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Theoretical modelling of the effect of photon lifetime on the output dynamics of Er-doped distributed feedback fibre lasers*

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By employing a simple model of describing three-level lasers, we have theoretically investigated the effect of photon lifetime on the output dynamics of Er-doped distributed feedback fibre lasers. And based on the theoretical analysis we have proposed a promising method to suppress self-pulsing behaviour in the fibre lasers.

Keywords: photon lifetime, self-pulsing, Er-doped fibre distributed feedback laser
PACC: 4265S, 4255N

1. Introduction

The instability has been observed and discussed in Er-doped ring lasers in which the photon lifetime is usually several hundred nanoseconds.[1−12] Methods to control and suppress the self-pulsing behaviour in this kind of laser have also been proposed and demonstrated. However, there are few papers to deal with the self-pulsing behaviour in distributed feedback (DFB) fibre grating lasers whose photon lifetime is on the order of several nanoseconds or even less. In testing the output characteristics of Er-doped fibre lasers, we find that Er-doped DFB fibre lasers always exhibit self-pulsing. Because of a big difference in photon lifetime between the two kinds of fibre lasers, few conclusions can be applied directly to explaining the behaviour in DFB fibre lasers. So it is necessary to investigate the DFB fibre lasers particularly.

In this paper, we theoretically investigate the relation between the self-pulsing behaviour in an Er-doped DFB fibre grating laser and the photon lifetime in cavity through a simple model of describing a three-level laser system, in which only the photon number in the cavity and the population inversion are employed. The dynamic behaviour of the laser is carefully investigated for different photon lifetimes, and a promising method of suppressing the self-pulsing behaviour is proposed.

2. Model

An erbium-doped fibre laser is generally treated as a three-level laser system. We start with the following standard rate equations of a three-level:[5]

$$\frac{\mathrm{d}q}{\mathrm{d}t} = [V_\mathrm{a}BN - (1/\tau_\mathrm{c})]q, \tag{1a}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = W_\mathrm{p}(N_\mathrm{t} - N) - 2BqN - \frac{N_\mathrm{t} + N}{\tau_2}, \tag{1b}$$

where N is the instantaneous population inversion; N_t is the total population; q is the total number of photons; W_p represents the pumping rate; τ_2 is the decay time of the population of lasing upper level. The exact meanings of other parameters in Eqs.(1a) and (1b) can be found in Ref.[5].

Then we normalize Eq.(1b) by multiplying both sides with τ_2 and dividing both sides by N_t to obtain

$$\frac{\mathrm{d}(N/N_\mathrm{t})}{\mathrm{d}(t/\tau_2)} = -(1 + \tau_2 W_\mathrm{p} + 2\tau_2 Bq)(N/N_\mathrm{t}) + \tau_2 W_\mathrm{p} - 1.$$

Denoting N/N_t by D, $\tau_2 W_\mathrm{p}$ by I_p, $2\tau_2 Bq$ by I_L, and changing the time variable from t to $\tau = t/\tau_2$ such that $\mathrm{d}t = \tau_2 \mathrm{d}\tau$, the above equation becomes

$$\dot{D} = -(1 + I_\mathrm{p} + I_\mathrm{L})D + I_\mathrm{p} - 1. \tag{1c}$$

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Similarly, normalize Eq.(1a) by multiplying both sides with $(2\tau_2 B)\tau_2$ and denote $N/N_{\rm t}$ by D, $2\tau_2 Bq$ by $I_{\rm L}$, $\tau_2 V_{\rm a} B N_{\rm t}$ by g, $\tau_2/\tau_{\rm c}$ by k and $\tau = t/\tau_2$, then Eq.(1a) will turn to

$$\dot{I}_{\rm L} = -kI_{\rm L} + gI_{\rm L}D. \tag{1d}$$

By setting Eqs.(1c) and (1d) to be zero, we can easily obtain the steady solution as follows:

$$D_{\rm s} = \frac{k}{g}, \tag{2a}$$

$$I_{\rm Ls} = (I_{\rm p} - 1)\frac{g}{k} - (I_{\rm p} + 1). \tag{2b}$$

The laser threshold can be determined to be

$$I_{\rm pth} = \frac{k+g}{g-k} = \frac{1+s}{1-s}, \tag{2c}$$

where $s = k/g$. Defining $I_{\rm p} = rI_{\rm pth}$, the steady solution of $I_{\rm L}$ can be rewritten as

$$I_{\rm Ls} = \frac{(r-1)(1+s)}{s(1-s)}. \tag{2d}$$

The photon lifetime in cavity has been included in parameter s.

