# Study on formation of hot image in focusing systems 

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#### Abstract

The evolution of hot-image formation in consideration of focusing lens effect is theoretically and numerically investigated. Through analysis of linear propagation theory with lens, it is shown that the location of hot image in focusing systems can be predicated through definite functional relation to hot image position in non-focusing systems, on condition that the distance between the hot image and the lens is relatively small. In addition through numerical calculation, specific variation characteristics of maximum intensity with the propagation distance representing hot-image formation process are presented, it is obtained that perturbing scatter size directly affects the location and corresponding peak intensity of the hot image. The lens influences the value of scatter size where minimum distance between hot-image plane and focus lens is generated. Finally, the effects of phase and amplitude modulation of scatter on hot image are also given. Results in this paper are of fair referenced value for controlling perturbing scatter and restraining the damage risk of optical components in focusing systems.


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## 1. Introduction

In high-power laser systems, hot image resulting from the nonlinear holography, is a paramount process that draws considerable attention, it is one of important factors that limit maximum output power available from solid-state laser. The formation of hot image originates from the scatter rooted in a strong background beam. After propagating a distance in free space, the beam goes through a second-order nonlinear medium, an intensified holographic image of the scatter downstream is produced in certain position. The peak intensity of hot image may be several times larger than the initial background beam, what's worse, costly optics may be damaged if intensities of hot image exceed damage threshold of materials even the anticipated average fluences should have been at the safe operation point. The physical mechanism manifesting the formation of the hot image [1] was first demonstrated by Hunt et al., afterwards many researchers constantly focused on the features of hot image. Widmayer et al. [2,3] successively presented the nonlinear formation of images of obscuration and phase errors experimentally, and the computer model was testified to be in good agreement with experimental results. Moreover, he revealed that phase scatters exerted a larger damage threat to optical components than the amplitudes ones. Xie et al. [4-6] developed a simple analytical method for describing hot image. In addition, some researchers

[^0]extended studies of the hot image in complicated systems comprising the cascaded nonlinear medium or with multiple obscurations and arrayed mechanical defects [7-10]. So far as we know, there are few studies about hot-image formation considering lens focusing effect. As is known, Lens in final optical system, which is one of the most important parts in laser drivers [11], is a critical component to converge incident beam onto a target used in high-energy-density physical experiments. Vacuum windows and crystals are in front of lens for isolation and frequency conversion, respectively, a grating and debris shields are situated at the back of lens. In such a structure, hot image may be produced behind the lens taking crystals or windows as nonlinear medium. Optical components just located in that position may suffer from risk of damage, so it is required to discuss hot image characters in consideration of lens focusing effect. In this paper, we present theoretical and numerical treatment for hot-image formation in focusing systems, the potential connection of perturbation scatter size with peak intensity and position of hot image is given, and changing trends of maximum intensity representing formation process of hot image is also demonstrated in detail. The results may be helpful for providing guide to control scatter size and minimizing the damage risks caused by the hot image.

## 2. Model and theoretical analysis

We simplify final optical system as an explicit model. Formation of hot image in this model is sketched in Fig. 1. Briefly, the scatter illuminated by an intense background beam is located in


Fig. 1. Sketch of final optical system.
plane A, after propagating a distance $d_{0}$, The scatter wave along with the background beam passes through a nonlinear medium E with thickness $L$, then the modulated beam goes on propagating a distance $d_{1}$ until it falls onto a lens with focal length $f$. After that, the beam keeps on propagating through certain distance in free space, a hot image with largest peak intensity is generated.

Assuming $\tau_{0}(x, y)$ as the transmission function of the scatter and taking $A(x, y) \exp (j k z)$ as optical field of the continuous background beam, the beam modulated by the scatter in plane $A$ is given by:
$E_{\mathrm{A}}(x, y, 0)=A_{0}(x, y, 0)[1+\tau(x, y)]=E_{\mathrm{A} 0}(x, y, 0)+E_{\mathrm{A} 1}(x, y, 0)$
where $\tau(x, y)$ is written as:
$\tau(x, y)=\tau_{0}(x, y)-1=\left\{\begin{array}{cc}a_{0} \exp (\mathrm{j} \phi)-1 & \text { inside the scatter area } \\ 0 & \text { outside the scatter area }\end{array}\right.$
where $a_{0}\left(0 \leq a_{0} \leq 1\right)$ and $\phi(0 \leq \phi \leq 2 \pi)$ denote the amplitude and the phase modulation coefficient of the scatter, respectively, the scatter is circular with radius $r_{\mathrm{s}}$. Supposing:

$$
\begin{equation*}
E_{\mathrm{A} 0}(x, y, 0)=A_{0}(x, y, 0), \quad E_{\mathrm{A} 1}(x, y, 0)=A_{0}(x, y, 0) \tau(x, y) \tag{3}
\end{equation*}
$$

