Numerical analysis of intermodal delay in few-mode fibers for mode division multiplexing in optical fiber communication systems*

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(Received 28 August 2011)

In order to achieve higher spectral efficiency, mode division multiplexing (MDM) in few-mode fibers is a new research area. The idea faces lots of technical issues including intermodal delay and mode coupling which limit the achievable length of the system. This paper is designated to complete the analysis of intermodal delay in step-index few-mode fibers. We analyze numerically all the parameters of fiber, which could impact intermodal delay in few-mode fibers and identify the conditions which can increase the number of multiplex modes without significant increase in maximum intermodal delay.

Document code: A **Article ID:** 1673-1905(2012)02-0138-4

DOI 10.1007/s11801-012-1110-7

Optical fiber communication transforms from conventional on-off keying to coherent optical domain with polarization multiplexing to increase information transmission rate[1]. Recently, another fundamental change has been conceived by adding another degree of freedom in information mapping with mode division multiplexing (MDM) which is also termed as spatial multiplexing in optical fiber communication[2]. The information theory reveals that adding a degree of freedom in information mapping can increase the information rate in multiplicative order[3]. Multimode fibers can transmit multiple modes with distinct propagation constants and unique spatial distributions of electric field on the cross section of fiber cores. The modes can be considered as orthogonal modes on spatial distribution of fiber cross sections[4]. If the orthogonal spatial modes are collectively stimulated in fiber, fiber core can be assumed as a bunch of separated transmission channels through the fiber.

This new theory has been experimentally tested and reported as a proof of principle[2,5,6]. Most of the experiments use two-mode fibers or few-mode fibers as transmission media to avoid excessive mode coupling[7], and employ multi-input multi-output (MIMO) signal processing in discrete domain at the receiver to remove propagation effects[8]. Few-mode fibers, in contrast to multimode fibers, allow limited number of modes to be guided depending upon normalized frequency of the fiber. All the reported experiments are with fewer distances[5]. Primarily, the length of the transmission link is limited by intermodal delay, intermode coupling and loss profiles of each mode[9]. This paper specifically focuses on intermodal delay in few-mode fibers. In the presence of mode coupling, larger intermodal delay restricts the performance of signal processing to counter the effects of mode coupling through propagation length[10]. So whatever transmission format has been used, intermodal delay is a parameter to be considered for overall system performance.

This paper is focused on theoretical and numerical study of intermodal delay in few-mode fibers, especially for MDM. Its dependence on physical parameters of step index few-

* This work has been supported by the National Basic Research Program of China (No.2010CB328300), the National Natural Science Foundation of China (Nos.61077050, 61077014 and 60932004), the BUPT Young Foundation (No.2009CZ07), Fundamental Research Funds for Central Universities, and the Open Foundation of State Key Laboratory of Optical Communication Technologies and Networks (WRI) (No.2010OCTN-02).

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mode fibers and its relevance on mode multiplexing density are explored. The formulations are reviewed to understand the phenomenon of modal delays in multimode fibers, especially in general and few-mode fibers. Numerical analysis is made on the basis of formulations.

In optical fibers, spatial modes are a set of distinct field distributions, which propagate through the fiber with specific propagation constants. The fields are the solutions of wave equation for optical fiber geometry. Although wave equations have exact solutions for cylindrical dielectric media after rigorous mathematical efforts[11,12], considerable simplifications can be achieved by applying weak guidance approximations, which can be described as

$$\Delta=\frac{n_1^2-n_2^2}{2n_1^2}\approx\frac{n_1-n_2}{n_1}\ , \tag{1}$$

where Δ is the fractional change in refractive indices at core-cladding interface, and n_1, n_2 are the refractive indices of the core and cladding, respectively. Simplified solutions of wave equations under approximation of Eq.(1) lead to linearly polarized (LP) modes[13-15].

