

Negative refraction in a honeycomb-lattice photonic crystal

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Received 19 September 2006; accepted 24 October 2006 by E.L. Ivchenko

Available online 9 November 2006

Abstract

In this paper we theoretically investigate a photonic crystal with dielectric rods in a honeycomb lattice. Two left-handed frequency regions are found in the second and third photonic band by using the plane wave expansion method to analyze the photonic band structure and equifrequency contours. Subwavelength imaging by the photonic crystal flat lens are systematically studied by numerical simulations using the multiple scattering method. Different from the photonic crystals with noncircular dielectric rods in air, this structure is almost isotropic at the optimal frequency for superlensing. As a comparison, flat slab focusing is also demonstrated at other frequencies in the two left-handed regions.

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PACS: 78.20.Ci; 41.20.JB; 42.70.Qs

Keywords: A. Photonic crystal; A. Multiple scattering; D. Left-handed materials

1. Introduction

A material with simultaneously negative permittivity and permeability, first introduced by Veselago in 1968 [1], has recently attracted renewed interest because of experimental progress [2–5]. Such a material is often called a Left-handed material (LHM), since the electric field, the magnetic field and the wave vector of an electromagnetic wave propagating in it form a left-hand system. Compared with conventional materials, many extraordinary electromagnetic phenomena could be expected in LHMs [1]. For example, negative refraction occurs because the phase velocity of the light propagating inside LHMs is pointed in the opposite direction to the energy flow. The conventional optical lens needs curved surfaces to form an image, whereas LHMs can focus a point source to produce a real image even for a flat slab. Ideally, a flat loss-less LHM slab, which is impedance-matched to its

surrounding, can be made into a perfect lens surpassing the diffraction limit [6].

Due to the absence of LHMs in nature, various approaches have been proposed to fabricate the equivalent metamaterial [2, 3,7,8]. Photonic crystal (PhC) is one of them due to its dispersion characteristics, which can be obtained by analyzing the photonic band structure and equifrequency contours (EFCs). Notomi's theoretical work indicated that left-handed behaviors in PhCs are possible in the regimes of negative group velocity above the first band near the Brillouin-zone (BZ) center [7]. Left-handed phenomena have been investigated theoretically and experimentally in a lot of two-dimensional (2D) triangular lattice PhCs such as rectangular dielectric cylinders in air [9], elliptical dielectric rods in air [10] and cylindrical air-holes in dielectric [11–13]. In this paper, we theoretically investigate a photonic crystal made from cylindrical dielectric rods arranged in a honeycomb lattice, which is a triangular lattice with two rods in each unit cell. Two left-handed frequency regions are found in the second and third band. Subwavelength imaging by the PhC flat lens are studied through dispersion characteristics analysis and numerical simulation of field patterns. As a comparison, flat

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slab focusing are also demonstrated at other frequencies in the two left-handed regions.

The rest of this paper is organized as follows. In Section 2, we discuss the dispersion characteristics of the honeycomb-lattice PhC. Numerical simulations of the field patterns for negative refraction and flat slab focusing are shown in Section 3. The conclusions are given in Section 4.

