OPTOELECTRONICS LETTERS Vol.8 No.3, 1 May 2012

Comparison study of several underwater light scattering phase functions*

HUANG You-wei（黄有为）******, **LIU Guang-rong**（刘广荣）, **JIN Wei-qi**（金伟其）, **DING Kun**（丁琨）, **LI Hai-lan**（李海兰）, **CAO Feng-mei**（曹峰梅）, and **WANG Xia**（王霞）

Key Laboratory of Photoelectronic Imaging Technology and System of Ministry of Education, Beijing Institute of Technology, Beijing 100081, China

(Received 22 December 2011)

Scattering phase function is assumed to be one of the most significant factors in the inherent optical properties (IOPs) of natural water. According to three criteria proposed for assessment, several commonly used empirical phase functions are compared with their related practical or theoretical scattering distributions in terms of fitting errors under the circumstances of typical seawater and single-component polydisperse systems. The optimal factors corresponding to the minimum fitting errors are also calculated. It is found that both the one-term Henyey-Greenstein (OTHG) and two-term Henyey-Greenstein (TTHG) phase functions agree well with the theoretical ones for small particles, while the Fouriner-Forand (FF) phase function can be used in the case of suspensions with large suspended particles. The fitting accuracy of OTHG is the worst, FF is better and TTHG is the best.

Document code: A **Article ID:** 1673-1905(2012)03-0233-4

DOI 10.1007/s11801-012-0054-2

Mie theory[1,2] is an analytical solution of Maxwell's equations for the scattering of electromagnetic radiation by spherical particles, but it is complex and time consuming. In practice, several simple empirical approximations are proposed and frequently used because of their mathematical simplicity[3-8].Although these approximations are mathematically convenient, they often give unrealistic phase functions, especially at small or large scattering angles. Thus, whether these empirical scattering phase functions (SPFs) approach the actual scattering distribution will directly affect the accuracy of the solutions. Jonasz and Fouriner[9] compared the Fouriner-Forand (FF) SPF and the measured values, and the results show that the agreements between the empirical approximations and the realistic SPF vary with water turbidities and evaluation criteria. However, few criteria have been proposed to assess the performance of empirical SPFs.

In this paper, an evaluation criterion based on the cumulative distribution function of SPF is proposed. On basis of the measured SPFs for typical clear and turbid seawater and the theoretical SPFs of four single-component polydisperse systems computed by Mie theory, the empirical SPFs are compared with the actual ones in terms of fitting accuracy. The results show that the proposed criterion is more suitable than the other two for the analysis of light forward scattering properties.

As for seawater, many constitutions, including sea salts, various dissolved organic matters (DOMs) and suspended particles, are involved in the interaction of light with naturally occurring bodies of water[9]. The suspended particles are always the dominant source of the scattering found in water. The scattering coefficient of certain water mainly depends on the size and concentration of the suspended particles in it. Given the same scattering coefficient, the angular distribution of scattering intensity is determined by the shape, size distribution and relative refractive index of the suspended particles[10].

In order to compare the empirical SPFs with the experimental or theoretical ones, the following criteria are introduced. Fouriner defines the logarithm of SPF as the comparison kernel, and results in the average derivation as follows[9]:

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* This work has been supported by the National "863" Program (No. 2006AA09Z207).

** E-mail: fronter@bit.edu.cn

$$\varDelta_{\mathrm{p1}}=\frac{100}{N}\sqrt{\sum_{i=1}^{N}\frac{\left|\log_{10}p_{\mathrm{e}}(\theta_i)-\log_{10}p_{\mathrm{r}}(\theta_i)\right|}{\sigma_{\mathrm{r}}^{2}}}\quad, \tag{1}$$

where N is the number of data points and σ_{r}^2 is the variance of $\log_{10}p_{\mathrm{r}}$ over the whole applicable data set population. p_{e} and p_{r} represent the empirical and referenced SPFs, respectively.

Mobley defines a sin(è) weighted root-mean-square (RMS) percentage difference between the empirical and referenced SPFs as a quantitative criterion, and normalizes the two SPFs by their average value at each scattering angle so as to prevent the RMS difference from being dominated by the large magnitudes of phase functions at small scattering angles[3]. The criterion is given as

$$\varDelta_{\mathrm{p2}}=200\sqrt{\frac{1}{\theta_{\max}-\theta_{\min}}\int_{\theta_{\min}}^{\theta_{\max}}\left\{\frac{p_{\mathrm{e}}(\theta)-p_{\mathrm{r}}(\theta)}{p_{\mathrm{e}}(\theta)+p_{\mathrm{r}}(\theta)}\sin\theta\right\}^{2}\mathrm{d}\theta}\quad, \tag{2}$$

where $è_{\max}$ and $è_{\min}$ represent the maximum and minimum scattering angles, respectively.

The normalized cumulative probability function of SPF is given and denoted by P(è), which can also be defined as the comparison kernel, and results in the following expression of criterion:

$$\varDelta_{\mathrm{p3}}=100\sqrt{\frac{1}{\theta_{\max}-\theta_{\min}}\int_{\theta_{\min}}^{\theta_{\max}}[P_{\mathrm{e}}(\theta)-P_{\mathrm{r}}(\theta)]^{2}\,\mathrm{d}\theta}\quad, \tag{3}$$

where P_{e} and P_{r} are the cumulative probability distributions with respect to p_{e} and p_{r}, respectively.