3. Stimulation of laser dynamics

Employing Eqs.(1c) and (1d), we can theoretically investigate the relation between the photon lifetime in cavity and the temporal stability of laser output. The parameters of the fibre are assumed to be as follows: the effective area A=2×10^{-11}m^2, the absorption cross section at pump wavelength $\sigma_{\rm p}$=3×10^{-25}m^2,[13] the emission cross section at laser signal wavelength $\sigma_{\rm s}$= 4.1×10^{-25}m^2,[14] the refractive index of the fibre core n=1.4544, the fibre grating length L=5cm, the normalized pump rate r=3, and the absorption coefficient at pump wavelength α=35dB/m. Further, assuming the photon lifetime in the laser to be varied between 1 and 100ns, we can obtain the simulation results of time-dependent D and $I_{\rm L}$ for different photon lifetimes.

The numerical calculations are given in Figs.1–4. Figure 1 shows the laser dynamics when the photon lifetime in the laser cavity is assumed to be 1ns. It is seen that the laser intensity $I_{\rm L}$ in Fig.1(a) is self-pulsing with 100% modulation depth in 500μs. The

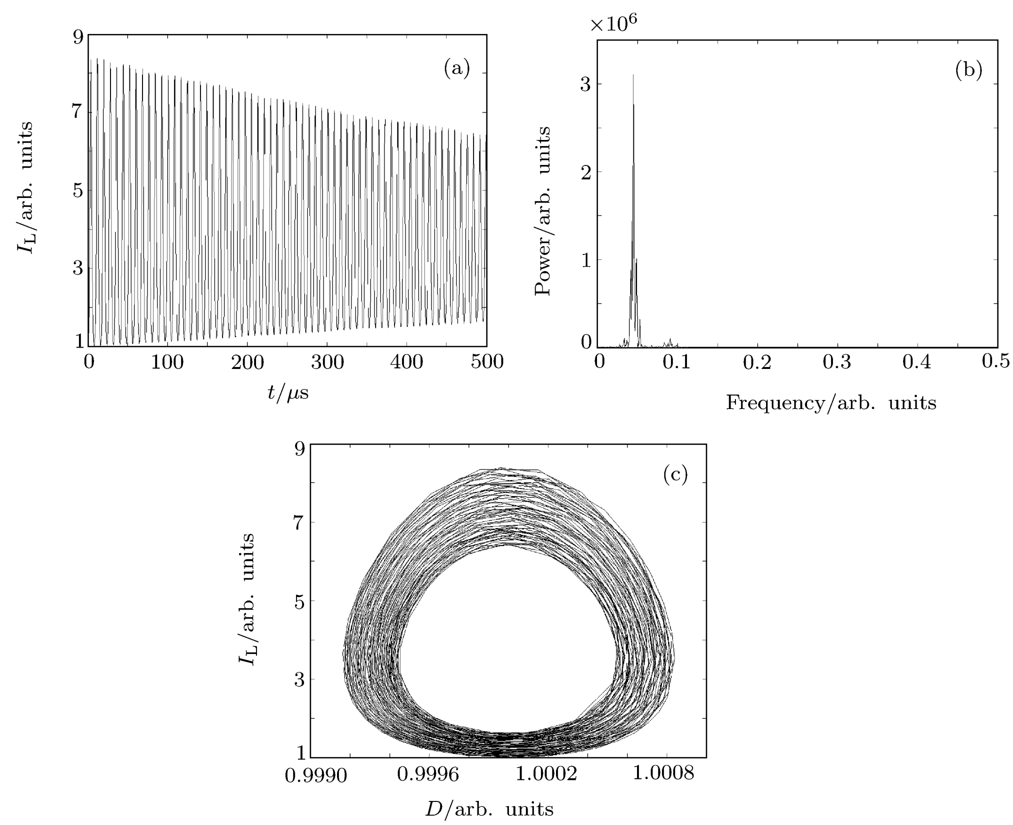

Fig.1. Laser dynamics with a photon lifetime of 1ns, showing instantaneous photon density in the cavity (a), Fourier power spectrum of the instantaneous photon density (b) and phase space between instantaneous photon density and instantaneous population inversion (c).