According to Fresnel diffraction integral theory, the field in front surface of nonlinear medium can be written as:

$$
\begin{align*}
& E_{\mathrm{L} 1}\left(x_{1}, y_{1}\right) \\
& \quad=\frac{\exp \left(j k d_{0}\right)}{j \lambda d_{0}} \iint E_{A}(x, y) \exp \left[j k \frac{\left(x_{1}-x\right)^{2}+\left(y_{1}-x\right)^{2}}{2 d_{0}}\right] d x_{1} d y_{1} \tag{4}
\end{align*}
$$

where $k$ is wave number in vacuum, $\lambda$ is wavelength of the incident beam, optical field designated as $E_{\mathrm{L} 2}\left(x_{2}, y_{2}\right)$ at the back surface of nonlinear medium can be obtained based on the nonlinear paraxial equation as follows:
$\nabla_{\perp}^{2} E+2 j k_{0} \frac{\partial E}{\partial z}=-\frac{2 k_{0}^{2}}{n_{0}} \Delta n E$
where $k_{0}$ is wave number in medium with refractive index $n_{0}$, $\Delta n=\gamma I$ is nonlinear refractive index variation compared to $n_{0} . \gamma$ is the nonlinear index coefficient of nonlinear medium. After propagating in free space $d_{1}$, the optical field becomes $E_{\mathrm{F}}\left(x_{3}, y_{3}\right), E_{\mathrm{F}}\left(x_{3}, y_{3}\right)$ can be obtained using the same equation as Eq. (4) by replacing $d_{0}$ and $E_{\mathrm{A}}$ with $d_{1}$ and $E_{\mathrm{L} 2}$, respectively, in consideration of corresponding coordinate changing. Transmission function of the focusing lens is $\exp \left[-j k\left(x^{2}+y^{2}\right) /(2 f)\right], f$ is the focal length of lens F . After a series of formula deduction and transformation, the optical field in plane $B$ can be deduced as:
$\mathrm{E}_{\mathrm{B}}\left(x_{\mathrm{B}}, y_{\mathrm{B}}\right)=\frac{1}{1-d_{2} / f} \exp \left[-\frac{j k\left(x_{\mathrm{B}}^{2}+y_{\mathrm{B}}^{2}\right)}{2\left(f-d_{2}\right)}\right] \xi_{0 B}\left(X_{\mathrm{B}}, Y_{\mathrm{B}}, Z_{\mathrm{B}}\right)$

$$
\begin{align*}
& \xi_{0 B}\left(X_{\mathrm{B}}, Y_{\mathrm{B}}, Z_{\mathrm{B}}\right) \\
& \quad=\frac{1}{j \lambda Z_{\mathrm{B}}} \iint E_{F}\left(x_{3}, y_{3}\right) \exp \left[j k \frac{\left(X_{\mathrm{B}}-x_{3}\right)^{2}+\left(Y_{\mathrm{B}}-y_{3}\right)^{2}}{2 Z_{\mathrm{B}}}\right] d x_{3} d y_{3} \tag{7}
\end{align*}
$$

$X_{\mathrm{B}}=\frac{x_{\mathrm{B}}}{1-d_{2} / f}, Y_{\mathrm{B}}=\frac{y_{\mathrm{B}}}{1-d_{2} / f}, Z_{\mathrm{B}}=\frac{d_{2}}{1-d_{2} / f}$
where $d_{2}$ is the distance between the lens and the observation plane B.

