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非局域非线性介质中厄米高斯光束的变分解

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摘要: 非局域非线性介质中的薛定谔方程很难用传统的方法得出精确解析解, 利用变分法系统研究了强非局域非线性介质中厄米高斯光束的传播问题。通过对非线性介质中响应函数的展开, 使得非线性薛定谔方程得以简化, 求解出高阶高斯光束孤子解。利用数值模拟研究了厄米高斯光束在介质中传播时束宽不变的问题, 结果显示当非局域程度非常大时, 解析解非常接近数值解。

关键词: 非局域非线性介质; 变分法; 孤子

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A variational solution of Hermite Gaussian beams in the strongly nonlocal nonlinear media

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Abstract: It is difficult to use the conventional method to obtain accurate analytical solution of the Schrodinger equation in the nonlocal nonlinear media. The propagation of Hermite-Gaussian (HG) beams in the strongly nonlocal nonlinear media is discussed with a variational method in this paper. The nonlinear Schrodinger equation can be simplified through expanding the response function in the nonlinear medium. The solution of high-order Gaussian beam soliton is obtained. The beam width of HG beam is unchanged when it propagates in the media by using numerical simulations. The results show that the analytical solution is closer to the numerical solution when the degree of the nonlocality is very large.

Keywords: nonlocal nonlinear media; variational approach; solitons

引 言

空间光孤子是指在介质里面传输时其束宽不变的光束, 近年来由于其独特的传输特性在世界范围内引起了极大的关注^[1-16]。Snyder 和 Mitchell 的研究引起了广泛的关注^[1]。Guo 等研究了非局域非线性薛定谔方程揭示了高斯孤子的大相移^[7]。Huang 等利用变分法研究了亚强非局域介质中的光束传输问题^[8]。Hu 等讨论了向列相液晶中非局域孤子的相互作用^[9]。Deng 等也得到了拉盖尔高斯光束的讨论并且获得了精确解析解^[11]。Bai 等利用变分法讨论了非局域非线性介质中的高斯光束的传播问题^[12]。

本文利用变分法解析出了(1+2)维非局域非线性薛定谔方程的解, 并给出了数值模拟。讨论了厄米

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高斯光束在非局域非线性介质里面的传输特性,得到了厄米高斯光束解析解。数值模拟显示厄米高斯光束能够传输一段较长的距离而束宽保持不变,随着非局域程度的增大,解析解愈加接近数值解。

1 非局域非线性薛定谔方程

光束在非局域非线性介质中的传输满足非局域非线性薛定谔方程

$$i \frac{\partial \psi}{\partial z} + \mu \nabla_{\perp}^2 \psi + \rho \psi \int_{-\infty}^{+\infty} R(r-r', z) |\psi(r', z)|^2 d^D r' = 0 \tag{1}$$

式中: $\psi(r, z)$ 为傍轴光束的慢变包络函数; $\mu=1/2k, \rho=k\eta, k$ 为介质中的波数; $\nabla_{\perp}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ 为横向拉普拉斯算符; η 为材料常数($\eta>0, \eta<0$ 表示自聚焦和自散焦介质); z 为光束的传输轴; r 为横向空间坐标; $R(r, z)$ 为介质的实对称响应函数,满足归一化条件 $\int_{-\infty}^{+\infty} R(r') d^D r' = 1$ 。式(1)由描述物理系统的 Lagrange 密度函数可推导出

$$L = \frac{i}{2} (\psi^* \frac{\partial \psi}{\partial z} - \psi \frac{\partial \psi^*}{\partial z}) - \mu (|\nabla_{\perp} \psi|^2) + \frac{1}{2} \rho |\psi|^2 \int_{-\infty}^{+\infty} R(r-r') |\psi(r', z)|^2 d^D r' \tag{2}$$

式中*表示复共轭。式(2)所对应的变分方程可写为

$$\delta \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} L(\psi, \psi^*, \psi_z, \psi_z^*, \psi_x, \psi_x^*, \psi_y, \psi_y^*) d^D r dz = 0 \tag{3}$$

介质的非局域程度为 $\alpha=w/w_m, w$ 和 w_m 分别是光束的宽度和介质的响应函数的特征响应长度。

2 变分方程

在(1+2)维介质中,假设式(1)存在厄米高斯型的近似试探解

$$\phi(x, y, z) = A(z) H_n[\frac{x}{w(z)}] H_m[\frac{y}{w(z)}] \exp[-\frac{x^2 + y^2}{2w^2(z)} + ic(z)(x^2 + y^2) + i\theta(z)] \tag{4}$$