Fourier spectrum which is obtained by the fast Fourier transform of I_L for this photon lifetime is depicted in Fig.1(b). There are only two discrete frequency components in the spectrum. In fact, if the evolution time is assumed to be longer in the simulation, we will find that the laser temporal output will relax down to a stable output in a few milliseconds, which can be judged from the corresponding phase space of D and I_L shown in Fig.1(c). So for this case a stable temporal output will be achieved after several-millisecond transit behaviour of self-pulsing.

For a photon lifetime of 2ns, laser output intensity dynamics as a function of time is given in Fig.2(a). It can be seen from Fig.2(a) that the amplitude of pulses is ascending with time, which is contrary to the tendency in Fig.1(a). The power spectrum of I_L in Fig.2(b) comprises discrete lines, of which the amplitude is larger than that for the case in which the photon lifetime is 1ns (see Fig.1(b)). From the phase space shown in Fig.2(c), it is obvious that the laser dynamics is characterized by a limit cycle behaviour, but the range of I_L is compressed, which means that the amplitude of the pulses is reduced. This can be clearly seen from a comparison between Fig.1(a) and Fig.2(a).

When the photon lifetime increases to 3ns, the dynamics of the laser shows some differences in amplitude of the pulses shown in Fig.3(a) compared with those in Figs.1(a) and 2(a). The amplitude reduces and then varies with time neither ascendingly nor descendingly, but irregularly. There is little difference that can be found in the power spectrum and phase space in this case compared with those in the case where the photon lifetime is 2ns, which means that the laser temporal output shows a limit cycle behaviour, too.