2. Dispersion characteristics analysis

The 2D PhC considered in this paper is formed by a honeycomb lattice of cylindrical rods with dielectric constant 9.2. The radius of the rods is $0.26a$, where a is the distance between nearest-neighbor rods. For this honeycomb structure, we only consider TM polarization so the electric field E_z is parallel to these rods. Using the plane wave expansion method [14], the photonic band structure as well as EFCs are calculated and plotted in Fig. 1. The frequency is normalized as a/λ . From the photonic band structure in Fig. 1(a), we can see that there are two bands (i.e. the second and third band) with distinct downward curvatures, which indicates that the PhC behaves in a left-handed manner [7,15]. The effective index n_{eff} in these two frequency regions (extending from 0.22 to 0.27 in the second band and from 0.37 to 0.41 in the third band) is shown in Fig. 2(a). The normalized frequency $\omega_0 = 0.24$, at which the effective index $n_{\text{eff}} = -1$, is the optimal frequency for superlensing [6]. Different from PhCs with noncircular rods [9,10], as shown in Fig. 1(b), the EFC of this PhC at the optimal frequency is very close to a circle. Considering the symmetry of a honeycomb lattice, in one sixth of the first BZ the angle-dependent effective index $n_{\text{eff}}(\omega_0, \theta)$ is calculated and plotted in Fig. 2(b). Because of near isotropy, $n_{\text{eff}}(\omega_0, \theta)$ hardly varies with the angle θ formed by the wave vector $\mathbf{k}(\omega_0)$ and ΓK . At the same time, the angle-dependent effective indexes $n_{\text{eff}}(\omega, \theta)$ for $\omega_1 = 0.26$ and $\omega_2 = 0.38$ are also plotted in Fig. 2(b) as a contrast.

3. Numerical simulations

To test the above analyses, numerical simulations are conducted to investigate the negative refraction and flat superlensing by using the multiple scattering method [16–18]. Considering the inactive Bloch modes along the direction of ΓM , which results from the symmetry of the eigenmodes of this structure, the interface between the PhC slab and free space is arranged along ΓK [14]. First, we consider a slit beam [19] at normalized frequency $\omega_0 = 0.24$ incident on a PhC slab with an angle $\alpha = 45^\circ$ to the interface normal. The propagation maps are shown in Fig. 3. It is obvious that the incident wave refracts along the opposite direction of the reflected wave, indicating that the effective index of the slab is $n_{\text{eff}} = -1$. Because the PhC slab is not impedance-matched to free space, at the interface the incident beam does not refract without reflections as described in Ref. [1,6]. In Fig. 3, the reflected waves are denoted by dotted arrows.

Negative refraction allows a flat slab to behave as a lens. Usually, there are two kinds of negative refraction and its

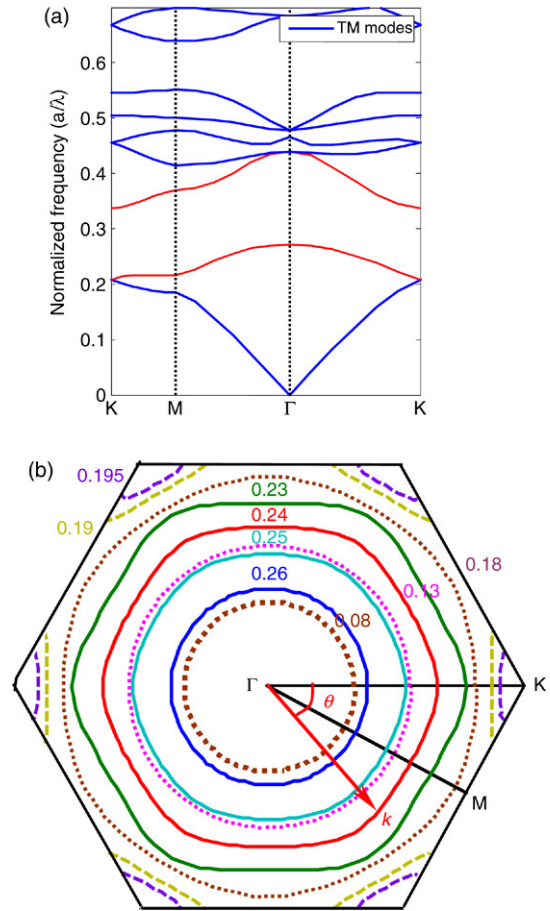


Fig. 1. (a) Photonic band structure and (b) EFCs for the 2D honeycomb lattice PhC studied in this paper.

