A rotating inertial navigation system with the rotating axis error compensation consisting of fiber optic gyros*

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An effective and flexible rotation and compensation scheme is designed to improve the accuracy of rotating inertial navigation system (RINS). The accuracy of single-axial RINS is limited by the errors on the rotating axis. A novel inertial measurement unit (IMU) scheme with error compensation for the rotating axis of fiber optic gyros (FOG) RINS is presented. In the scheme, two couples of inertial sensors with similar error characteristics are mounted oppositely on the rotating axes to compensate the sensors error. Without any change for the rotation cycle, this scheme improves the system's precision and reliability, and also offers the redundancy for the system. The results of 36 h navigation simulation prove that the accuracy of the system is improved notably compared with normal strapdown INS, besides the heading accuracy is increased by 3 times compared with single-axial RINS, and the position accuracy is improved by 1 order of magnitude.

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The rotating inertial navigation system (RINS) is a kind of strapdown inertial navigation system (SINS), in which the sensor and system errors are modulated by the periodical rotating of inertial measurement unit (IMU). The rotation modulation technology was proposed by Levinson[1,2] in 1980. And then, American developed a series of RINS consisting of optical gyro, such as MARLIN, MK39, MK49, and AN/WSN[3,4]. The accuracy of the system is improved greatly when the rotation modulation technology is adopted.

The effective and optimized rotation scheme is the key factor to improve the accuracy of the system. Yuan[5] analyzed the error characteristics of 2-position and 4-positon schemes in single-axial rotation, and proposed the improved 8-position and 16-position rotation-dwell schemes for dual-axial rotation. Wen, Huang and Zhang[6-9] researched the compensation principle of single-axial and dual-axial rotation schemes, and indicated that the error on the rotating axis can't be compensated in the single-axial scheme. And the relevant research team[10,11] developed the prototype systems with different rotation schemes, and tested them on sea.

A novel scheme based on single-axial rotation is proposed in this paper. In the proposed scheme, two couples of inertial sensors with similar error characteristics are mounted on the opposite direction of the rotation axes. The error on the rotation axis, which can't be modulated in the single-axial rotation scheme, can be eliminated or reduced by the sensors. The system can operate as a single-axial RINS while a couple of sensors on the rotating axis are out of order.

Coordinate systems are defined before the analysis. n is the navigation frame of reference, i is the inertial frame, b is the vehicle body frame, and p is rotation frame, which is concurrent with b frame when IMU does not rotate. The inertial sensors are mounted in p. The error equations of RINS are presented as

$$\dot{\phi} = -\omega_{\mathrm{in}}^{\mathrm{n}} \times \phi + \delta\omega_{\mathrm{in}}^{\mathrm{n}} - \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}} \boldsymbol{C}_{\mathrm{p}}^{\mathrm{b}} \delta\omega_{\mathrm{ip}}^{\mathrm{p}} , \tag{1}$$

$$\delta\dot{v} = f^{\mathrm{n}} \times \phi + \boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}} \boldsymbol{C}_{\mathrm{p}}^{\mathrm{b}} \delta f_{\mathrm{ip}}^{\mathrm{p}} - (2\omega_{\mathrm{ie}}^{\mathrm{n}} + \omega_{\mathrm{en}}^{\mathrm{n}}) \times \delta v - (2\delta\omega_{\mathrm{ie}}^{\mathrm{n}} + \delta\omega_{\mathrm{en}}^{\mathrm{n}}) \times v - \delta g , \tag{2}$$

where ϕ is attitude error and δv is velocity error of the system. $\delta\omega_{\mathrm{ip}}^{\mathrm{p}}$ and $\delta f_{\mathrm{ip}}^{\mathrm{p}}$ represent the measurement errors of the inertial sensors (gyro and accelerometer), and $\boldsymbol{C}_{\mathrm{p}}^{\mathrm{b}}$ and $\boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}$ are the transformation matrices between different reference frames. The subscripts in the equations represent the relationship between the coordinate systems.

