



Distortion of ultrashort laser pulses in a spatial filter

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ABSTRACT

By means of Collins diffraction integral, the propagation expression of an ultrashort laser pulse passing through a spatial filter is derived. The effects of the magnification of spatial filter on pulse broadening and distortion of pulse-front for the ultrashort optical pulse are analytically deduced and numerically simulated. It is found that pulse broadening and propagation time difference of a laser pulse getting through spatial filter is proportional to the magnification. As a conclusion, in an ultrashort pulse laser system with a large aperture, the effect of pulse broadening and distortion of pulse-front should be considered.

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1. Introduction

For using chirped pulse amplification (CPA), high power laser system has greatly developed in several decades and achieved orders of magnitude higher peak power than laser systems could generate before the invention of CPA. Due to rapid advances in using of ultrashort laser pulse, the study of their spatial and temporal distribution on propagation in free space and optical systems has become a subject of interest [1–4]. Ultrashort pulse focused by a chromatic lens has been extensively studied [5–8]. Even more, interact of chromatic and spherical aberration in focusing an ultrashort pulse has been analyzed [9,10].

In high power laser system, dispersion of optical component of a spatial filter will stretch pulse duration, induce pulse-front distortion and great temporal modulations. Up to now, there still no quantitative study about the dispersion effect of a spatial filter on the temporal distortion of an ultrashort pulse. In this paper, we present our results on the spatial dependence of temporal behavior of an ultrashort pulse due to dispersion in a pair of lenses and its impact on the pulse-front distortion in a spatial filter. The analytic deduction and numerical simulations of temporal behaviors for an ultrashort laser pulse propagating through a spatial filter are performed.

2. Spatial and temporal distribution of laser pulse

A typical spatial filter is depicted by Fig. 1, which have a pair of lenses with dispersion for an ultrashort pulse. RP0, RP1 and RP2 are focal plane, input reference plane and output reference plane, respectively. L_1, L_2, L_3 and L_4 are distances between reference planes and lenses. According to Collins diffraction integral and condition of one-dimension, the ray transfer matrix of optical system between RP0 and RP1 plane can be expressed as [11]:

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 1 & L_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{k_b}{f_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} 1 - \frac{L_2 k_b}{f_1} & L_1 \left(1 - \frac{L_2 k_b}{f_1}\right) + L_2 \\ -\frac{k_b}{f_1} & 1 - \frac{k_b L_1}{f_1} \end{bmatrix} \quad (1)$$

where

$$k_b = \frac{k_l - k_a}{k_a} \quad (2)$$

$$f_1 = f_{10}(n_0 - 1) \quad (3)$$

Considering dispersion up to the second order, the wave-number inside the lens is given by

$$k_l = \frac{\omega}{c} n(\omega) \approx k_0 n_0 [1 + \beta_1 \Delta\omega + \beta_2 (\Delta\omega)^2] \quad (4)$$

with

$$\beta_1 = \frac{1}{\omega_0} + \frac{1}{n_0} \frac{dn}{d\omega} \Big|_{\omega=\omega_0} \quad (5)$$

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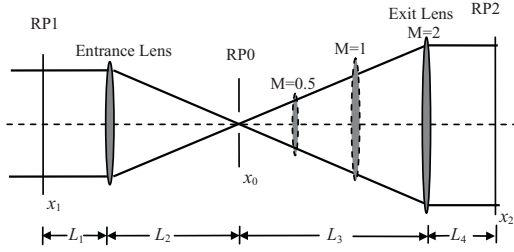


Fig. 1. A schematic of spatial filter with different magnifications.

$$\beta_2 = \frac{1}{\omega_0 n_0} \frac{dn}{d\omega} \Big|_{\omega=\omega_0} + \frac{1}{2n_0} \frac{d^2n}{d\omega^2} \Big|_{\omega=\omega_0} \quad (6)$$

The parameter f_{10} denotes the focal length of the entrance lens for center frequency ω_0 . The value of $n(\omega)$ is the frequency-dependent refractive index of lens material. k_a is the wave number in air. Under the same propagation algorithm, ray transfer matrix of optical system between RP2 plane and RP0 plane can be written as:

$$\begin{aligned} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} &= \begin{bmatrix} 1 & L_4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{k_b}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & L_3 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 - \frac{L_4 k_b}{f_2} & L_3 \left(1 - \frac{L_4 k_b}{f_2}\right) + L_4 \\ -\frac{k_b}{f_2} & 1 - \frac{k_b L_3}{f_2} \end{bmatrix} \end{aligned} \quad (7)$$

where $f_2 = f_{20}(n_0 - 1)$, and f_{20} is the focal length of the exit lens for center frequency ω_0 .