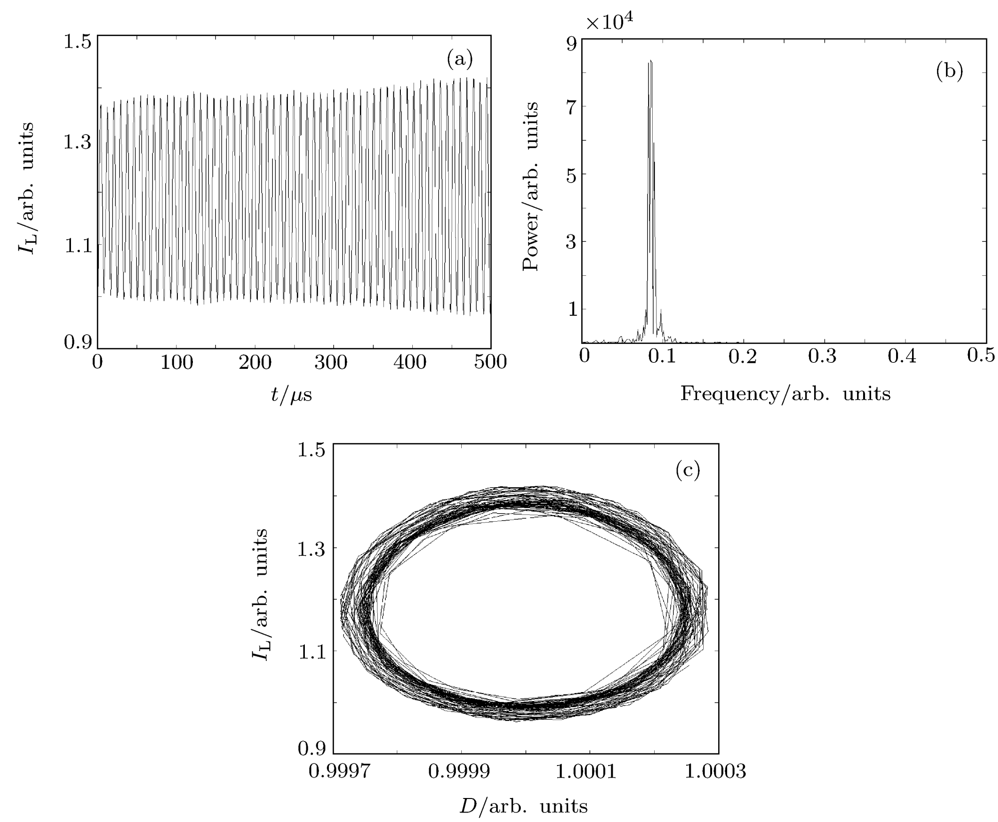

Fig.2. Laser dynamics with a photon lifetime of 2ns, showing instantaneous photon density in the cavity (a), Fourier power spectrum of the instantaneous photon density (b) and phase space between instantaneous photon density and instantaneous population inversion (c).

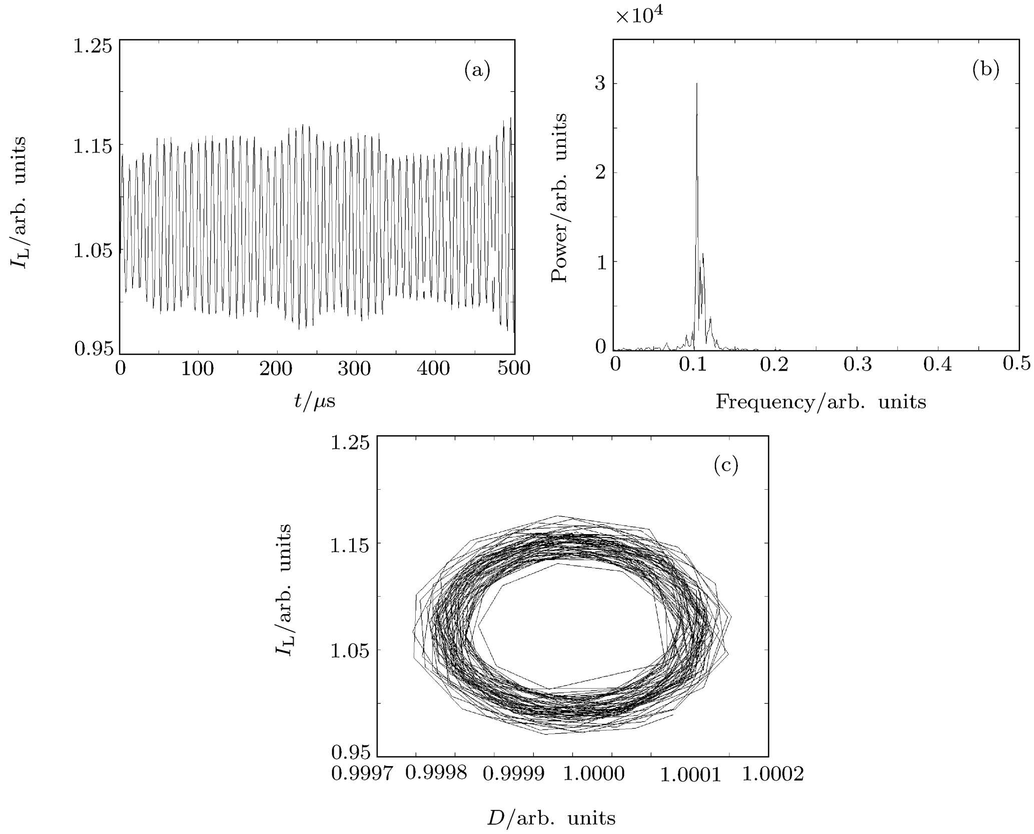

Fig.3. Laser dynamics with a photon lifetime of 3ns, showing instantaneous photon density in the cavity (a), Fourier power spectrum of the instantaneous photon density (b) and phase space between instantaneous photon density and instantaneous population inversion (c).