## 3. Simulations and comparison

In this section, the properties of hot image in plane B are numerically simulated and systematically analyzed, First of all, we estimate the location of hot image in focusing systems theoretically. From Eqs. (6-8), it can be seen that the field in focusing systems is calculated using the Fresnel diffraction integral by the coordinate transformation adding quadratic phase factor and position variation factor denoting the lens convergence. The space field distribution at $d_{2}$ is similar to that at $\mathrm{Z}_{\mathrm{B}}$ in the non-focusing systems, the actual intensity is determined by two items, the equivalent diffraction item and position variation factor. The equivalent diffraction item poses a scaling transformation on the position of hot image compared to non-focusing case and variation factor item can make the position of hot image push away from the lens relative to scaling distance. However, when the difference $\left(d_{0}-d_{1}\right)$ is very small compared to $f$, the position variation factor has small influence on location of hot image, the shifting can be neglected. While equivalent diffraction item plays a major role, then in this case the position of hot image in focusing systems can be derived using the equation as follows:
$Z_{\mathrm{FI}}=\frac{f *\left(Z_{\mathrm{FN}}-d_{1}\right)}{f+\left(Z_{\mathrm{FN}}-d_{1}\right)}$
where $Z_{\mathrm{FI}}$ is the distance between hot image plane and focusing lens. $Z_{\mathrm{FN}}$ is the distance of hot image plane and the nonlinear medium in non-focusing systems with the same original optical field. It's important to note that Eq. (9) is a rough estimate of position of hot image with its limited application conditions. In fact, the lens convergence has more or less effects on the location of hot image.

Based on the split-step Fourier method in nonlinear medium E with Eq. (5) and the linear propagation with Eqs. (4-7). The parameters of the nonlinear medium are taken as follows: the thickness of the medium $L=2 \mathrm{~cm}$, refractive index $n_{0}=1.48$, the nonlinear index coefficient $\gamma=3.6 \times 10^{-16} \mathrm{~cm}^{2} / \mathrm{W}$; the incident background beam with wavelength $\lambda=0.531 \mathrm{um}$ is assumed as super-Gaussian distribution with order 8 and beam radius $r_{\mathrm{w}}=0.6 \mathrm{~cm}$, peak intensity $I_{0}=3 \mathrm{GW} / \mathrm{cm}^{2}$. The distance $d_{0}$ is 100 cm and the distance $d_{1}$ is 30 cm . The focal length of lens $f$ is 500 cm . The area of the sampling region is designated as $0.8 \mathrm{~cm} \times 0.8 \mathrm{~cm}$ and divided into $2048 \times 2048$ grid of points. A scatter is located at the center of background beam and is supposed to be a translucent scrap. When amplitude modulation coefficient $a_{0}=0.8$ and phase modulation coefficient $\phi=0.5 \pi$. The scatter has smaller radius $r_{\mathrm{s}}=0.05 \mathrm{~mm}$ and 0.04 mm , the peak intensity along the propagation distance $d_{2}$ is shown in Fig. 2a. We can see that, there exists maximum intensity in some distance. Maximum peak intensity can reach $4.9 \mathrm{GW} / \mathrm{cm}^{2}$ at nearly $d_{2}=60.6 \mathrm{~cm}$ as $r_{\mathrm{s}}$ equals to 0.05 mm . The distance the maximum peak intensity located with $r_{\mathrm{s}}=0.04 \mathrm{~mm}$ is nearly equal to that with $r_{\mathrm{s}}=0.05 \mathrm{~mm}$. The intensity distribution with $d_{2}=60.6 \mathrm{~cm}$ is illustrated in Fig. 2b. It can be clearly shown that there is a striking bright spot in the center of the background beam. This spot


Fig. 2. (a) Peak intensity along the propagation $d_{2}$ with different small scatter size, (b) field distribution at maximum peak intensity, $a_{0}=0.8, \phi=\pi / 2$.
is hot image resulting from the original scatter field it may be a potential threat to the optical components placed here. As we know that the position of the hot image has tight relation to the distance between original scatter plane and nonlinear medium. In general, when the beam propagates in non-focusing systems, the distance between the hot image plane and nonlinear medium approximately equals to the distance between original scatter plane and nonlinear medium, on condition that the nonlinear medium is very thin and the size of the scatter is very small. Here, as the beam propagates in focusing systems with the same condition, the distance can also be approximately deduced from Eq. (9). $Z_{\mathrm{FN}}$ of the non-focusing system is $d_{0}-L$. As we calculated above, when $r_{\mathrm{s}}=0.05 \mathrm{~mm}$, the $Z_{\mathrm{FN}}$ is 98 cm , thus, $Z_{\mathrm{FI}}$ is 59.8 cm , it can clearly seen that $Z_{\mathrm{FI}}$ is very close to the result by numerical simulation.

