Theoretical study on modulating group velocity of light in photonic crystal coupled cavity optical waveguide*

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We present a novel mechanism,which is formed by periodically changing the radii of dielectric rods in the middle row of a photonic crystal, to control and stop light. Using the Bloch theory and coupled-mode theory, the dispersion characteristic of such a photonic crystal coupled cavity optical waveguide is obtained. We also theoretically demonstrate that the group velocity of a light pulse in this system can be modulated by dynamically changing the refractive index or radii of the selected dielectric rods, and the light stopping can be achieved.

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In recent years, controlling the propagation speed of light and making optical pulses drastically slow down have attracted considerable attention because of their applications for optical delay lines, optical buffer and optical memories[1,2]. The basic working principle of controlling light is to change the group velocity v_g using the variation of the refractive index $dn/d\omega$.

There are two major approaches to reduce the group velocity of light coherently, which employ either electronic or optic resonances[3-16]. The method of electronic resonances has many severe defects and deficiencies. Only a few special and delicate electronic resonances available in nature possess the required properties. All the demonstrated bandwidths are too small to be used for most purposes. Moreover, it is very difficult to implement such schemes on-chip with integrated optoelectronics. So controlling the group velocity of light by using optical resonance is very interesting.

Great progress has been made in photonic structures for controlling the optical pulse speed. Light pulses with group velocity of $10^{-2}c$ have been observed in the photonic crystal waveguide or the micro-ring coupled resonator optical waveguide. Nevertheless, such static structures are fundamentally limited by the delay-bandwidth product[17] and the group velocity of light pulses can not be reduced to zero. To overcome this constraint, dynamic photonic structures have been proposed to stop and store light pulses[18-20]. The basic idea is that the system supports an initial state with a sufficiently large bandwidth to accommodate the input pulse bandwidth. Then dispersion properties of the system are dynamically tuned so that the bandwidth of the pulse is compressed to zero and the pulse can be stopped.

In Refs.[18,19], a light-stopping system with a waveguide side coupled to tunable resonators is proposed, in which the photon pulse is kept in the resonators on the side of the waveguide when the zero group velocity is reached. Sunil Sandhu et al[20] showed that the dynamic process allows the pulse to be stopped by using loss tuning of coupled-resonator delay lines. In this paper, we present a novel mechanism, which is formed by periodically changing the radii of dielectric rods in the middle row of the photonic crystal, to control and stop light by refractive index dynamic change.

In this section, we investigate the dependence of the resonant frequency of the photonic crystal point defect cavity on the radius and the refractive index of the dielectric rods. The square lattice photonic crystal with the dielectric constant ε=11.7 and the radius r =0.2a (a is the lattice constant, and a=1 μm is taken for the sake of convenient calculation) is shown in Fig.1(a). The band diagram of this structure calculated using the plane wave expansion method is shown in Fig.1(b). The TM mode in this photonic crystal has a very narrow band gap, while the TE mode has two relatively wide band gaps. The normalized frequency of the TE-mode band

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gaps ranges from 0.29 to 0.42 and from 0.72 to 0.74, respectively. So if this kind of photonic crystal structure is used to make a waveguide, the TE mode should be selected as the guided mode.