During the navigation calculation, the sensor errors $\delta\omega_{\mathrm{ip}}^{\mathrm{p}}$ and $\delta f_{\mathrm{ip}}^{\mathrm{p}}$ are transformed from p frame to n frame with the transformation matrices $\boldsymbol{C}_{\mathrm{p}}^{\mathrm{b}}$ and $\boldsymbol{C}_{\mathrm{b}}^{\mathrm{n}}$. Assuming that the IMU

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rotates with z axis[10], the $\boldsymbol{C}_{\mathrm{p}}^{\mathrm{b}}$ is shown as

$$\boldsymbol{C}_{\mathrm{p}}^{\mathrm{b}}(t)=\begin{bmatrix}\cos\omega t & -\sin\omega t & 0\\ \sin\omega t & \cos\omega t & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{3}$$

The sensor errors can be expressed as $\delta\omega_{\mathrm{ip}}^{\mathrm{p}}=[\delta\omega_{\mathrm{ip}}^{\mathrm{p}x},\delta\omega_{\mathrm{ip}}^{\mathrm{p}y},\delta\omega_{\mathrm{ip}}^{\mathrm{p}z}]$. Substitute Eq.(3) into Eq.(1), and then we can get the third item of the right hand of Eq.(1) as following:

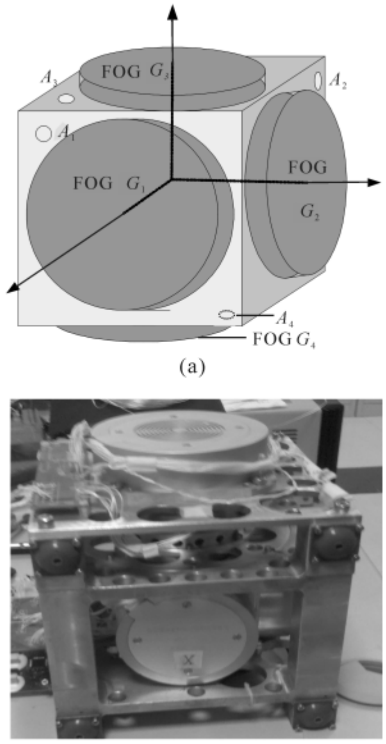
$$[\delta\omega_{\mathrm{ip}}^{\mathrm{p}x}\cos(\omega t)-\delta\omega_{\mathrm{ip}}^{\mathrm{p}y}\sin(\omega t)\quad \delta\omega_{\mathrm{ip}}^{\mathrm{p}x}\sin(\omega t)+\delta\omega_{\mathrm{ip}}^{\mathrm{p}y}\cos(\omega t)\quad \delta\omega_{\mathrm{ip}}^{\mathrm{p}z}]^{\mathrm{T}}.$$

The errors in the x and y axes are modulated to sine or cosine function, which can be eliminated by the integral process in the whole period during navigation calculation. But the error in the rotating axis z does not change compared with the SINS. Besides, the additional error caused by the couple of the scale factor error of the sensor and the rotary speed is brought into the RINS, which can limite the system accuracy badly.

In the dual-axial rotation scheme, the errors in three orthogonal axes can be compensated, but the rotation control is complex and hard to implement. So the system reliability decreases subsequently.

The error compensation scheme is designed on the basis of the single-axial RINS. The system is composed of IMU, rotation control component and angular measurement device. The rotation control component drives the IMU rotating with the 4-position rotation-dwell scheme[4]. The angular measurement device feeds the IMU back to rotation control component to implement the expected rotation. The main difference between the proposed scheme and the single-axial RINS is the architecture of the IMU, which is shown as Fig.1. In the architecture, two couples of gyros and accelerometers are mounted on the rotating axes with opposite directions. The outputs of the opposite gyros and accelerometers are fused for navigation calculation. The error on the rotating axis can be reduced or eliminated consequently if the error characteristics of the opposite sensors are similar. Besides, the errors in the x and y axes are modulated and eliminated by the IMU rotation.