式中: $A(z)$ 和 $\theta(z)$ 分别为光束复振幅的大小和相位; $H_n(x), H_m(x)$ 为厄米特多项式; $w(z)$ 和 $c(z)$ 分别为束宽和波前曲率。依变分原理

$$\delta \int_{-\infty}^{+\infty} L_z dz = 0 \tag{5}$$

可得 $\delta L_z / \delta p_i = 0, p_i$ 为 $A(z), \theta(z), w(z), c(z)$ 等。

其中

$$L_z = \int_{-\infty}^{+\infty} L dx dy \tag{6}$$

介质的响应函数 $R(r-r')=R(x-x', y-y')$,由于式(2)中最后一项是卷积项在积分处理时很难处理,为此将 $R(r-r')$ 分两步作泰勒展开。具体做法:先对 $R(x-x', y-y')$ 在 $x'=0, y'=0$ 处作泰勒展开,再对展开的各项在 $x=0, y=0$ 处进一步作泰勒展开,近似取到二阶,得到

$$R(x-x', y-y') \approx R_0 - \frac{1}{2}(x-x')^2 \gamma_x - \frac{1}{2}(y-y')^2 \gamma_y \tag{7}$$

式中: $R_0=R(0,0); \gamma_x=-R^{(2,0)}(0,0); \gamma_y=-R^{(0,2)}(0,0)$ (假设响应函数是圆对称的,则 $\gamma_x=\gamma_y=\gamma$)。式(1)对应的 Lagrange 密度式(2)可以简化为

$$L = \frac{i}{2} (\psi^* \frac{\partial \psi}{\partial z} - \psi \frac{\partial \psi^*}{\partial z}) - \mu (|\nabla_{\perp} \psi|^2) + \frac{1}{2} \rho |\psi|^2 \cdot \int_{-\infty}^{+\infty} [R_0 - \frac{1}{2} \gamma (x-x')^2 - \frac{1}{2} \gamma (y-y')^2] |\psi(x', y', z)|^2 dx' dy' \tag{8}$$

将试探解式(4)代入式(8)并对 x, y 积分得

$$L_z = -2^{n+m} n! m! A^2 \pi [(n+m+1)w^4 \frac{dc}{dz} + w^2 \frac{d\theta}{dz}] - 2^{n+m} n! m! (n+m+1) \mu A^2 \pi (1 + 4c^2 w^4) + \frac{1}{2} \pi^2 2^{2(n+m)} (n! m!)^2 \rho A^4 [R_0 w^4 - (n+m+1) \gamma w^6] \tag{9}$$

由式(5)和式(9)可得到试探解中各个参量所满足的欧拉方程

$$\frac{\delta L_z}{\delta A} = 0 \Rightarrow -(n+m+1)\omega^4 \frac{dc}{dz} - \omega^2 \frac{d\theta}{dz} - (n+m+1)\mu(1+4c^2\omega^4) + 2^{n+m}n!m!\pi\rho A^2\omega^4[R_0 - (n+m+1)\gamma\omega^2] = 0 \quad (10)$$

$$\frac{\delta L_z}{\delta \omega} = 0 \Rightarrow -2(n+m+1)\omega^2 \frac{dc}{dz} - \frac{d\theta}{dz} - 8(n+m+1)\mu c^2\omega^2 + 2^{n+m}n!m!\pi\rho A^2\omega^2[R_0 - \frac{3}{2}(n+m+1)\gamma\omega^2] = 0 \quad (11)$$

$$\frac{\delta L_z}{\delta c} = 0 \Rightarrow -8\mu c A^2\omega^4 + \frac{d}{dz}(A^2\omega^4) = 0 \quad (12)$$

$$\frac{\delta L_z}{\delta \theta} = 0 \Rightarrow \frac{d}{dz}(2^{n+m}n!m!\pi A^2\omega^2) = 0 \quad (13)$$

进而求得各个参量的具体表达式

$$\omega^2(z) = \omega_0^2[\cos^2(\beta_0 z) + \Lambda \sin^2(\beta_0 z)] \quad (14)$$

$$c(z) = \frac{\beta_0 k(\Lambda - 1)\sin(2(\beta_0 z))}{4a_0^2[\cos^2(\beta_0 z) + \Lambda \sin^2(\beta_0 z)]} \quad (15)$$

$$\theta(z) = -(n+m+1)\arctan[\sqrt{\Lambda} \tan(\beta_0 z)] + (n+m+1)\left[\frac{(\Lambda-1)\rho\gamma\omega_0^2 P_0}{8\beta_0}\sin(\beta_0 z) - \frac{(\Lambda+1)\rho\gamma\omega_0^2 P_0 z}{4}\right] + \rho R_0 P_0 z \quad (16)$$