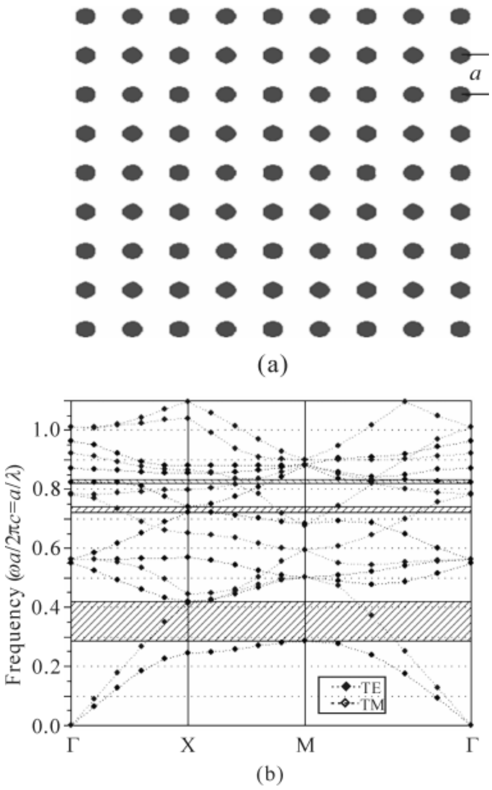

Fig.1(a) Photonic crystal structure; (b) Band structures of TE and TM modes

As shown in Fig.2(a), a point defect cavity is introduced, as the radius of one of the dielectric rods is changed to *R* (not equal to *r*). The FDTD numerical method is used to calculate the variation of the 4×4 super-lattice band structure with *R* changing from 0 to 0.5*a*, as shown in Fig.2(b). It can be seen that very narrow conduction bands appear in the band gap of the complete structure. The frequencies corresponding to these conduction bands are the defect cavity resonant frequencies. It can be also seen from the figure that the normalized resonant frequency shifts from 0.38 to lower frequency, as *R* increases from 0. When *R* increases to 0.16*a*, the resonant frequency disappears at the lower edge of the band gap. While *R* is in the range of 0.16*a*–0.23*a*, there is no resonant frequency. When *R* increases to 0.23*a*, a conduction band appears at the upper edge of the band gap, and the resonant frequency moves to lower frequency continuously with the increase of radius. When *R* increases to 0.42*a*, a conduction band appears again at the upper edge of the band gap, and the resonant frequency still moves to lower frequency with the increase of radius. It can be seen that the normalized frequency in the range of 0.28–0.42 corresponds to two or three radii. This indicates that in this range even though the radius of the dielectric rod in the defect cavity is taken to be different values, the same resonant frequency could be obtained.

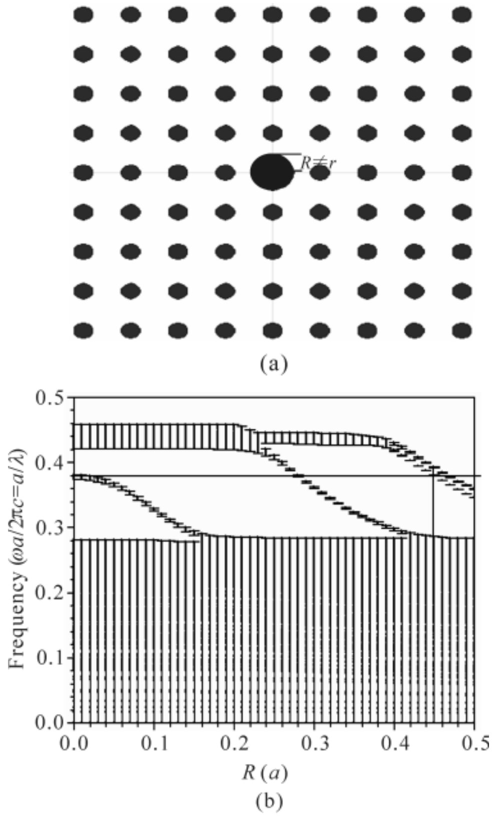

Fig.2 (a) Point defect cavity; (b) Normalized frequency as a function of the radius of the dielectric rod

Fig.3 shows the variation of the defect cavity normalized resonant frequency with the refractive index detuning *D* of the dielectric rod in the defect cavity. We can see that the

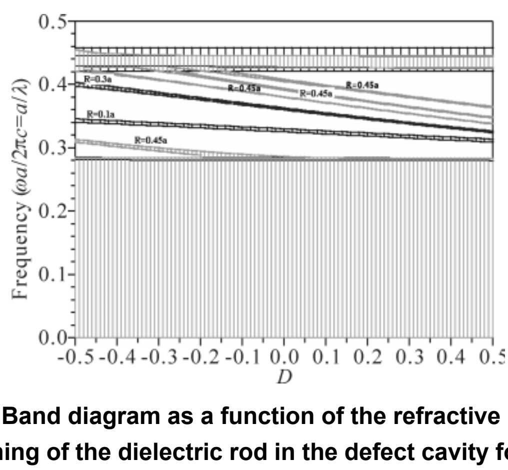

Fig.3 Band diagram as a function of the refractive index detuning of the dielectric rod in the defect cavity for the dielectric rod with different sizes (*R* = 0.1*a*, *R* = 0.3*a*, and *R*=0.45*a*)