A great change in laser dynamics occurs for the case in which the photon lifetime is 4ns as shown in Fig.4. The laser output exhibits trains of pulses with a short-time continuous output between two trains of pulses. The amplitude between trains of the pulses is irregular. However, it is worth noting that the amplitude of the pulses is further compressed in the case of a longer photon lifetime. More frequency components can be found in the low frequency domain of I_L power spectrum, which is shown in Fig.4(b). As shown in Fig.4(c), the phase space indicates that the trajectory changes along a spiral path. Assuming a longer time to be used in the simulation, we find that the trajectory does not approach a fixed point. The spiral path is not smooth because interrupt change can be seen in the phase space which corresponds to the transition area between the pulse clusters in Fig.4(a).

Completely irregular behaviour in laser dynamics occurs for the case in which the photon lifetime is 15ns, and it is indicates in Fig.5(a). From the power spectrum in this case we can see that, of the differences between Fig.5(b) and Fig.1(b)–Fig.4(b), it is the most prominent that there exist not only low frequency components but also high frequency components, i.e. I_L is composed of broadband spectra. The trajectory in phase space is chaotic as shown in Fig.5(c). A similar behaviour can be observed for all the cases when the photon lifetime varies between 15 and 100ns. Again, the amplitude of the pulses decreases to less than 1% of the steady state value.

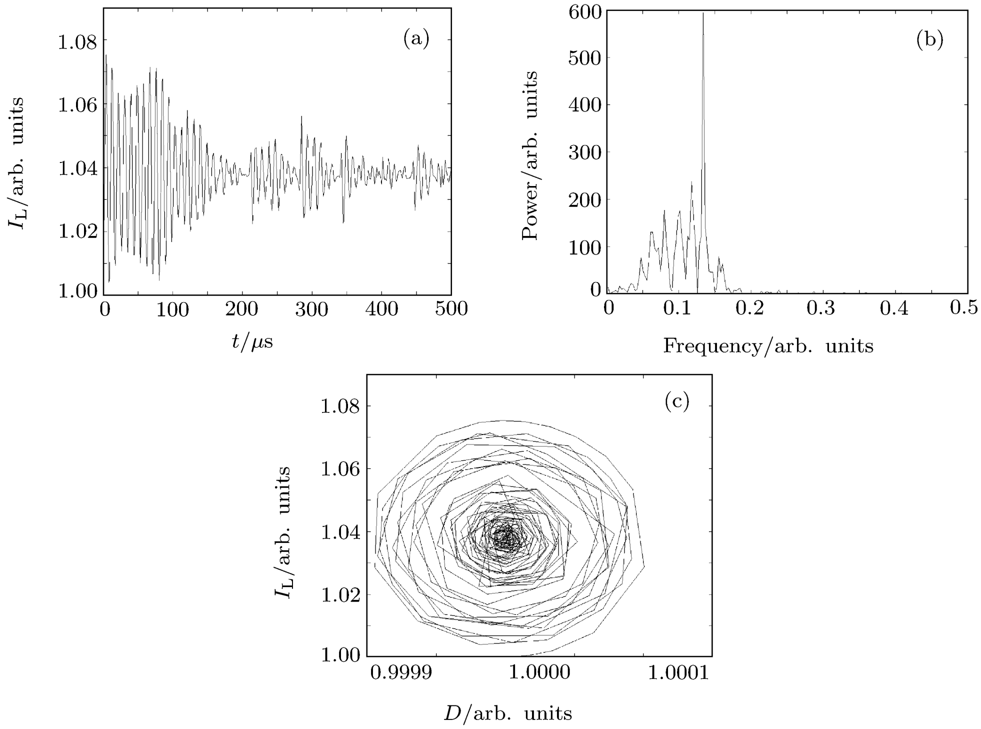

Fig.4. Laser dynamics with a photon lifetime of 4ns, showing instantaneous photon density in the cavity (a), Fourier power spectrum of the instantaneous photon density (b) and phase space between instantaneous photon density and instantaneous population inversion (c).