We find that as the size of scatter increases the position of hot image and maximum peak intensity of the hot image changes, as shown in Fig. 3. There exist two spikes along the propagation distance with the increasing of the scatter size, when $r_{\mathrm{s}}$ is 0.133 mm , the front spike is larger than the back spike, the hot image is located at $d_{2}=56.2 \mathrm{~cm}$ with maximum peak intensity $6.36 \mathrm{GW} / \mathrm{cm}^{2}$ at the front spike. However, when the $r_{\mathrm{s}}$ reaches 0.2 mm , the position of hot image move to $d_{2}=67.9 \mathrm{~cm}$ with maximum peak intensity


Fig. 3. Peak intensity along the propagation $d_{2}$ with larger scatter size, $a_{0}=0.8$, $\phi=\pi / 2$.


Fig. 4. Features of hot image with relation of scatter size, $a_{0}=0.8, \phi=\pi / 2$. (a)Position of hot image. (b) Peak intensity corresponding to the hot image with relation.
$7.03 \mathrm{GW} / \mathrm{cm}^{2}$ at the back spike. We figure out the position of hot image denoted as $Z_{\mathrm{F}}$ and corresponding peak intensity with the relation of scatter size as shown in Fig. 4. It can be seen that distance $Z_{\mathrm{F}}$ is basically stable at about 60.63 cm when $r_{\mathrm{s}}$ is less than 0.05 mm , afterwards, $Z_{\mathrm{F}}$ starts to decrease and achieves the minimum distance nearly 54.4 cm at $r_{\mathrm{s}}=0.15 \mathrm{~mm}$ and then rapidly rises with scatter increase. The rapid increase of distance $Z_{\mathrm{F}}$ results from the shift of hot image from the front spike to back spike.


Fig. 5. Features of hot image with relation of phase modulation, $r_{\mathrm{s}}=0.05 \mathrm{~mm}$, $a_{0}=0.8$, (a)position of hot image. (b) Peak intensity corresponding to the hot image.


Fig. 6. Features of hot image with relation of amplitude modulation. $r_{s}=0.05 \mathrm{~mm}$, $\phi=\pi / 2$. (a) Position of hot image. (b) Peak intensity corresponding to the hot image.

The maximum peak intensity keeps going up with the $r_{\mathrm{s}}$ increase. As $r_{\mathrm{s}}$ reaches 0.28 mm , the maximum peak intensity goes up to $9.36 \mathrm{GW} / \mathrm{cm}^{2}$. It's noted that near the $r_{\mathrm{s}}=0.15 \mathrm{~mm}$ corresponding to above-mentioned minimum distance, maximum peak intensity takes on slowly increase tread. We don't go on increasing the $r_{\mathrm{s}}$, as contributions of the diffraction and nonlinear process compete with each other, the optical field become complicated, moreover, generally researches of the hot image focus on the small scatter. Fig. 4a and b verify that the position of hot image has tight relation to the size of scatter. Compared to the non-focusing systems, the focusing lens not only shortens the distance of hot image from the nonlinear medium in some proportion, but also significantly effects relative peak intensity of the two spikes, thus, influences the minimum distance at which position of hot image switches between two spikes as $r_{\mathrm{s}}$ changes.

As is known to all,the phase modulation coefficient of scatter can also affact the formation of hot image, so next we discuss the relation of position and intensity of hot image to the phase modulation, the results are shown in Fig. 5a and b. It can be found that the position of hot image denoted also as $\mathrm{Z}_{\mathrm{F}}$ changes litte, around 60 cm distance. The phase modulation has a great influence on
the intensity of hot image. The intensity of hot image increases at first,and reaches maximum with phase $0.83 \pi$,then decreases with the increase of the phase modulation. so we can see that the phase modulation provides slight impacts on the location of hot image, but can have a great impact on the maximum peak intensity. In this case, the focusing lens effects can be ingnored as the phase modulation changes.we also obtain the effects of the amplitude modulation coefficient on formation of hot image indicated in Fig. 6 a and b the results are the same as that of the phase modulation.

## 4. Conclusion

We analyze the formation of hot image in focusing systems, we discuss the effect of the size and modulation coefficients of scatter on location and peak intensity of hot image. The focusing lens has great impacts on the location of hot image, the position of hot image can be estimated using a conversion formula relative to nonfocusing systems, and then add a small shift. As size of scatter changes, the lens influences the value of $r_{\mathrm{s}}$ where hot image is clostest to focusing lens. The results may be helpful for the design of final target systems in high power laser drivers.

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