Generation of four-photon GHZ states based on interaction between a four-level atom and two bimodal cavities*

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A simple scheme is presented for generating four-photon Greenberger-Horne-Zeilinger (GHZ) states with interaction between a four-level atom and two bimodal cavities. In the proposed protocol, the quantum information is encoded on Fock states of the cavity fields. The detection of the atom can collapse the cavity to the desired state. The experimental feasibility of our proposal is also discussed.

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Cavity quantum electrodynamics (QED) is an ideal candidate for implementing quantum information processing[1-10]. The reason is based on the following two points. First, photons are ideal carriers for fast and reliable communication over long distances, and the atoms are good memorizers for storing and processing quantum information. Thus the combination of atoms and photons can be useful in quantum computation. Second, the atoms trapped in a high-Q cavity have long decoherence time[11]. Entanglement of two or more particles is the most interesting characteristic of quantum mechanics. Entangled states not only are recognized as an essential ingredient for testing the foundation of quantum mechanics, but also have many significant applications in quantum information processing (QIP). Generally, the more particles can be entangled, the more clearly nonclassical effects are exhibited, and the more useful the states are for quantum applications[12]. Thus the generation and manipulation of multipartite entangled states are very important tasks in QIP and have been attracting much attention. Dür et al[13] showed that there are two inequivalent classes of tripartite entanglement states, i.e., the GHZ class and the W class, under stochastic local operations and classical communications. GHZ type of entangled state has many interesting properties. For example, the three-particle GHZ state is maximally stable against noise, maximally violates Bell inequalities, and can be used to implement perfect teleportation[14]. Hence, the preparation of the GHZ entangled state has become a critical technique in QIP. Meng et al[15] proposed a scheme for preparing an N-atom GHZ entangled state through the interaction between N atoms and a cavity. Although many schemes for tripartite entangled states have been studied[16,17], the reports about preparing multiphoton (N>3) GHZ state are very few.

In this paper, we propose a scheme for the generation of four-photon GHZ states via cavity QED. The proposed scheme is more simple and experimentally feasible, because we only employ one four-level atom and two cavities. The considered system is an interaction between a four-level atom and two bimodal cavities. The level structure of the atomic configuration is shown in Fig.1. The atom states are denoted by $|g\rangle$, $|i\rangle$, $|j\rangle$ and $|e\rangle$, with the relevant Bohr frequencies ω_g, ω_i, ω_j and ω_e, respectively. As illustrated in Fig.1, let the atom pass through a high-Q bimodal cavity with two mode circular frequencies ω_a and ω_b, and the modes with annihilation/creation operators a /a^+ and b /b^+ interact with the transitions $|g\rangle \leftrightarrow |j\rangle$, and $|i\rangle \leftrightarrow |e\rangle$, respectively. The two atomic transitions $|g\rangle \leftrightarrow |j\rangle$ and $|i\rangle \leftrightarrow |e\rangle$ resonantly couple to the cavity modes with coupling constants g_a and g_b. Simultaneously, the classical laser field with Rabi frequency Ω_L resonantly drives the dipole-forbidden atomic transition $|i\rangle \leftrightarrow |j\rangle$.

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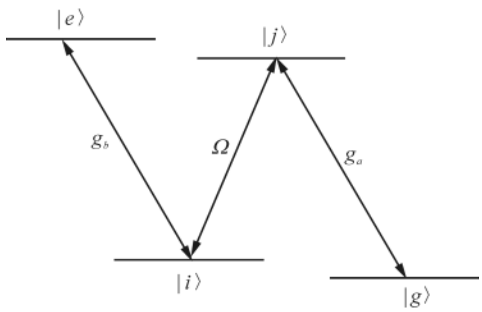

Fig.1 Involved level structure of the four-level atom and the corresponding atomic transition