As shown in Fig.1, the RINS consisting of 4 FOGs and 4 accelerometers is constructed to prove the validity of the error compensation scheme. The sensors G_1 and A_1 are mounted orthogonally with G_2 and A_2 as a block and the sensor block is fixed on the axis of the motor. G_3 and A_3 are fixed on the horizontal plate on the top of the IMU, and G_4 and A_4 are mounted on the opposite plate at the bottom of the IMU.



(a)

(b)

Fig.1(a) Schematic diagram and (b) photo of the IMU architecture

In this scheme, outputs of the four couples of gyros and accelerometers, which are measured in frame of reference, are used for navigation calculation. We mark the outputs of four gyros with $\omega_{\mathrm{ip}\,x}^{\mathrm{p}},\omega_{\mathrm{ip}\,y}^{\mathrm{p}},\omega_{\mathrm{ip}\,z^{+}}^{\mathrm{p}}$ and $\omega_{\mathrm{ip}\,z^{-}}^{\mathrm{p}}$, and mark those of four paccelerometers with f_x^{p}, f_y^{p}, $f_{z^{+}}^{\mathrm{p}}$ and $f_{z^{-}}^{\mathrm{p}}$. On the rotating axis of fiber optic gyros (FOG), the same angular rate and acceleration are measured by two couples of opposite sensors. Simply we can average the outputs of the opposite sensors to compensate the error in the rotating axis direction. Therefore the angular rate and acceleration in the rotation frame can be expressed as $[\omega_{\mathrm{ip}\,x}^{\mathrm{p}},\omega_{\mathrm{ip}\,y}^{\mathrm{p}},(\omega_{\mathrm{ip}\,z^{+}}^{\mathrm{p}}-\omega_{\mathrm{ip}\,z^{-}}^{\mathrm{p}})/2]^{\mathrm{T}}$ and $[f_x^{\mathrm{p}},f_y^{\mathrm{p}},(f_{z^{+}}^{\mathrm{p}}-f_{z^{-}}^{\mathrm{p}})/2]$. The outputs are measured in rotation frame, and should be transformed to navigation frame during navigation. So that can be presented as follows according to Eq.(3)

$$\begin{aligned}\omega_{\mathrm{ib}\,1}^{\mathrm{b}}&=\omega_{\mathrm{ip}\,1}^{\mathrm{p}}\cos(\omega t)-\omega_{\mathrm{ip}\,2}^{\mathrm{p}}\sin(\omega t)\\ \omega_{\mathrm{ib}\,2}^{\mathrm{b}}&=\omega_{\mathrm{ip}\,1}^{\mathrm{p}}\sin(\omega t)+\omega_{\mathrm{ip}\,2}^{\mathrm{p}}\cos(\omega t)\\ \omega_{\mathrm{ib}\,3}^{\mathrm{b}}&=(\omega_{\mathrm{ip}\,3}^{\mathrm{p}}-\omega_{\mathrm{ip}\,4}^{\mathrm{p}})/2\ ,\end{aligned} \tag{4}$$

$$\begin{aligned}f_1^{\mathrm{b}}&=f_1^{\mathrm{p}}\cos(\omega t)-f_2^{\mathrm{p}}\sin(\omega t)\\ f_2^{\mathrm{b}}&=f_1^{\mathrm{p}}\sin(\omega t)+f_2^{\mathrm{p}}\cos(\omega t)\\ f_3^{\mathrm{b}}&=(f_1^{\mathrm{p}}-f_2^{\mathrm{p}})/2\ .\end{aligned} \tag{5}$$

$\omega_{\mathrm{ib}}^{\mathrm{b}}$ and f^{b} are angular rate and acceleration expressed in the b frame, respectively, which are used for the navigation calculation after the coordinate transformation.

