



Study on impact of spatial filter on a hot image through medium with gain



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ABSTRACT

The evolution of hot image formation through medium with gain in consideration of the effect of spatial filter is theoretically and numerically investigated. Based on the linear diffraction theory and small-scale self-focusing theory of Bespalov and Talanov, intensity distribution of hot image in conjugate plane is derived analytically. Then, the peak intensity of hot image for different medium gain and different pinhole sizes are discussed in detail, the results show theoretical analysis is mostly approximate to the numerical simulations, furthermore, it is found that suppressing effect on peak intensity ratio with small gain coefficient is larger than that with bigger ones for determined pinhole size.

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1. Introduction

In high power laser systems, hot image resulting from the nonlinear holography is a paramount process that draws considerable attention; it is one of predominant factors that limit maximum output power available from solid-state laser. Once plaguing the safe operation of high power system, damage caused by hot image has not clearly understood for several decades because of its queer characteristic of hard tracks. The formation of hot image originates from the scatter embedded in a strong background beam. After propagating a distance in free space, the scatter wave interferes with the strong background beam and produces interference pattern, as they enter a second order nonlinear medium, an intensity-dependent term is imposed onto the phase front of the incident beam owing to the functionality of the nonlinear medium, then the phase modulated beam proceed to propagate in free space, subsequently a conjugate wave to the initial scatter wave is generated and converges to produce an intensified holographic image of the scatter downstream in corresponding position. So hot image phenomena is also nicknamed nonlinear holographic imaging. The peak intensity of hot image may be several times larger than the initial background beam, What's worse, costly optics may be damaged if intensities of hot image exceed damage threshold of materials even the anticipated average fluences should have been at the safe operation point. The physical mechanism exposing the formation of the hot image [1] is first demonstrated by Hunt et al. afterwards

many researches on the character of hot image is constantly pursued. Widmayer et al. [2,3] successively provided the nonlinear formation of images of obscuration and phase errors experimentally, and the computer model is proved to be in good agreement with experimental results. Moreover, he revealed that phase scatter pose a larger damage threat to optical components than the amplitudes ones. Xie et al. [4–6] developed an analytical method for hot image. Peng et al. [7] numerically investigated the evolution of hot image in high power systems with a single thick medium. In addition, some researchers extended studies of the hot image in complicated systems comprising the cascaded nonlinear medium and in special conditions in which the multiple obscurations and arrayed mechanical defects [8–10] are introduced. Peng et al. [11] analyzed the restraining effect of spatial filters on hot image with nonlinear medium of no gain, as we know, in high power systems spatial filters are most commonly used between amplifying chains for filtering out high frequency, so it is required to discuss the impacts of spatial filter on a hot image through medium with gain, in this paper, we present theoretical and numerical treatment for hot image formation considering the spatial filter effect. It may be helpful for designing the high power systems and minimizing the damage risks coming from the hot image.

2. Model and theoretical analysis

The principle suppressing effect of spatial filter on a hot image is sketched in Fig. 1. Briefly, the scatter illuminated by an intense background beam is located in plane A, after propagating a distance d_0 , The scatter wave along with the background beam passes through the spatial filter, which is composed of two lens with the

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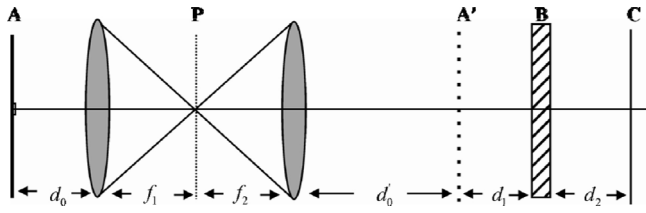


Fig. 1. Sketch of the spatial filter and formation of hot image.

focal length f_1 and f_2 , respectively, a pinhole is positioned on the common focus plane of two lens. Then the beams go on propagating until they fall onto a nonlinear medium with gain coefficient g_0 , the thickness of the medium is L , as the beams propagate through certain distance in free space, a hot image with large peak intensity is generated.