式中: $\Lambda = P_c/P_0$; $\beta_0 = (\gamma\eta P_0)^{1/2}$; 输入功率 $P_0 = \int_{-\infty}^{+\infty} |\psi(x, y, z)|^2 dx dy = 2^{n+m}n!m!\pi A^2(z)\omega^2(z) = 2^{n+m}n!m!\pi A_0^2\omega_0^2$ 。输入功率与横向阶数有关。将式(14)~(16)代入试探解式(4)可得到厄米高斯光束的具体表达式。当输入功率小于临界功率即 $\Lambda > 1$ 或输入功率大于临界功率 $\Lambda < 1$ 时,光束在传输过程中形成周期振荡结构即呼吸子。呼吸子的周期可由 $T = \pi/\beta_0 = \pi/(\gamma\eta P_0)^{1/2}$ 式求出。当输入功率等于临界功率即 $\Lambda = 1$ 时可形成(1+2)维的厄米高斯孤子。定义厄米高斯光束的束宽 $w_n(z)$ 为光强降为最大值 1/e 的最外侧点到轴的距离,则

$$w_n(z) = \sqrt{2n+1}\omega(z) \quad (17)$$

可知当介质的响应宽度 w_m 一定时,随着厄米高斯光束阶数 n 的增大,介质的非局域程度在减小(α 增大),强非局域的前提条件不再成立,因此要使解析解接近数值解,非局域程度必须取较大的值(即 α 取较小的值)。当 $n=0, m=0$ 时

$$\psi(x, y, z) = \frac{\sqrt{P_0}}{\sqrt{\pi}\omega_0(z)} \exp\left[-\frac{x^2+y^2}{2\omega_0^2(z)} + ic(z)(x^2+y^2) + i\theta_0(z)\right] \quad (18)$$

$w_0(z)$ 和 $\theta_0(z)$ 分别对应于零阶厄米高斯光束的束宽及复振幅相位。

图 1 给出了(1+2)维不同阶数的厄米高斯孤子传输过程中的强度分布图。图 2 和图 3 给出了解析解与数值解的对比图。图中采取了归一化坐标

$$Z = z/k\omega_0^2, X = x/\omega_0, Y = y/\omega_0, \Psi = k\omega_0 \eta^{1/2} \psi \quad (19)$$

在数值模拟中假设响应函数为高斯函数

$$R(x, y) = \frac{1}{(\sqrt{2\pi}\omega_m)} \exp\left(-\frac{x^2+y^2}{2\omega_m^2}\right) \quad (20)$$

由图 2 和图 3 的比较可知随着非局域程度

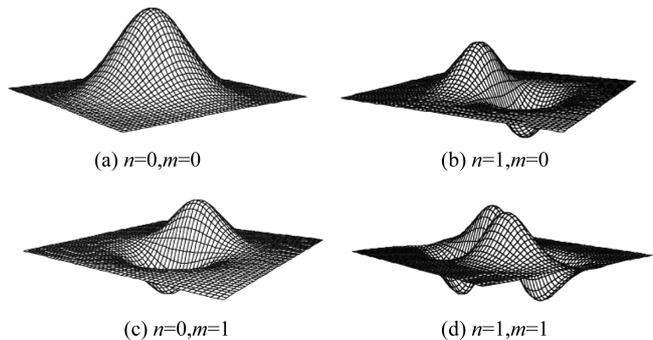


图 1 (1+2)维不同阶数的厄米高斯孤子传输过程中的强度分布图

Fig. 1 The intensity distributions mode Hermite-Gaussian beams in the media

的增加,解析解更加接近数值解。当 $P_0 < P_c$ 时,衍射作用强于非线性作用,光束束宽演化趋势为先展宽后压缩至初始宽度,形成呼吸子;当 $P_0 > P_c$ 时,衍射作用弱于非线性作用,光束先压缩后展宽至初始宽度,形成呼吸子。两种情况下的呼吸子的周期可由 $T = \pi/\beta_0 = \pi/(\gamma\eta P_0)^{1/2}$ 求出。当 $P_0 = P_c$ 时, $w_n(z) = w_n(0)$, 光束的束宽不变,即形成光孤子解

$$\psi_s(x, y, z) = \frac{\exp(i\varphi z)}{(2^{n+m}n!m!)^{1/2}\pi^{1/2}\omega_0^3(\gamma\eta)^{1/2}k} H_n\left[\frac{x}{\omega_0}\right] H_m\left[\frac{y}{\omega_0}\right] \exp\left[-\frac{x^2+y^2}{2\omega_0^2}\right] \quad (21)$$