To determine how many modes can be guided in fiber for given physical parameters, we need eigen value equations for guided modes. Under approximations of Δ<<1, the eigen value equation of step index fiber is described as[13]

$$V(1-b)^{1/2}\frac{J_{l+1}[V(1-b)^{1/2}]}{J_l[V(1-b)^{1/2}]}=V(b)^{1/2}\frac{K_{l+1}[V(b)^{1/2}]}{K_l[V(b)^{1/2}]}\ , \tag{2}$$

where the normalized frequency V is defined as

$$V=ak_0\sqrt{n_1^2-n_2^2}\ , \tag{3}$$

where k_0=2π/λ is a free space wave number, a is the radius of the core, and b is the normalized propagation constant. J_l and K_l are the Bessel's function and modified Bessel's function of l order.

$$b=\frac{\beta^2-n_2^2k_0^2}{n_1^2k_0^2-n_2^2k_0^2}=\frac{(\beta/k_0)^2-n_2^2}{n_1^2-n_2^2}\ , \tag{4}$$

where β is the propagation constant for a specific mode. Numerical solution of Eq.(2) is adopted to compute b for given value of V and l = 0, 1, 2⋯. For each value of l, we may have multiple solutions corresponding to m=1, 2⋯. So numerically, we can get β_{lm} for any LP_{lm} mode and get its field distributions. After computing β_{lm} and b for a particular mode, we can analyze dispersion. As β is dependent on wavelength, a spectral component of input pulse with a certain value of λ can travel along the fiber with the time delay (group delay) of $T_g=L/V_g$, where $V_g=\left(\frac{d\beta}{d\omega}\right)^{-1}$. V_g and T_g are group velocity and group delay of a particular mode, respectively. Group delay of a particular mode for unit length can also be expressed as a function of wavelength (λ) and fiber parameters (V)[16] as follows,

$$T_g(\lambda,V)=\frac{1}{c}\{N_1\cdot f(bV)+N_2[1-f(bV)]+N_2\Delta[f(bV)-b]\}\ , \tag{5}$$

where $N=n-\lambda\frac{dn}{d\lambda}$, and $f(bV)=1/2\left[\frac{d(bV)}{dV}+b\right]$.

For multimode or few-mode fibers, the difference of group delay between one mode and another mode is called intermode delay, which can be described as ΔT (intermodal) = $T_{g_1}-T_{g_2}$.

Because intermodal delay is the dominant factor in multimode or few-mode fibers[9], we may treat group delay expression of Eq.(5) by assuming negligible material dispersion and setting N_1=n_1 and N_2=n_2, and we get

$$t_g=\frac{n_2}{c}\left[1+\Delta\frac{d(bV)}{dV}\right]. \tag{6}$$

Contrary to chromatic dispersion, modal dispersion is independent of changes in wavelength[17].

According to the above theoretical background, we study the effects of different physical parameters on intermode dispersion in few-mode fibers. We look for the conditions which can be adopted to keep modal dispersion in desired limits and increase the number of multiplex modes. The analysis is based on the general solutions of field equations for step index few-mode fibers to compute normalized propagation constants, group delays and finally group delay differences. We focus our analysis on group delay difference between any two guided modes on the variation of physical parameters of fiber separately. MATLAB is used for the computation of modes, group delays and group delay differences.

To start with analysis, we set Δ = 0.0107 for fused silica (SiO_2) fiber, and the wavelength is chosen as 1.550 μm. So V is changed by varying the core radius of fiber from 0 μm to 15 μm. Numerically computing normalized propagation constants and ignoring degenerate modes for simplicity of analysis, we use Eq.(6) for calculating group delays (T_g) for each mode followed by group delay differences.

The group delay differences between the fundamental mode and the five selected modes are shown in Fig.1. To calculate group delay, we use fundamental mode as benchmark for every comparison, because every mode multiplexing is supposed to add further modes with fundamental mode. It can be seen that we have two trends for mode delay plots. One is the region close to the cutoff value of higher mode in delay equation and the other is far from the cutoff value. Near the cutoff value, the group delay difference has sharp slope with small variations of radius. Away from the cutoff value, the group delay difference is with smooth response.