Assuming $\tau_0(x,y)$ as the transmission function of the scatter and taking $A(x,y)\exp(jkz)$ as optical field of the continuous background beam, the beam modulated by the scatter in plane A is given by

$$E_A(x, y, 0) = A_0(x, y, 0)[1 + \tau(x, y)] = E_{A0}(x, y, 0) + E_{A1}(x, y, 0) \quad (1)$$

where $\tau(x,y)$ is written as

$$\tau(x, y) = \tau_0(x, y) - 1 = \begin{cases} a_0 \exp(j\phi) - 1 & \text{inside the scatter area} \\ 0 & \text{outside the scatter area} \end{cases} \quad (2)$$

Due to basic properties of image relaying of spatial filter incident beams are reimaged subsequently in plane A', let setting $f=f_1=f_2$, the relation between project distance d_0 and image distance d'_0 is deduced as $d'_0 = 2f - d_0$ by transfer matrix method. Optical field in plane A' can be given by

$$E_{A'}(x_{A'}, y_{A'}) = \frac{\exp(jkS)}{(j\lambda f)} \iint F_A\left(\frac{x_p}{\lambda f}, \frac{y_p}{\lambda f}\right) T_p(x_p, y_p) \times \exp\left[-\frac{jk}{f}(x_p x_{A'} + y_p y_{A'})\right] dx_p dy_p \quad (8)$$

From Eq. (8), the Fourier transform spectrum of $E_{A'}$ is obtained.

$$F_{A'}(f_x, f_y) = F_A(f_x, f_y) \times T_p(\lambda f f_x, \lambda f f_y) = [F_{A0}(f_x, f_y) + F_{A1}(f_x, f_y)] T_p(\lambda f f_x, \lambda f f_y) \quad (9)$$

where $F_{A0}(f_x, f_y)$ and $F_{A1}(f_x, f_y)$ are Fourier transform spectrum of optical field $E_{A0}(x, y)$ and $E_{A1}(x, y)$, respectively. It is clearly shown from Eq. (9) that effect of spatial filter on incident beam is cutting off high frequency components with shutting frequency $\sqrt{f_{xc}^2 + f_{yc}^2} = a_c/(\lambda f)$.

Starting from plane A', the beams go through a distance d_1 , then they are injected into the nonlinear medium with gain. As described in Ref. [4], the optical field of the scattered wave away from the rear surface of the nonlinear medium d_2 can be derived based on Bspalov and Talanov (BT) theory and linear transfer diffraction formula, namely that is

$$E_{C1}(x, y, z) = \mathcal{F}^{-1} \{ F_{C1,r} + jF_{C1,i} \} = \exp\left[\frac{\beta - \alpha}{2}L\right] \mathcal{F}^{-1} \left\{ \begin{aligned} & j \left[\left(\frac{k_0 g}{q_{\perp}^2} + \frac{q_{\perp}^2}{4k_0 g} \right) \sinh(gL) \right] \times (F_{A1,r} T_p - jF_{A1,i} T_p) e^{-j(\theta_2 - \theta_1)} \\ & + \left[\cosh(gL) + j \left(\frac{k_0 g}{q_{\perp}^2} - \frac{q_{\perp}^2}{4k_0 g} \right) \sinh(gL) \right] \times (F_{A1,r} T_p + jF_{A1,i} T_p) e^{-j(\theta_2 + \theta_1)} \end{aligned} \right\} \quad (10)$$

where $a_0(0 \leq a_0 \leq 1)$ and $\phi(0 \leq \phi \leq 2\pi)$ denote the amplitude and the phase modulation coefficient of the scatter. Supposing

$$E_{A0}(x, y, 0) = A_0(x, y, 0), \quad E_{A1}(x, y, 0) = A_0(x, y, 0)\tau(x, y) \quad (3)$$