Two kinds of SPFs are chosen to be the referenced SPFs for convenient comparison: the average SPFs of the clear and turbid seawater collected by Jonasz, the theoretical SPFs of four single-component polydisperse systems computed by Mie theory with suspended particles of calcium carbonate.

The most carefully made and widely cited light scattering distributions of seawater are given by Petzold. The similarity among the shapes of the measured volume scattering functions from different waters suggests that it is reasonable to define a kind of typical SPFs for typical seawater. A geometric average of 108 data sets of measured SPFs from a computer-readable data collection is compiled by Jonasz. According to Eqs.(1) and (2), the optimal fitting factors and errors are shown in Tab.1. It's noted that the data superscripted by "*" are magnified by 100 times. It is shown that the fitting error of FF is the minimum. Although the one-term Henyey-Greenstein (OTHG) has the fewest variables, its fitting accuracy is the worst.

Tab.1 Optimal fitting factors and errors of SPFs for typical seawater

Typical	FF				OTHG			TTHG				
seawater	$\varDelta_{\mathrm{p1}}$	$\varDelta_{\mathrm{p2}}$	m	n	$\varDelta_{\mathrm{p1}}$	$\varDelta_{\mathrm{p2}}$	g	$\varDelta_{\mathrm{p1}}$	$\varDelta_{\mathrm{p2}}$	g_1	g_2	α
Clear	2.63*		3.39	1.09	3.66*		0.96	3.30*		0.99	0.89	0.70
		9.48	3.46	1.07		14.6	0.96		12.4	0.97	-0.95	0.99
Turbid	2.11*		3.53	1.08	3.55*		0.95	2.13*		0.99	0.87	0.67
		2.81	3.41	1.11		5.20	0.95		4.95	0.99	0.88	0.65

The empirical SPFs of four single-component polydisperse suspensions with particles of calcium carbonate are compared with their theoretical ones so as to evaluate the performance of empirical SPFs. The particle size distributions of four suspensions shown in Fig.1 are measured by the laser particle size analyzer. The four kinds of suspensions are denoted by C1, C2, C3 and C4, respectively. The corresponding particle medium diameters D_{50} are 0.501 ìm, 2.351 ìm, 12.44 ìm and 16.69 ìm, respectively.

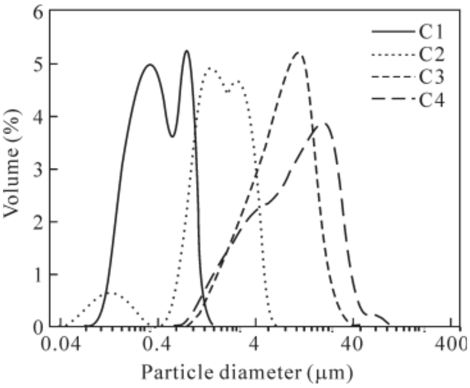


Fig.1 Size distribution of suspensions

The theoretical SPFs of four suspensions are shown in Fig.2. The related optimal fitting factors and errors are given in Tab.2. The effects of particle refractive index on SPFs are shown in Fig.3. The typical n is chosen to be 1.04, 1.12, 1.20 and 1.28, respectively.

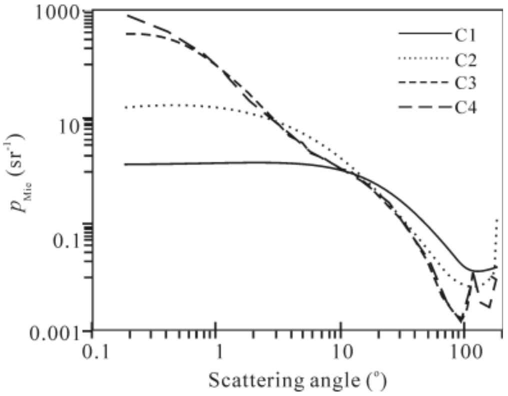
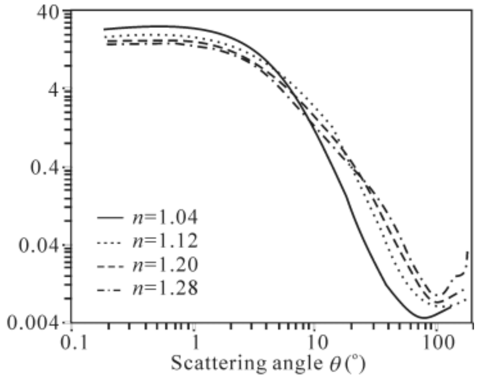