In terms of Collins diffraction formula, the field amplitude at image plane behind the exit lens is

$$\begin{aligned} E_2(x_2, z; \omega) &= \sqrt{\frac{i}{\lambda B_1}} \sqrt{\frac{i}{\lambda B_2}} \exp(ik_a(L_1 + L_2 + L_3 + L_4)) \\ &\quad + i(k_l - k_a)(d_1 + d_2) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx_1 dx_0 E_1(x_1, z) \\ &\quad \times V(\omega) T(x_0) \exp \left[\frac{ik_a}{2B_1} (A_1 x_1^2 - 2x_1 x_0 + D_1 x_0^2) \right] \\ &\quad \times \exp \left[\frac{ik_a}{2B_2} (A_2 x_0^2 - 2x_2 x_0 + D_2 x_2^2) \right] \end{aligned} \quad (8)$$

where $E_1(x_1, z)$ is spatial profile of the initial input beam, $V(\omega)$ is amplitude of a monochromatic plane wave, and $d_{1,2}$ are the thickness of the lens pair along the optical axis. Aperture function of the pinhole with radius a_0 yields

$$T(x_0) = \begin{cases} 1, & \text{if } |x_0| \leq a_0 \\ 0, & \text{others} \end{cases} \quad (9)$$

From relations (1)–(9), within the approximation of $L_1 = 0$ and $L_4 = 0$, we can obtain the amplitude distribution of output field

$$\begin{aligned} E_2(x_2, z; \omega) &= \sqrt{\frac{i}{\lambda B_1}} \sqrt{\frac{i}{\lambda B_2}} \exp(ik_a(L_1 + L_2 + L_3 + L_4) + i(k_l - k_a)(d_1 + d_2)) \int_{-1}^{+1} \int_{-1}^{+1} dx_1 dx_0 E_1(x_1, z) V(\omega) \\ &\quad \exp \left[\frac{ik_0}{2f_{10}} x_0^2 \left(1 + \frac{\Delta\omega}{\omega_0}\right) \right] \exp \left[\frac{-ik_0(1 + (\Delta\omega/\omega_0))x_1 x_0}{f_{10}} \right] \exp \left(-i\frac{u_1}{2} x_1^2 \right) \exp \left[\frac{-ik_0 a_1^2}{2f_{10}} x_1^2 \Delta\omega \left(b_1 - \frac{f_{10}}{L_2 \omega_0} + b_2 \Delta\omega \right) \right] \\ &\quad \exp \left[\frac{ik_0}{2f_{20}} x_0^2 \left(1 + \frac{\Delta\omega}{\omega_0}\right) \right] \exp \left[\frac{-ik_0(1 + (\Delta\omega/\omega_0))x_2 x_0}{f_{20}} \right] \exp \left(-i\frac{u_2}{2} x_2^2 \right) \exp \left[\frac{-ik_0 a_2^2}{2f_{20}} x_2^2 \Delta\omega \left(b_1 - \frac{f_{20}}{L_3 \omega_0} + b_2 \Delta\omega \right) \right] \end{aligned} \quad (10)$$

where

$$u_1 = a_1^2 k_0 \left(\frac{1}{f_{10}} - \frac{1}{L_2} \right) \quad (11)$$

$$u_2 = a_2^2 k_0 \left(\frac{1}{f_{20}} - \frac{1}{L_3} \right) \quad (12)$$

$$b_1 = \frac{1}{\omega_0} + \frac{1}{(n_0 - 1)} \frac{dn}{d\omega} \Big|_{\omega=\omega_0} \quad (13)$$

$$b_2 = \frac{1}{\omega_0(n_0 - 1)} \frac{dn}{d\omega} \Big|_{\omega=\omega_0} + \frac{1}{2(n_0 - 1)} \frac{d^2n}{d\omega^2} \Big|_{\omega=\omega_0} \quad (14)$$

$a_{1,2}$ are the radii of the lens pair.

Introducing the numerical aperture of the lens pair $F_1 = a_1/f_{10}$, $F_2 = a_2/f_{20}$ and the magnification $M = f_{20}/f_{10}$, the amplitude distribution of the output field in the time domain can be derived from inverse Fourier transform of Eq. (10) as