According to Collins formula theory, the field in the pinhole plane P is obtained as

$$E_p(x_p, y_p) = \frac{\exp[jk(x_p^2 + y_p^2)/(2f_0)]}{j\lambda f_1} F(x_p, y_p) \quad (4)$$

where k is wave number, λ is wavelength of the incident beam, supposed $F_A(f_x, f_y)$ is Fourier transform spectrum function of optical field $E_A(x, y, 0)$, f_x, f_y are components of spatial frequency in x and y directions, respectively by substituting f_x with $x_p/(\lambda f_1)$, $F(x_p, y_p)$ in Eq. (4) can be written as

$$F(x_p, y_p) = F_A\left(\frac{x_p}{\lambda f_1}, \frac{y_p}{\lambda f_1}\right) \exp\left[-j\pi\lambda d_0 \left(\left(\frac{x_p}{\lambda f_1}\right)^2 + \left(\frac{y_p}{\lambda f_1}\right)^2 \right)\right] \quad (5)$$

The transmission of the pinhole is described as

$$T_p(x_p, y_p) = \begin{cases} 1 & |r_p| = \sqrt{x_p^2 + y_p^2} \leq a_c/2 \\ 0 & |r_p| = \sqrt{x_p^2 + y_p^2} \geq a_c/2 \end{cases} \quad (6)$$

where a_c is diameter of the pinhole, then after passing through the pinhole, optical field is expressed as

$$E'_p(x_p, y_p) = E_p(x_p, y_p) \times T_p(x_p, y_p) \quad (7)$$

where $F_{A1,r}$ and $F_{A1,i}$ are expressed as the real part and image part of $F_{A1}(f_x, f_y)$ respectively, \mathcal{F}^{-1} is defined as reverse Fourier transform function, $q_x = 2\pi f_x$, $q_y = 2\pi f_y$, and $q_{\perp} = \sqrt{q_x^2 + q_y^2}$ is transverse spatial frequency, $g = \sqrt{q_{\perp}^2(q_c^2 - q_{\perp}^2)}/(4k_0^2)$ is denoted as the growing rate corresponding to q_{\perp} , in which $q_c = \sqrt{4k_0^2 \gamma l / n_0}$ is critical point of growing spatial frequency, k_0 is wave number in medium with refractive index n_0 , $\theta_2 = (q_{\perp}^2 d_2 / (2k))$, $\theta_1 = (q_{\perp}^2 d_1 / (2k))$. From Eq. (10), it is easy to see that the term on the right hand containing $(F_{A1,r} T_p - jF_{A1,i} T_p)$ is related to the hot image, and the other term containing $(F_{A1,r} T_p + jF_{A1,i} T_p)$ can provide the minimum intensity in some place. Therefore, intensity distribution in hot image plane satisfying the relation $d_2 = d_1$ is given by

$$I = I_0 \exp\left[\frac{\beta - \alpha}{2}L\right] \times \left| 1 + \mathcal{F}^{-1} \left\{ j \left[\left(\frac{k_0 g}{q_{\perp}^2} + \frac{q_{\perp}^2}{4k_0 g} \right) \sinh(gL) \right] \times (F_{A1,r} T_p - jF_{A1,i} T_p) e^{-j(\theta_2 - \theta_1)} \right\} \right|^2 \quad (11)$$

3. Simulations and comparison

In this section, the effect of the spatial filter on hot image is numerically simulated by using split-step Fourier method in nonlinear medium, besides, we also give the analytical results according to Eq. (11), the parameters of the nonlinear medium are taken as follows: the thickness of the medium $L=15$, refractive index $n_0=1.528$, the nonlinear index coefficient $\gamma=3 \times 10^{-16} \text{ cm}^2/\text{W}$; the incident background beam with