Fig.2 Mie SPFs of particles

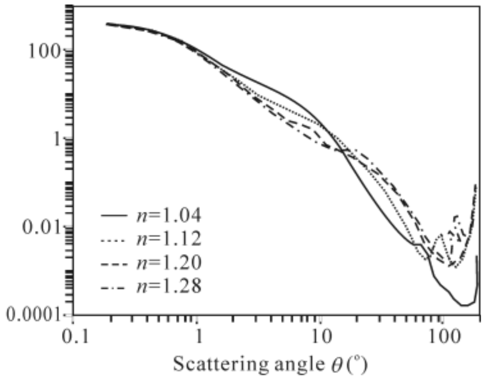
It can be found in Fig.3 and Tab.2 that the forward scattering ratio of SPF depends on the proportion of large particles in polydisperse systems. For polydisperse suspensions with constrained size distributions, the two-term Henyey-Greenstein (TTHG) owns the best fitting accuracy. The performance of OTHG is better than that of FF for suspensions of small particles, and vice versa for suspensions of large particles. For small particles, when the refractive index increases, the forward scattering ratio of SPF decreases and the backscattering ratio increases. For larger particles, the backscattering ratio increases with the particle refractive index, while the values of SPFs at small scattering angles are almost unchanged.

According to Tabs.1 and 2, although given different minimum fitting errors and optimal fitting factors, the three evaluation criteria show almost the same trend in fitting accuracy for the same group ($\ddot{A}_{p3}$ in group C2 excluded). It means that the three criteria can all be used to evaluate the performance of empirical SPFs based on the same referenced SPFs. Because of the effect of σ_r^2, $\ddot{A}_{p1}$ is only suitable for the comparison of the same set of data. However, $\ddot{A}_{p2}$ and $\ddot{A}_{p3}$ are applicable to compare different sets of data due to their normalized operations. In addition, it is known by analyzing the focus of the three evaluation criteria that $\ddot{A}_{p1}$ emphasizes the fitting accuracy at forward and backward scattering angles, $\ddot{A}_{p2}$ emphasizes the accuracy of the total scattering coefficient, while $\ddot{A}_{p3}$ emphasizes the fitting accuracy at forward scattering angles. For many reasons in underwater photography, the portion of the light that is scattered in the forward direction with respect to the direction of the incident light is of primary importance. Thus, $\ddot{A}_{p3}$ can be used to evaluate the influence of empirical SPFs on image blur effect caused by the forward scattering light.

An evaluation criterion based on the cumulative probability distribution is proposed. Together with the other two criteria proposed previously, the minimum fitting errors and the optimal fitting factors are computed for the three kinds of empirical SPFs and their related referenced SPFs, which are chosen to be the average SPFs of the clear and turbid seawater and the theoretical SPFs of four single-component polydisperse systems with different sizes of suspended particles of calcium carbonate. Besides, the influence of the



(a) C2 group



(b) C3 group

Fig.3 Relationship between SPFs and relative refractive index

Tab.2 Optimal fitting factors and minimum errors for the SPFs

NO.	D_{50} (im)	FF			OTHG		TTHG			
		$\ddot{A}_{p1}/\ddot{A}_{p2}/\ddot{A}_{p3}$	m	n	$\ddot{A}_{p1}/\ddot{A}_{p2}/\ddot{A}_{p3}$	g	$\ddot{A}_{p1}/\ddot{A}_{p2}/\ddot{A}_{p3}$	g_1	g_2	á
C1	0.501	3.41	3.98	1.19	2.57	0.69	1.58	0.71	-0.43	0.98
		11.50	4.05	1.19	6.96	0.70	3.54	0.72	-0.52	0.98
		5.22	4.22	1.19	0.52	0.70	0.41	0.70	-0.60	0.99
C2	2.351	0.57	3.76	1.19	0.78	0.85	0.40	0.89	-0.33	0.96
		13.20	3.66	1.19	20.50	0.85	6.34	0.90	-0.29	0.95
		2.93	3.76	1.19	1.78	0.87	0.82	0.88	-0.50	0.97
C3	12.44	9.28*	3.44	1.19	10.00*	0.89	8.50*	0.93	-0.81	0.99
		33.10	3.32	1.19	35.10	0.93	29.00	0.96	-0.39	0.98
		1.89	3.51	1.19	3.61	0.92	1.10	0.99	0.85	0.41
C4	16.69	5.34*	3.40	1.19	5.69*	0.90	4.96*	0.92	-0.94	0.99
		33.70	3.30	1.19	35.40	0.94	29.80	0.96	-0.41	0.98
		2.02	3.50	1.19	3.79	0.93	1.25	0.99	0.85	0.42

particle refractive index on the shape of the SPFs is also analyzed. It is concluded that OTHG and TTHG SPFs suit small particle cases, while FF SPF suits large particle cases. Furthermore, $\ddot{A}_{\mathrm{p}1}$ emphasizes the backscattering effect, $\ddot{A}_{\mathrm{p}2}$ emphasizes the total scattering distribution, while $\ddot{A}_{\mathrm{p}3}$ emphasizes the forward scattering effect.

As the increasing urgent demands for marine resources exploitation, underwater optical imaging technology develops rapidly in recent years. The effects of forward and backward scattering on underwater image degradation are considered to be the main sources determining the detecting range and image quality for underwater optoelectronic imaging equipments. The work in this paper can provide the fundamentation for the research of underwater optical imaging theory and technology.

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