The Hamiltonian (assuming $\hbar = 1$) of the whole system under consideration is[18]

$$H = H_{\text{free}} + H_{\text{ac}} + H_{\text{d}} \ , \tag{1}$$

where

$$H_{\text{free}} = \omega_a a^+ a + \omega_b b^+ b + \sum_A \omega_A \,|\, A >< A \,|, (A = g, i, j, e) \ , \tag{2}$$

$$H_{\text{ac}} = g_a \,|\, j >< g \,|\, a + g_b \,|\, e >< i \,|\, b + h.c., \tag{3}$$

$$H_{\text{d}} = \Omega(\mathrm{e}^{\mathrm{i}\omega t} \,|\, i >< j \,| + \mathrm{e}^{-\mathrm{i}\omega t} \,|\, j >< i \,|) \ , \tag{4}$$

where, H_{free} is the free energy of the atom and the cavity fields, H_{ac} represents the interaction energy between the atom and the cavity modes, and H_{d} describes the interaction between the atom and the laser fields. In this study, we must think over the case of the strong classical field. In order to demonstrate the modification of the system dynamics by the strong classical field, we define the new dressed state as

$$|+>= \frac{1}{\sqrt{2}}(|\, i > + \,|\, j >) \ , \tag{5}$$

$$|->= \frac{1}{\sqrt{2}}(|\, i > - \,|\, j >) \ . \tag{6}$$

After defining the above atomic basis, the Hamiltonian of the whole system in the interaction picture can be written as $H_1 = H_0 + H_{\text{i}}$, where

$$H_0 = \Omega\,(|+><+| - |-><-|) \ , \tag{7}$$

$$H_{\text{i}} = \frac{g_a}{\sqrt{2}} a(|+> - |->) < g \,| + \frac{g_b}{\sqrt{2}} b \,|\, e > (<+| + <-|) + h.c. \ . \tag{8}$$

The time evolution of this system is determined by Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}\,|\Psi(t)>}{\mathrm{d}t} = H_1 \,|\Psi(t)> \cdot \tag{9}$$

Perform the unitary transformation

$$|\Psi(t)>= \mathrm{e}^{-\mathrm{i}H_0 t}\,|\Psi'(t)> \ , \tag{10}$$

and then we can obtain

$$\mathrm{i}\frac{\mathrm{d}\,|\Psi'(t)>}{\mathrm{d}t} = H_{\text{i}}'|\Psi'(t)> \ , \tag{11}$$

where

$$H_{\text{i}}' = \frac{g_a}{\sqrt{2}} a(\mathrm{e}^{\mathrm{i}\Omega t}\,|+> - \mathrm{e}^{-\mathrm{i}\Omega t}\,|->) < g \,| + \frac{g_b}{\sqrt{2}} b \,|\, \mathrm{e} > (\mathrm{e}^{-\mathrm{i}\Omega t} <+| + \mathrm{e}^{\mathrm{i}\Omega t} <-|) + h.c \ . \tag{12}$$

In the strong driving regime, with the choice of $\Omega >> \{g_a, g_b\}$ and under the rotating wave approximation, we can neglect the effect of rapidly oscillating terms with high frequencies. Using the time averaging method[19], we can further obtain an effective Hamiltonian[10]

$$H_{\text{eff}} = -\lambda ab \,|\, e >< g \,| - \lambda a^+ b^+ \,|\, g >< e \,| \ , \tag{13}$$

where $g_a g_b / \Omega = \lambda$ is defined by the effective coupling coefficient. For simplicity, we assume that the cavities are initially prepared in $|1,1\rangle_1$ and $|0,0\rangle_2$, and the atom is initially in $|g\rangle$. According to Eq.(13), we solve the corresponding Schrödinger equation and obtain expression of the state. We send the atom through the first cavity, so after an interaction time t_1, the state evolves into

$$|\psi(t_1)\rangle = [\cos(\lambda t_1)|g\rangle|1,1\rangle_1 + \mathrm{i}\sin(\lambda t_1)|e\rangle|0,0\rangle_1]|00\rangle_2 . \tag{14}$$

Then we let the atom pass through the second cavity, and after an interaction time t_2, the atom-cavity system evolves into the state as

$$|\psi(t_1+t_2)\rangle = \cos(\lambda t_1)|1,1\rangle_1|g\rangle|0,0\rangle_2 + \mathrm{i}\sin(\lambda t_1)|0,0\rangle_1 \times [\mathrm{i}\sin(\lambda t_2)|g\rangle|1,1\rangle_2 + \cos(\lambda t_2)|e\rangle|0,0\rangle_2]. \tag{15}$$