Furthermore, the architecture improves the redundancy and fault tolerance of system. The system can operate as a single-axial RINS while a couple of sensors on the rotating axis malfunction. The corresponding measurement data on rotating axis are shown in Tab.1.

Tab.1 Measurement data on rotating axis adopted by the rotating SINS

	No fault	A_3 fault	A_4 fault
No fault	$\omega_{\mathrm{ib}3}^{\mathrm{b}}=(\omega_{\mathrm{ip}3}^{\mathrm{p}}-\omega_{\mathrm{ip}4}^{\mathrm{p}})/2$ $f_3^{\mathrm{b}}=(f_1^{\mathrm{p}}-f_2^{\mathrm{p}})/2$	$\omega_{\mathrm{ib}3}^{\mathrm{b}}=(\omega_{\mathrm{ip}3}^{\mathrm{p}}-\omega_{\mathrm{ip}4}^{\mathrm{p}})/2$ $f_3^{\mathrm{b}}=-f_2^{\mathrm{p}}$	$\omega_{\mathrm{ib}3}^{\mathrm{b}}=(\omega_{\mathrm{ip}3}^{\mathrm{p}}-\omega_{\mathrm{ip}4}^{\mathrm{p}})/2$ $f_3^{\mathrm{b}}=f_1^{\mathrm{p}}$
G_3 fault	$\omega_{\mathrm{ib}3}^{\mathrm{b}}=-\omega_{\mathrm{ip}4}^{\mathrm{p}}$ $f_3^{\mathrm{b}}=(f_1^{\mathrm{p}}-f_2^{\mathrm{p}})/2$	$\omega_{\mathrm{ib}3}^{\mathrm{b}}=-\omega_{\mathrm{ip}4}^{\mathrm{p}}$ $f_3^{\mathrm{b}}=-f_2^{\mathrm{p}}$	$\omega_{\mathrm{ib}3}^{\mathrm{b}}=-\omega_{\mathrm{ip}4}^{\mathrm{p}}$ $f_3^{\mathrm{b}}=f_1^{\mathrm{p}}$
G_4 fault	$\omega_{\mathrm{ib}3}^{\mathrm{b}}=\omega_{\mathrm{ip}3}^{\mathrm{p}}$ $f_3^{\mathrm{b}}=(f_1^{\mathrm{p}}-f_2^{\mathrm{p}})/2$	$\omega_{\mathrm{ib}3}^{\mathrm{b}}=\omega_{\mathrm{ip}3}^{\mathrm{p}}$ $f_3^{\mathrm{b}}=-f_2^{\mathrm{p}}$	$\omega_{\mathrm{ib}3}^{\mathrm{b}}=\omega_{\mathrm{ip}3}^{\mathrm{p}}$ $f_3^{\mathrm{b}}=f_1^{\mathrm{p}}$

The error characteristics on the horizontal axes are similar to those of the single-axis RINS, which is described in Refs.[6] and [7].

The error on the rotating axis is reduced because of the data fusion of the opposite sensors. Assume that the true angular rate in the rotating axis is $\omega_{\mathrm{ip}}^{\mathrm{p}}$, and the errors of the opposite gyros are $\delta\omega_{\mathrm{ip}3}^{\mathrm{p}}$ and $\delta\omega_{\mathrm{ip}4}^{\mathrm{p}}$. Consequently, the practical outputs of gyro G_3 and G_4 are as follows

$$\omega_{\mathrm{ip}3}^{\mathrm{p}}=\omega_{\mathrm{ip}}^{\mathrm{p}}+\delta\omega_{\mathrm{ip}3}^{\mathrm{p}}\,, \tag{6}$$

$$\omega_{\mathrm{ip}4}^{\mathrm{p}}=-\omega_{\mathrm{ip}}^{\mathrm{p}}+\delta\omega_{\mathrm{ip}4}^{\mathrm{p}}\,. \tag{7}$$