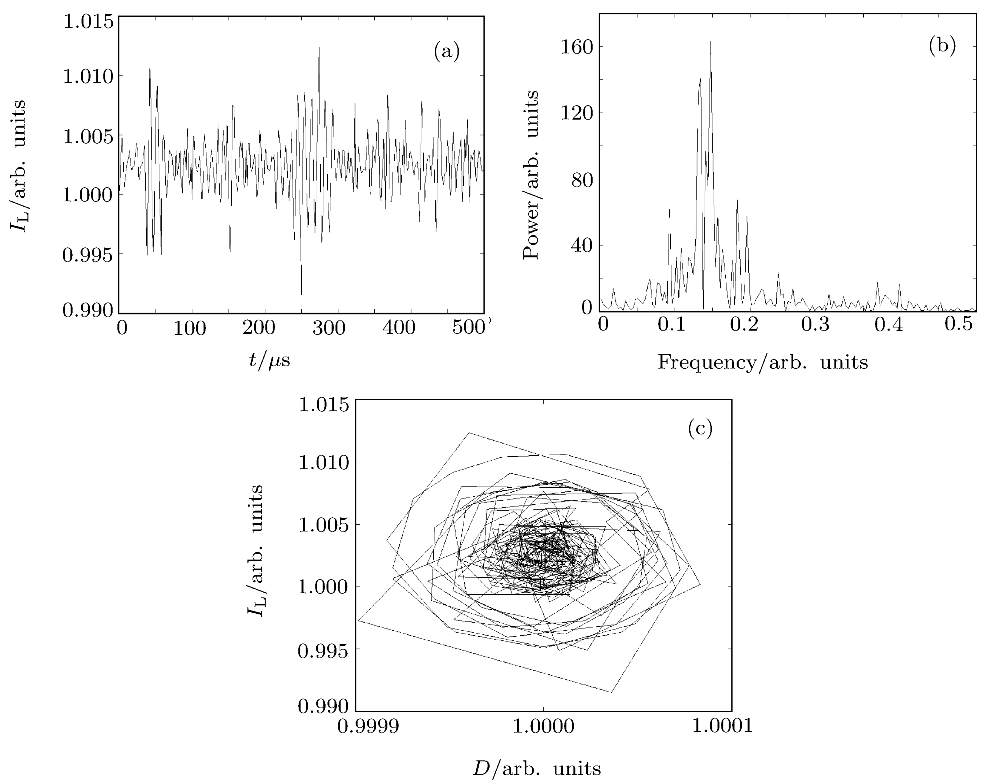

Fig.5. Laser dynamics with a photon lifetime of 15ns, showing instantaneous photon density in the cavity (a), Fourier power spectrum of the instantaneous photon density (b) and phase space between instantaneous photon density and instantaneous population inversion (c).

From the above figures, it is worth noting that with the increase of photon lifetime in cavity, the range of I_L oscillation gradually reduces from about 1.4 times the corresponding steady state value of I_L for the case where the photon lifetime is 2ns to less than 1.015 times the steady state value of I_L for the case where the photon lifetime is 15ns. However, the pulses become much more irregular with photon lifetime increasing. So we can conclude with confidence that a longer photon lifetime is favourable for suppressing the self-pulsing behaviour of laser in that it can help to reduce the output modulation amplitude. But a short photon lifetime, such as 1ns in our simulation, can help to suppress the self-pulsing behaviour completely after initial self-pulsation in the first several milliseconds although the amplitude of the transit pulses is large. In the following we will analyse the value range of photon lifetime that we can achieve in the Er-doped phase-shifted DFB fibre lasers.

It is well-known that the cavity decay rate of the laser cavity, which is inversely proportional to the photon lifetime, can be written as

$$\gamma_c = 2\alpha_0 c + \ln(1/R)/T, \tag{3}$$

where α_0 represents the loss in the cold cavity, which means that the absorption of the active media in the cavity is not taken into account. For the Er-doped phase-shifted DFB fibre laser, α_0 comprises mainly two parts: one is the background loss at the lasing wavelength of the fibre used to fabricate the fibre grating and the other is the UV induced loss at the lasing wavelength resulting from the fibre grating fabrication. And we assign R to the fibre grating reflection coefficient and T to the cavity round-trip time which is related to the effective fibre grating length by

$$T = 2n_{\text{eff}} L_c/c, \tag{4}$$

where n_{eff} is the effective index of the fibre core; L_c is the effective length of the fibre grating, given by $L_c = 1/\kappa$[14] with κ being the fibre grating coupling coefficient; c is the speed of light in vacuum.