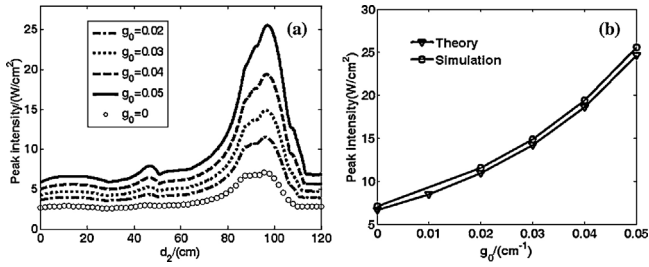


Fig. 2. (a) Peak intensity evolution with the distance d_2 and (b) the peak intensity in hot image plane versus different gain g_0 when the pinhole size is infinite.

wavelength $\lambda=1.053 \mu\text{m}$ is assumed as super-Gaussian distribution with order 8 and beam radius $r_w=1 \text{ cm}$, peak intensity $I_0=2 \times 10^9 \text{ W/cm}^2$. A scatter is located at the center of background beam and is supposed to be a translucent scrap with radius $r_s=0.02 \text{ cm}$, amplitude modulation coefficient $a_0=0.8$, and phase modulation coefficient $\phi=0.6\pi$. The focal length of lens pairs of spatial filter f is 500 cm, project distance d_0 , and imaging distance d'_0 is set as 300 and 700 cm, respectively. The interval between imaging plane and front surface of nonlinear medium $d_1=100 \text{ cm}$. The area of the sampling region is designated as $4 \text{ cm} \times 4 \text{ cm}$ and divided into 2048×2048 grid of points.

Fig. 2(a) gives the peak intensity evolution with the distance between the rear surface of nonlinear medium and the observation plane as the pinhole size is assumed infinite. It can be found that the peak intensity get larger as gain coefficient of nonlinear medium increase because of the increasing intensity of background beams. The maximum peak intensity related to the hot image with different gain appears in approximately the same location, that is, d_2 of hot image plane is approximately equal to 95 cm. Fig. 2(b) shows peak intensity in hot image plane with different gain using numerical simulation and theoretical analysis based on Eq. (11), respectively, it is easily seen that the results obtained by the two different methods basically match each other, the theory results are slightly smaller than numerical simulation owing to the small signal approximation in BT theory. The maximum peak intensity with $g_0=0.05/\text{cm}$ reaches up to six times the primary intensity, and 5.3 times corresponding to $g_0=0.04/\text{cm}$. Fig. 3(a) and (b) illustrate the two and one dimensional intensity distribution separately. It can be clearly shown that there is a striking bright spot in the center of the beam.

As the pinhole is used to filter out the high frequency of beam, it can be obtained from Fig. 4(a) that the maximum intensity with pinhole size 1.9 cm became smaller than that with the pinhole infinite. That is easy to understand that the filtering effect of spatial filter makes the beam smoothing and lower nonlinear growth in nonlinear medium. By the way, the locations related to the hot image plane move close to the rear surface of the nonlinear medium, and with different gain the distance d_2 of hot image plane have minor difference, as $g_0=0.05/\text{cm}$, $d_2=87 \text{ cm}$, whereas $g_0=0.02/\text{cm}$, $d_2=89 \text{ cm}$. Similarly, the peak intensity in hot image

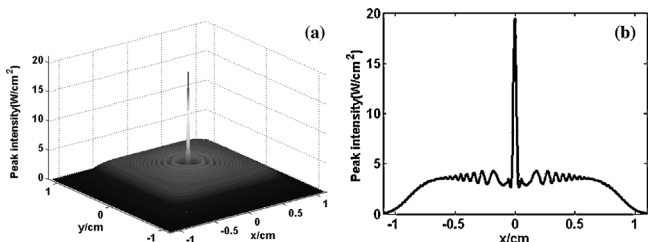


Fig. 3. Intensity distribution in hot image plane in (a) x - y dimensions and (b) in x direction with $g_0=0.04/\text{cm}$ when the pinhole size is infinite.