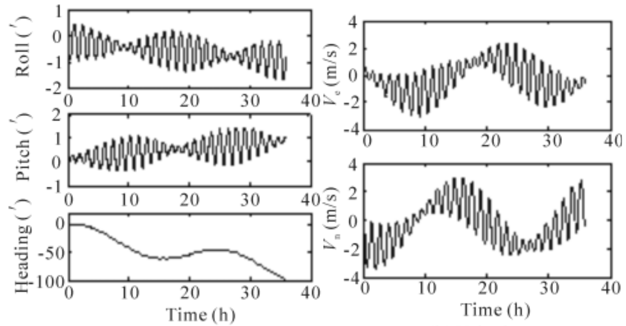
According to Eq.(4), the angular rate used for navigation calculation can be written as

$$\omega_{\mathrm{ib}3}^{\mathrm{b}}=\omega_{\mathrm{ip}}^{\mathrm{p}}+(\delta\omega_{\mathrm{ip}3}^{\mathrm{p}}-\delta\omega_{\mathrm{ip}4}^{\mathrm{p}})/2\,. \tag{8}$$

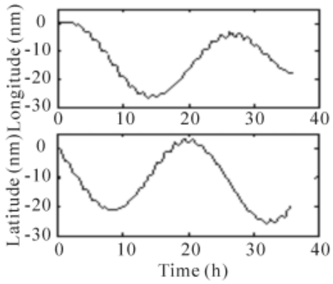
So the error on the rotating axis in the proposed scheme is ($\delta\omega_{\mathrm{ip}3}^{\mathrm{p}}-\delta\omega_{\mathrm{ip}4}^{\mathrm{p}}$)/2, which decreases compared with $\delta\omega_{\mathrm{ip}3}^{\mathrm{p}}$ and $\delta\omega_{\mathrm{ip}4}^{\mathrm{p}}$. At the optimum condition, the error approaches to zero if the error characteristics of the opposite gyros are similar. Meanwhile the additional error caused by the gyro error and the rotary speed consequently reduces. Subsequently, the accuracy of the system increases greatly while the compensation scheme is performed on the rotating axis.

SINS, single-axial RINS and RINS with error compensation are simulated respectively. The bias instabilities of the two horizontal gyros are 0.05 °/h and their RWC is 0.05 °/h. The bias instabilities of the two opposite gyros in the rotating axis are 0.0001°/h, and their RWC is 0.0005 °/h. The accelerometer bias is set as 50 μg with white noise of 25 μg. The scale factor errors of all sensors are set as 50 ppm, and alignment errors are set as 2 s. The shake situation is the typical motion of ship. The IMU rotates with the 4-position rotation-dwell scheme. The rotation speed is set as 1°/s, and the dwell time at each position is 60 s. The navigation time is set as 36 h at the interval of 1 s.

Figs.2 to 4 are the system errors of the SINS, the single-axial RINS and the RINS with compensation, respectively.



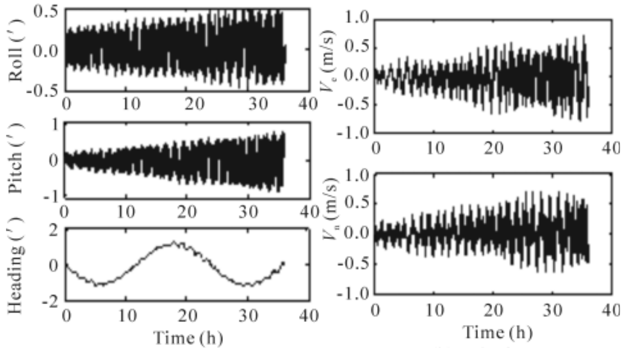
(a) Attitude error (b) Velocity error



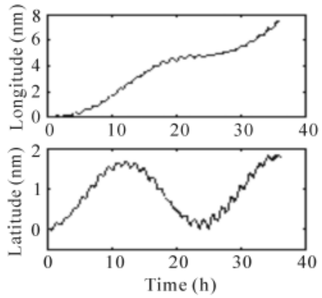
(c) Position error

Fig.2 Errors of the SINS

As shown in Figs.2-4, the accuracy of the proposed scheme is improved compared with the SINS. For the single-axial RINS, the attitude and velocity errors of the system are equal before and after the compensation, but its heading error reduces to 0.5′ from 2′ and the position error decreases to 1 nm from 10 nm while the compensation scheme is adopted.