式中 $\varphi = R_0/\gamma k \omega_0^4 - (n+m+1)/k \omega_0^2$ 。当 $n=0, m=0$ 时,即为高斯孤子

$$\psi_{0s}(x, y, z) = \frac{\exp(i\varphi z)}{\pi^{1/2}\omega_0^3(\gamma\eta)^{1/2}k} \exp\left[-\frac{x^2+y^2}{2\omega_0^2}\right] \quad (22)$$

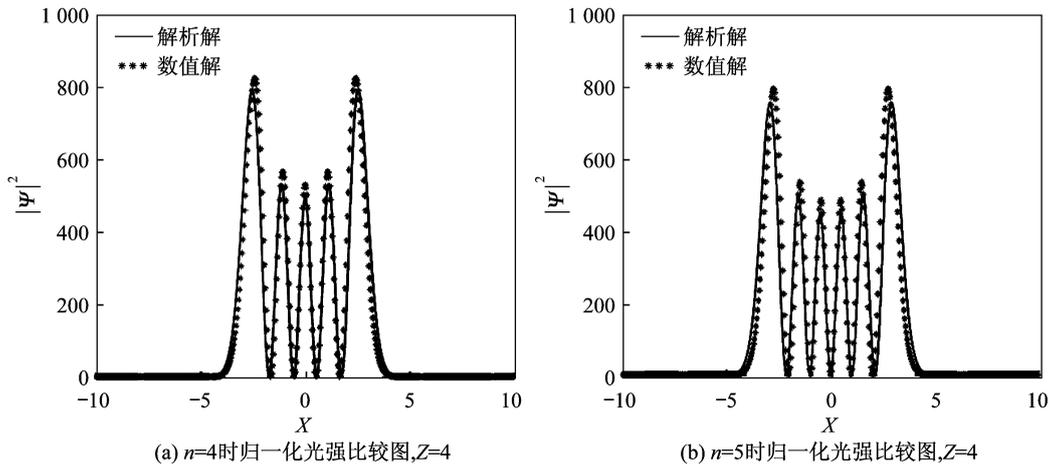


图 2 (1+2)维高阶孤子传输过程中的强度分布图, $a=0.1$
 Fig. 2 Comparison between the analytical solution (solid curves) and numerical solution (dotted curves) of (1+2)-dimensional, $a=0.1$

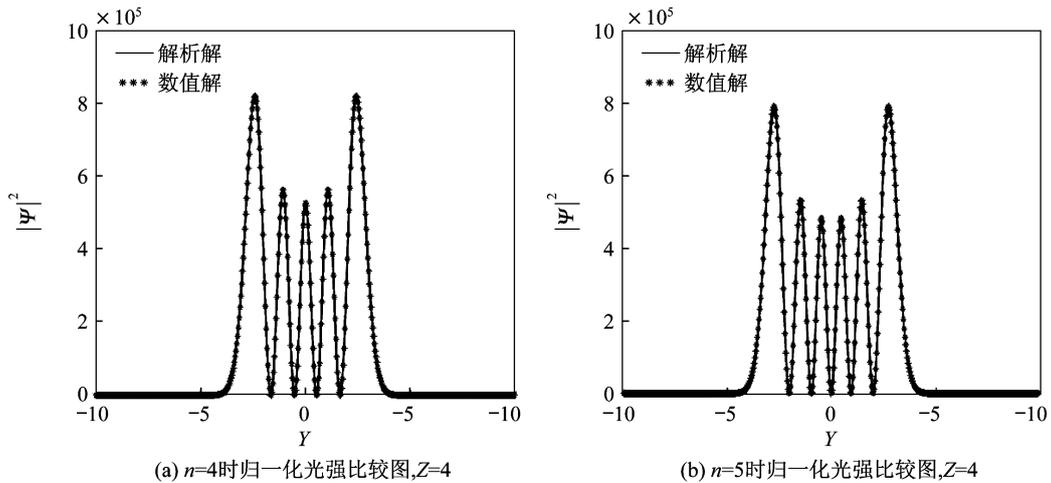


图 3 (1+2)维高阶孤子传输过程中的强度分布图, $a=0.01$
 Fig. 3 Comparison between the analytical solution (solid curves) and the numerical solution (dotted curves) of (1+2)-dimensional, $a=0.01$

3 结 论

用解析的方法研究了非局域非线性介质中光束的传输特性,通过对非线性介质中响应函数的展开,

使得非线性薛定谔方程得以简化,得到光束各参量在传输过程中的演化规律,求解出高阶高斯光束孤子解。通过解析解与数值模拟比较,发现随着非局域程度的增加,解析解更加接近数值解。

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