The typical parameter values for the Er-doped phase-shifted DFB laser are given in Table 1.

Table 1. Typical values of parameters in the Er-doped phase-shifted DFB fibre laser.

parameters	typical value
α_b/dB	5.6×10^{-3}
α_{UV}/dB	0.01–1.0
c/(m/s)	3×10^8
T/ps	130
R/dB	20–33

Figure 6 gives the photon lifetime computed by using expression (3) and the parameters listed in Table 1. Figure 6(a) shows the photon lifetime versus UV induced loss with the assumed R=30dB and T=130ps, and Figure 6(b) gives the value of photon lifetime versus the reflectivity of the fibre grating with the assumption of α_{UV}=0.5dB.

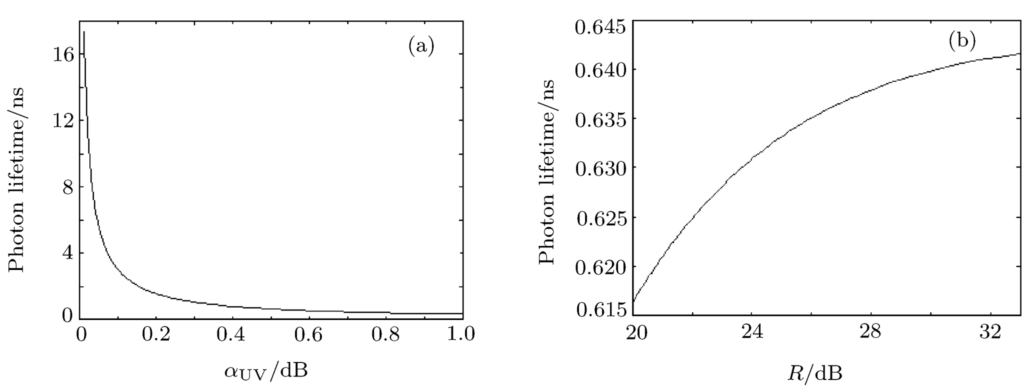

Fig.6. Photon lifetime versus UV induced loss (a) and reflective coefficient of the phase-shifted fibre grating (b).

In Fig.6(a), when the UV induced loss is 0.01–1dB for the grating, the corresponding photon lifetimes are in a range of 0.31–17.37ns. Figure 6(b) indicates that when the fibre grating reflection coefficient varies between 20 and 33dB, the corresponding value of photon lifetime is in a range of 0.616–0.64ns. A comparison between Figs.6(a) and 6(b) shows that the photon lifetime in laser cavity is mainly determined by the UV induced loss during fibre grating fabrication. So it is a promising method to suppress the self-pulsing behaviour in the laser by introducing UV induced loss as large as possible during fibre grating fabrication. But this is in contradiction to the fact that the laser will deteriorate with more UV induced loss[11] or even cannot lase. So there should be considered a compromise between UV loss and laser performances.

4. Conclusion

We have theoretically investigated the effect of photon lifetime on the dynamics of an Er-doped phase-shifted DFB fibre grating laser. When the photon lifetime in cavity increases from 2 to 15ns, the amplitude of the output pulses reduces down to an acceptable level, less than 1.5% of the total output power. It can be concluded that a longer photon lifetime can help to decrease the self-pulsing amplitude for the fibre lasers of this kind. However, a short photon lifetime, 1ns in our simulation, is favourable for suppressing the self-pulsation in the laser, for the self-pulsing behaviour can be completely suppressed after initial large-amplitude pulses. And by analysing the decay rate of cavity, we find that the photon lifetime in cavity is determined mainly by the UV induced loss during fibre grating fabrication. A smaller UV induced loss is favourable for achieving a longer photon lifetime. So minimizing the UV induced loss during fabrication will help to suppress the laser output instability with respect to the pulse amplitude. An appropriately high UV loss can help to achieve a short photon lifetime for stable operation. In fact, by appropriately increasing the pump rate, continuous-wave (CW) operation of the laser can also be achieved, of which the simulation results will be presented elsewhere.

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