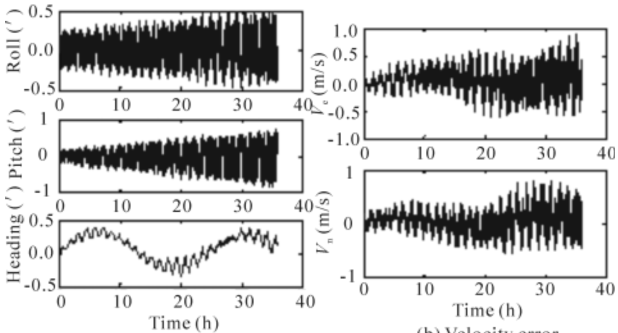


(a) Attitude error (b) Velocity error

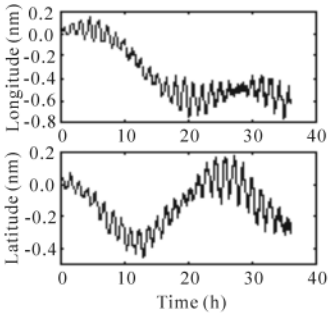


(c) Position error

Fig.3 Errors of the single-axial RINS



(a) Attitude error (b) Velocity error

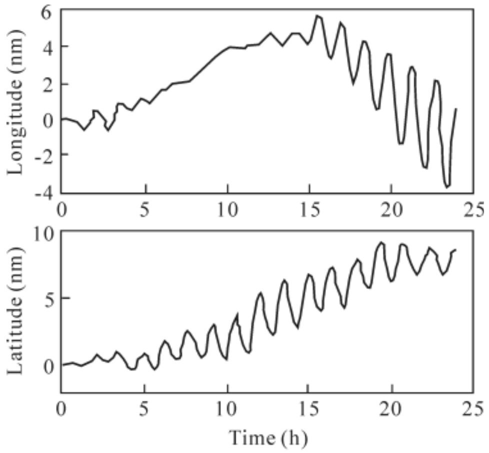


(c) Position error

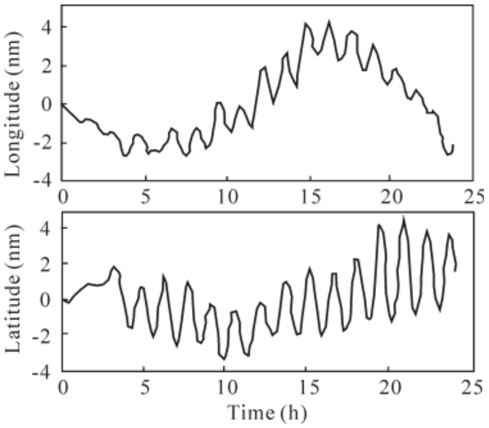
Fig.4 Errors of the RINS with rotating axis error compensation

The 24-hour static tests for the RINS with and without the rotating axis error compensation are carried out. Their position errors are illustrated in Fig.5. Compared with the results in Fig.5(a), the position error of RINS in Fig.5(b) reduces to 5.6 nm from 11.7 nm in 24 h. The accuracy of the system is improved by one time when the compensation scheme of the rotating axis is adopted.

The conclusion can be drawn that the proposed compensation scheme reduces the errors in the rotating axis and subsequently increases the accuracy of the system. Besides, the architecture of the IMU in the proposed scheme improves the redundancy and fault tolerance of system.



(a) Without error compensation



(b) With the proposed compensation scheme

Fig.5 Position errors of the RINS

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