$$\begin{aligned} E_2(x_2, z; t) &= \exp[ik_0(L_1 + L_2 + L_3 + L_4) + ik_0(n_0 - 1)(d_1 + d_2)] \\ &\quad \times \int_{-\infty}^{+\infty} d(\Delta\omega) V(\omega) \sqrt{\frac{i}{\lambda B_1}} \sqrt{\frac{i}{\lambda B_2}} \\ &\quad \int_{-1}^{+1} \int_{-1}^{+1} dx_1 dx_0 E_1(x_1, z) \exp \left[\frac{ik_0}{2f_{10}} x_0^2 \left(1 + \frac{\Delta\omega}{\omega_0}\right) \right] \\ &\quad \times \exp \left[\frac{ik_0}{2f_{20}} x_0^2 \left(1 + \frac{\Delta\omega}{\omega_0}\right) \right] \exp \left(-i\frac{u_1}{2} x_1^2 \right) \exp \left(-i\frac{u_2}{2} x_2^2 \right) \\ &\quad \times \exp \left[\frac{-ik_0(1 + (\Delta\omega/\omega_0))x_1 x_0}{f_{10}} \right] \exp \left[\frac{-ik_0(1 + (\Delta\omega/\omega_0))x_2 x_0}{f_{20}} \right] \\ &\quad \times \exp[i\Delta\omega(t + \tau_0 - \tau_1 x_1^2 - \tau_2 x_2^2)] \exp[i(\Delta\omega)^2(\delta_0 - \delta_1 x_1^2 - \delta_2 x_2^2)] \end{aligned} \quad (15)$$

where

$$\tau_0 = \frac{L_1 + L_2 + L_3 + L_4}{c} + b_1 k_0(n_0 - 1)(d_1 + d_2) \quad (16)$$

$$\tau_1 = \frac{k_0 a_1^2}{2f_{10}} \left(b_1 - \frac{1}{\omega_0} \right) \quad (17)$$

$$\tau_2 = \frac{1}{2} k_0 M a_1 F_2 \left(b_1 - \frac{1}{\omega_0} \right) \quad (18)$$

$$\delta_0 = b_2 k_0(n_0 - 1)(d_1 + d_2) \quad (19)$$

$$\delta_1 = \frac{k_0 a_1^2}{2f_{10}} b_2 \quad (20)$$

$$\delta_2 = \frac{1}{2} k_0 M a_1 F_2 b_2 \quad (21)$$

The values of τ_0 , τ_1 , τ_2 and δ_0 , δ_1 , δ_2 are determined by the lens parameters taken at center frequency. The phase terms that include δ_0 , δ_1 and δ_2 are results of the group-velocity dispersion (GVD) in lens material.

The term $(\Delta\omega)^2 \delta_0$ is responsible for a phase modulation caused by an ultrashort pulse traveling through the lens pair of thicknesses d_1 and d_2 . The phase term $(\Delta\omega)^2(\delta_1 x_1^2 + \delta_2 x_2^2)$ is caused by the focusing behavior of the lens pair. The term $\Delta\omega \tau_0$ is a certain time delay resulting from pulse transmission along the optical axis. The

$$\text{delay term } F_{PTD} = \Delta\omega(\tau_1 x_1^2 + \tau_2 x_2^2) \text{ is the factor of the propagation time difference (PTD) which is the time delay between the pulse front and the phase front passing through spatial filter. When the}$$

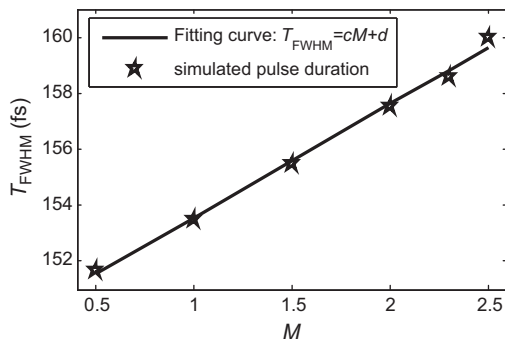


Fig. 2. Pulse broadening of an ultrashort laser pulse as a function of the magnification of spatial filter.

aperture of the entrance lens is constant, F_{PTD} is proportional to magnification of spatial filter

$$F_{PTD} = \Delta\omega(\tau_1 x_1^2 + \tau_2 x_2^2) \propto aM + b \quad (22)$$

where a and b are parameters with relation to a_1 and other constant factors. Similarly, the factor F_τ of pulse broadening in Eq. (15) can be defined as

$$F_\tau = (\Delta\omega)^2(\delta_1 x_1^2 + \delta_2 x_2^2) \propto cM + d \quad (23)$$

where c and d are parameters in relation to a_1 and other constant factors.

From the obvious linear relations in Eqs. (22) and (23), we can estimate the PTD and pulse duration are proportional to the magnification of the spatial filter. Under a small-signal approximation, we can describe the temporal behavior of laser pulse for various M in a linear system of spatial filter. The theoretical result can show a clear physical image for temporal transformation characteristic of an ultrashort pulse getting through a spatial filter in a high power laser system.