cavity resonant frequency decreases with the increase of the refractive index of the dielectric rods. When R=0.1a, the normalized frequency decreases from 0.34 to 0.32; when R= 0.3a, the normalized frequency decreases from 0.4 to 0.35; when R = 0.45a, the normalized frequency decreases from 0.43 to 0.36. Moreover, the slops of the three curves within the band gap increase orderly. It indicates that the greater the radius R, the greater change range of the resonant frequency and the more sensitive to the refractive index detuning.

Our proposed photonic crystal coupled cavity optical waveguide for modulating and stopping light is shown Fig.4, where the radii of small dielectric rods and large ones are R_1 and R_2, respectively. We assume a_i and b_i to represent the field amplitudes in the ith small and big dielectric rods, respectively. Since the system has translational symmetry along the X direction, the following relationships can be obtained according to the Bloch theory

$$a_{i-1} = \mathrm{e}^{-\mathrm{i}K2a} a_i, a_{i+1} = \mathrm{e}^{\mathrm{i}K2a} a_i,$$
$$b_{i-1} = \mathrm{e}^{-\mathrm{i}K2a} b_i, b_{i+1} = \mathrm{e}^{\mathrm{i}K2a} b_i \quad , \tag{1}$$

where K is the wave vector. The dynamics of the field amplitudes a_i and b_i can be described using the coupled mode theory as

$$\frac{\mathrm{d}a_i}{\mathrm{d}t} = (\mathrm{i}\omega_\mathrm{A} - \gamma_\mathrm{A})a_i + \mathrm{i}\kappa b_{i-1} + \mathrm{i}\kappa b_i \quad , \tag{2}$$

$$\frac{\mathrm{d}b_i}{\mathrm{d}t} = (\mathrm{i}\omega_\mathrm{B} - \gamma_\mathrm{B})b_i + \mathrm{i}\kappa a_i + \mathrm{i}\kappa a_{i+1} \quad , \tag{3}$$

where ω_A, ω_B, γ_A and γ_B are the resonant frequencies and loss coefficients for small and large cavities, respectively, and κ is the coupling coefficient between cavities. The eigenfrequencies $\omega_{\pm,k}$ of the structure with a wave vector k can be given by

$$\omega_{\pm,k} = \frac{\omega_\mathrm{A} + \omega_\mathrm{B}}{2} + \mathrm{i}\frac{\gamma_\mathrm{A} + \gamma_\mathrm{B}}{2} \pm \sqrt{4\kappa^2 \cos^2(Ka) + \left(\frac{\omega_\mathrm{A} - \omega_\mathrm{B} + i\gamma_\mathrm{A} - i\gamma_\mathrm{B}}{2}\right)^2} = \omega_0 + \mathrm{i}Y \pm \sqrt{4\kappa^2 \cos^2(Ka) + \left(\frac{-\Delta\omega + \mathrm{i}\gamma_\mathrm{A} - \mathrm{i}\gamma_\mathrm{B}}{2}\right)^2} \quad . \tag{4}$$

Group velocity is defined as the derivative of ω with respect to K

$$v_\mathrm{g} = \frac{\mathrm{d}\omega}{\mathrm{d}K} \quad . \tag{5}$$

The band diagram of the structure is shown in Fig.5 according to Eq.(4), assuming that this structure is energy lossless. We can find that when the resonant frequency separation $\Delta\omega=\omega_\mathrm{B} - \omega_\mathrm{A}= 0$, there are two dispersion curves which have a point of intersection at the point of K=0.25. The bandwidth of the structure as well as the group velocity is relatively large. When ω_B is changed to make $\Delta\omega$ larger, the bandwidth becomes narrower (Fig.5(b)). Upon further increase of $\Delta\omega$ (Fig.5(c)), $\omega - \omega_0$ of both bands are equal for all k and the dispersion curves become a straight line parallel to the K axis, where the group velocity of the signal is zero.