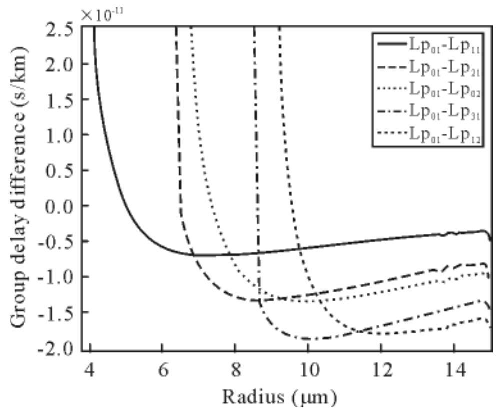

Fig.1 Group delay difference of higher order modes (LP_{11}, LP_{21}, LP_{31}, LP_{12}, LP_{02}) and fundamental mode (LP_{01})

However it must be careful about the cutoff values of the next higher modes in keeping group delay difference in smooth response region. Another point of interest in Fig.1 is the intersection point of two lines, e.g., LP_{01-11} and LP_{01-21}. At this point, group delay differences of LP_{01}–LP_{11} and LP_{01}–LP_{21} are the same, so V of a few-mode fiber can be figured out, which can increase mode multiplexing by adding another mode without significant change in mode dispersion.

Mode multiplexing density is a parameter which represents how many modes are used in MDM. Ultimately, this parameter can lead to the increase in information transmission capacity of fiber per wavelength. As we know, increasing the V value lets higher modes be guided through the fiber, so we increase the V value to accommodate higher modes and calculate the maximum delay difference between the fundamental mode and the highest allowed mode. The relationship can be seen in Fig.2.

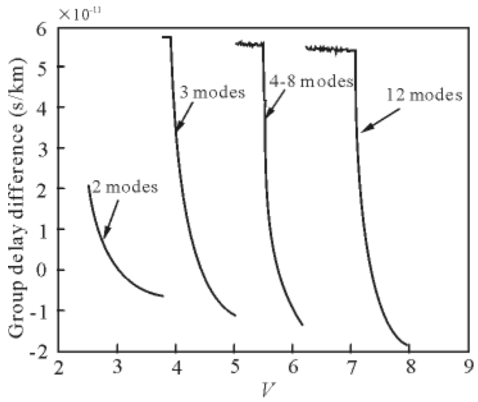

Fig.2 Group delay difference between the highest allowed guided mode and fundamental mode with different *V* parameters

Fig.2 shows mode multiplexing of 2, 3, 8 and 12 modes. All cases shown in Fig.2 are related to certain region of V for their existence. Within the region, there are steeper responses with small variations of V values. Mode multiplexing density can be increased by precisely controlling V parameter of fiber at specific wavelength.

The impact of Δ on group delay difference is shown in Fig.3. We can see that by decreasing the value of Δ, the maximum value of group delay difference is decreased. It is because of more adherences to weakly guiding conditions of $\Delta \ll 1$. The modes tend to propagate nearly along the center axis of core, so the relative difference between mode delays is also observed on decline.

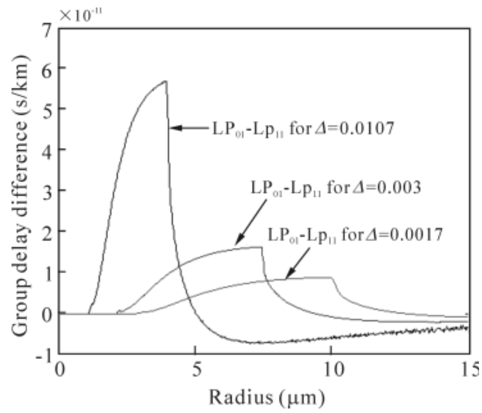

Fig.3 Group delay difference for various *Δ*

In this paper we exclusively cover the intermodal delay for few-mode step index fibers with mode multiplexed transmission. The impacts of fiber parameters on mode multiplexed transmission in terms of mode delays are studied. In this analysis, the conditions are figured out, in which the mode multiplexing density can be increased by controlling V parameter of fiber. We introduce a new observation parameter of mode multiplexing density while looking at intermode delay plots.

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