3. Numerical results

In order to analyze the value of pulse broadening and distortion of the pulse front, we need to discuss far field distribution of the ultrashort laser pulse getting through the spatial filter. The spatial and temporal distribution of laser pulse in the far field can be described by Fourier transform of output field in RP2 plane. In the following, we discuss the temporal behavior of the far field behind spatial filter for a Gaussian-shaped input ultrashort pulse

$$V(\omega) = \frac{T_0}{2\sqrt{\ln 2}} \exp \left[-\frac{(\Delta\omega)^2 T_0^2}{(8 \ln 2)} \right] \quad (24)$$

where T_0 is the incident pulse duration. Initial beam profile is the sixth-order super-Gaussian. Eq. (15) has been numerical integrated with the following parameters: $\lambda_0 = 1053$ nm, $L_1 = 10$ mm, $a_0 = 0.014$ mm, $a_1 = a_2 = 10$ mm, $T_0 = 150$ fs, $dn/d\omega|_{\omega=\omega_0} = 0.0059$ fs, $d^2n/d\omega^2|_{\omega=\omega_0} = 0.00016$ fs², $f_{10} = f_{20} = 70$ mm, $n_0 = 1.457$.

Fig. 2 illustrates the effect of various magnifications on pulse broaden of the ultrashort laser pulse propagating through dispersive lens pair of the spatial filter. It is observed from Fig. 2 that pulse duration of the ultrashort laser pulse is proportional to the magnification of spatial filter. The simulation result is in agreement with Eq. (23).

In general, for an ultrashort laser pulse, marginal rays of lens travel faster than axially propagating part. This kind of time difference and GVD can cause temporal pulse broadening in the field behind the spatial filter. With the larger aperture of lens material, pulse broadening for an ultrashort laser pulse getting through a spatial filter becomes wider.

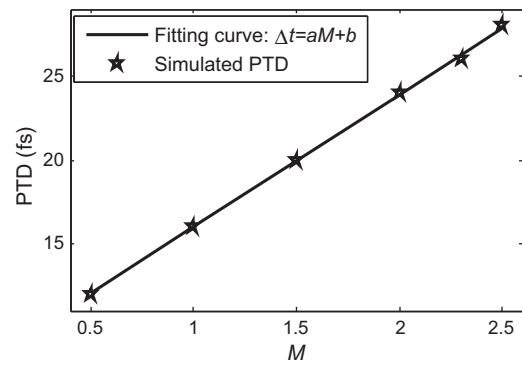


Fig. 3. PTD as a function of the magnification of spatial filter.

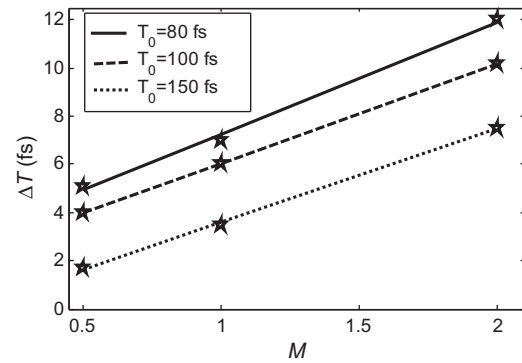


Fig. 4. ΔT depending on M for different initial pulse durations T_0 .

The time propagation difference PTD is depicted in Fig. 3 for various M . With increasing M , the PTD increases linearly in output field of the spatial filter. The numerical simulation results of PTD for various M are in accord with the predicting result of theoretical calculation shown in Eq. (22). The temporal difference is mainly caused by the time delay between the partial laser pulses which get through different parts of the lens aperture.

For showing clear physical picture, the relative pulse broadening induced by dispersion in spatial filter is defined as $\Delta T = T_{FWHM} - T_0$. Fig. 4 shows that the pulse broadening in the output field of spatial filter as a function of the parameter M for various T_0 . It is obvious that the temporal stretching increases rapidly with decreasing T_0 , and the pulse broadening will be up to 15% of the total pulse duration for shorter pulse of 80 fs. Thus, the effect of pulse broadening should be considered for an ultrashort pulse.

4. Conclusions

The pulse broadening and pulse front distortion become very important in designing an optical system for an ultrashort laser pulse. In this paper, the propagation of an ultrashort pulse through a spatial filter has been described based on analytical deduce of Collins diffraction integral. The clear expression for the temporal characteristic of an ultrashort laser pulse propagating through a spatial filter has been derived. Numerical simulation results display pulse stretching and propagation time difference of a laser pulse getting through a spatial filter is proportional to M . With the increase of M , the PTD and pulse broadening increase rapidly in output field of spatial filter. For beam expanded system, the pulse front distortion and pulse broadening will completely destruct the original pulse's qualities.

In a high power laser system, the effects of spatial distortion induced by both magnification of spatial filter and the diameter of

pinhole in the focusing plane always have been analyzed in detail [12]. Actually, if both the ultrashort pulse and large aperture of laser system are considered simultaneously, the coupling modulation effect between spatial and temporal distortions will be given a further study.

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