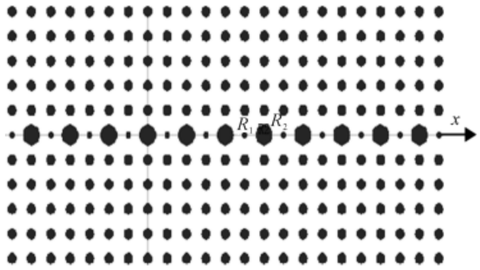

Fig.4 Schematic illustration of the photonic crystal coupled resonator optical waveguide

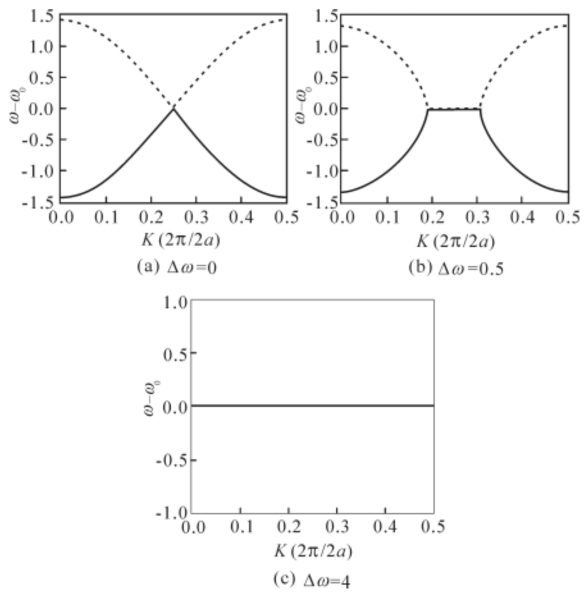

Fig.5 Dispersion curves of the photonic structure shown in Fig.3 corresponding to different Δω

The dispersion can be controlled by adjusting the radius or refractive index of the resonant cavity, according to the front discussion on the relationship between the resonant frequency of the defect cavity and the radius as well as the refractive index of the dielectric rods, and then the group velocity can be modulated. R_1 and R_2 are chosen to be 0 and 0.45a, respectively. The reason for this choice is that when R_2 is changed, the cavity with the radius of R_1 will not be affected because of the absence of the dielectric rods(R_1=0), while the larger the radius, the more sensitive of the resonant frequency to the change of the refractive index. As a result, R_2 should be as large as possible.

Our system presented above satisfies the basic principles given in Ref.[18] for controlling and stopping light: at the beginning, the resonant frequencies of small and large cavities are the same, so that an optical pulse with large bandwidth is allowed to be coupled into the waveguide. After the pulse is fully in the waveguide, the resonant frequency separation $\Delta\omega$ is changed by slowly modulating the refractive index of the big dielectric rods, and the group velocity of the pulse is reduced. If the modulation range is large enough, the speed of light can be reduced to zero. The process is reversible. When the same index modulation of the big dielectric rods is repeated in reverse, the dispersion curve returns to its original shape, and the pulse is released.

Similarly, the modulation of the pulse group velocity can also be achieved by changing the radii of the large dielectric rods. Furthermore, from Figs.2 and 3, it can also be seen that the resonant frequency of the defect cavity is more sensitive to the radius variation than the refractive index variation. To achieve the same modulation effect, the range of radius required to change is smaller. However, it is indeed a great challenge to change the radius of the dielectric rods flexibly. Relatively speaking, changing the refractive index is easier.

In summary, we have investigated a point defect cavity which is obtained by changing the radius of a dielectric rod in the center of a photonic crystal. It is shown that the resonant frequency of such a point defect cavity can be modulated by changing the radius or the refractive index of the dielectric rod. Then through periodically changing the radii of dielectric rods in a row of the photonic crystal, a photonic crystal coupled cavity optical waveguide is formed. Based on the Bloch theory and coupled mode theory, the dispersion characteristic of the waveguide is derived. It is theoretically demonstrated that in such a system, the group velocity of a light pulse can be modulated by changing the radius or refractive index of the big dielectric rods and the light stopping can be achieved. Our presented system will be a good reference for light-modulating devices.

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