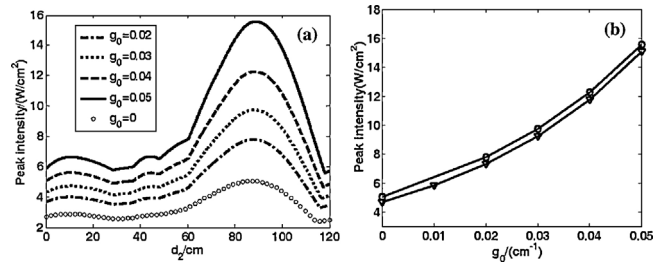


Fig. 4. (a) Peak intensity evolution with the distance d_2 and (b) the peak intensity in hot image plane versus different gain g_0 when the pinhole size is 1.9 cm.

plane increases linearly with the growing gain g_0 . And the theory analysis is nearly the same as the numerical results as shown in Fig. 4(b) identical to Fig. 2(b). However, the maximum peak intensity with $g_0=0.05/\text{cm}$ drops to 3.5 times the primary intensity, and 3.2 times corresponding to $g_0=0.04/\text{cm}$.

In next part, we concentrate on the suppressing effect on peak intensity in hot image plane with different gain coefficient. In order to compare filtering impact of pinhole size with different gain coefficient, a normalized parameter defined as relative ratio of peak intensity with fixed pinhole size to that with infinite pinhole size for correspondent gain coefficient is used. Fig. 4(a) gives the relative intensity ratio as a function of gain coefficient with different pinhole size. From the curve, we can see that decreasing effect of peak intensity ratio with small gain coefficient is larger than that with bigger ones for determined pinhole size. The reason may be that as the gain coefficient becomes bigger, the nonlinear effect in nonlinear medium with gain becomes larger. If we filter out some high frequency of beam, the decreasing nonlinear effect with large gain coefficient may be larger than that with small ones. In addition, we can find that the restraining effect is greater with smaller pinhole size in fixed gain coefficient, which can be clearly seen from Fig. 5(b). As the pinhole size reaches up to a certain value, the restraining effect is negligible. For gain coefficient 0.04/cm and 0/cm, when pinhole size is 3.2 cm, we can see that the relative intensity ratio achieves nearly one as shown in Fig. 5(b), as the pinhole size reduces to 1.22 cm, the relative intensity ratio correspondingly decreases to 0.44 and 0.34 for gain coefficient 0.04/cm and 0/cm, respectively. But this doesn't mean that we go on reducing the pinhole size to decrease the peak intensity of hot image. Smaller pinhole size can cause more energy lost. So the pinhole size should be chosen considering the balance of energy lost and suppressing effect.

In conclusion, the restraining effect of the spatical filter on hot image through nonlinear medium with gain is researched detailedly, the theoretical analysis based on linear diffraction theory and BT theory is also given. Through numerical simulations, we obtained the theory results are fairly near the numerical results. In addition, it is found that decreasing effect of peak intensity ratio

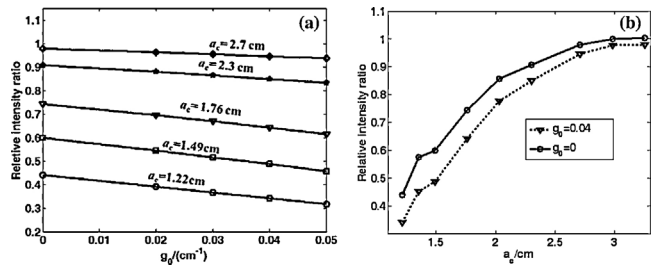


Fig. 5. (a) Peak intensity ratio versus gain coefficient with different pinhole size a_c . (b) peak intensity ratio related to different pinhole size with gain coefficients 0.04/cm and 0, respectively.

with small gain coefficient is larger than that with bigger ones for determined pinhole size because the nonlinear effect in nonlinear medium with gain becomes larger as the gain coefficient become bigger. If some high frequency of beam is suppressed, the decreasing nonlinear effect with large gain coefficient may be larger than that with small ones. The result may be helpful to optimize function of spatial filter in high power systems if the optical components appear some defects.

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