By choosing $\lambda t_1 = \frac{\pi}{4}$ and $\lambda t_2 = \frac{\pi}{2}$, we can obtain the state as

$$|\psi(t_1+t_2)\rangle = \frac{\sqrt{2}}{2}\left[|g\rangle(|1,1\rangle_1|0,0\rangle_2 - |0,0\rangle_1|1,1\rangle_2) + \mathrm{i}|e\rangle|1,1\rangle_1|0,0\rangle_2\right] . \tag{16}$$

We can perform a measurement on the atom, if the atom is detected in the state $|g\rangle$, and the cavity field collapses into the following states

$$|\psi(t_1+t_2)\rangle_g \to \frac{\sqrt{2}}{2}(|1,1\rangle_1|0,0\rangle_2 - |0,0\rangle_1|1,1\rangle_2) \ , \tag{17}$$

where the states $|\psi(t_1+t_2)\rangle_g$ are four-photon GHZ states.

Finally, we give a brief discussion about the experimental feasibility of the proposed scheme within cavity QED. As with any proposal for quantum computing implementation, its success ultimately depends on being able to complete many coherent dynamics during the decoherence time, so the atomic and cavity lifetime should be larger than the interaction time of the atoms with the cavity fields. As a possible implementation using ^{87}Rb, the above atomic level structures can be obtained[20]. For example, the states $|g>$ and $|i>$ are$|F=2, m=0>$ and $|F=2, m=2>$ of $5S_{1/2}$, respectively, and $|j>$and $|e>$ are $|F=2\ m=-1>$ and $|F=2, m=1>$ of $5P_{1/2}$, respectively. Therefore, based on cavity QED techniques, the proposed scheme might be realizable.

In summary, we propose a simple method for generating four-photon GHZ states via cavity QED based on the interaction between a four-level atom and two cavity modes. Compared with previous schemes, our proposal is more simple and experimentally feasible, since the detection of atom can collapse the cavity to the desired state.

References

[1] Raimond J. M. and Brune M., Rev. Mod. Phys. **73**, 65 (2001).

[2] Deng Xiao-hui, Wang Wen-wei, Zhang Deng-yu and You Kaiming, Physica B **406**, 656 (2011).

[3] Wang Xin-wen, Xia Li-xin, Wang Zhi-yong and Zhang Dengyu, Optics Communication **283**, 1196 (2010).

[4] Lü Xin-you, Phys. Rev. A **78**, 032305 (2008).

[5] C. E. Lopez, G. Romero and J. C. Retamal, Phys. Rev. A **81**, 062114 (2010).

[6] Wang Xin-wen and Yang Guo-jian, Phys. Rev. A **79**, 062315 (2009).

[7] Wang Xin-wen and Yang Guo-jian, Phys. Rev. A **79**, 064306 (2009).

[8] Han Feng, Man Zhong-xiao and Xia Yun-jie, Journal of Optoelectronics ·Laser **21**, 470 (2010). (in Chinese)

[9] Zhao Feng and Li Jing-ling, Journal of Optoelectronics • Laser **21**, 1383 (2010). (in Chinese)

[10] ZHAO Nan, PEI Chang-xing, QUAN Dong-xiao and SUN Xiao-nan, Journal of Optoelectronics • Laser **21**, 1411 (2010). (in Chinese)

[11] Tang Shi-qing, Zhang Deng-yu and Xie Li-jun, Chin. Phys. Lett. **26**, 20310 (2009).

[12] Wang Xin-wen, Shan Yong-guang, Xia Li-xin and Lu Maowang, Phys. Lett. A **36**, 7 (2007).

[13] Dür W, Vidal G. and Cirac J I, Phys. Rev. A **62**, 062314 (2000).

[14] Khrennikov A, Contextual Approach to Quantum Fomalism, Berlin:Springer, 2009.

[15] Meng Fan-yu, Zhu Ai-dong, Yeon Kyu-Hwang and Yu Seong-Cho, Phys. Scr. **81**, 015009 (2010).

[16] Zheng Shi-biao, Phys. Rev. Lett. **87**, 230404 (2001).

[17] Wu Chun-feng, Chen Jing-ling, Kwek L C and Oh C. H, Phys. Rev. A **73**, 012310 (2006).

[18] Xiao Yunfeng, Phys. Rev. A **74**, 044303 (2006).

[19] James D F, Fortschr. Phys. **48**, 823 (2000).

[20] Maunz P, Nature **428**, 50 (2004).