8) code in ITU-T G.975.1 at the third iteration for the *BER* of 10^{-12}. Furthermore, the performance analysis shows that the novel RS BTC has the excellent characteristics, such as the shorter component code and the rapider encoding/decoding speed, and both the complexity to implement the software/hardware and the time delay from the encoding/decoding can greatly be reduced. Therefore, the novel RS BTC can be better used in high-speed long-haul optical communication systems. In addition, the design and implementation of the novel RS BTC are also discussed.

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