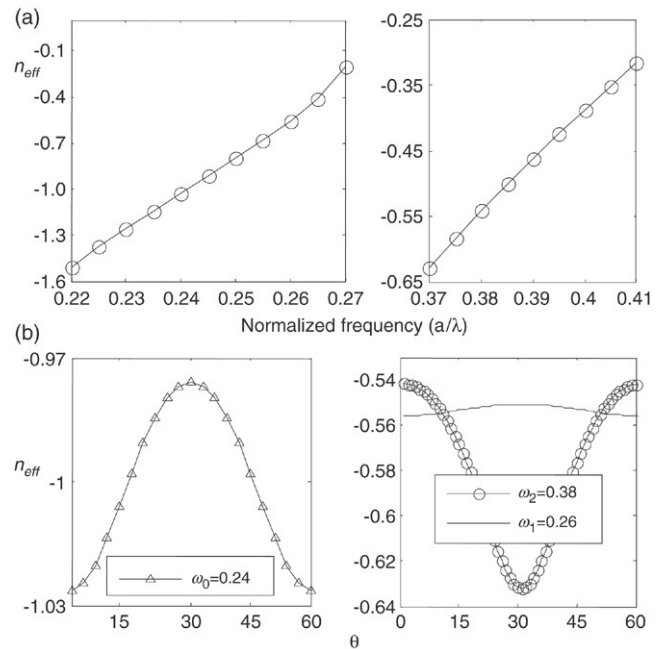


Fig. 2. (a) Effective index in these two left-handed regions. (b) The angle-dependent effective index $n_{\text{eff}}(\theta)$ at three normalized frequency $\omega_0 = 0.24$, $\omega_1 = 0.26$ and $\omega_3 = 0.38$.

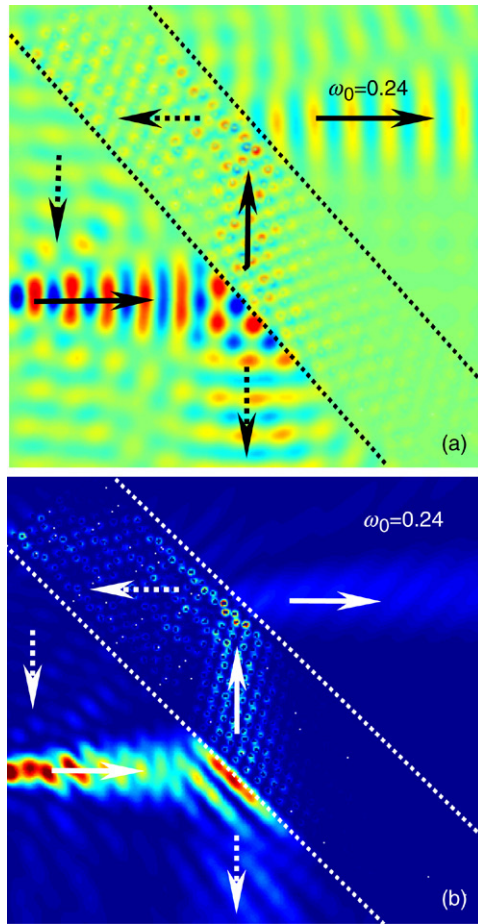


Fig. 3. Negative refraction for an incident beam at normalized frequency $\omega_0 = 0.24$ with an incident angle $\alpha = 45^\circ$. (a) Snapshot of the electric field. (b) Intensity distribution.

resulting focusing effect in PhCs [8,10,20,21]. One is the left-handed behavior as described by Veselago. The other negative refraction is realized by anisotropy, where the phase velocity makes an acute angle with the energy flow. Selecting the proper interface, a flat slab lens also can be made by 2D PhCs in a right-handed frequency region [22–25].

In the following, we demonstrate a left-handed superlens made from this honeycomb-lattice PhC at the optimal frequency. A $14a$ thick and $50a$ wide PhC slab is taken as a sample. First, a point source at normalized frequency $\omega_0 = 0.24$ is placed at $d_o = 6.5a$ away from the first interface of the PhC slab. As shown in Fig. 4, an image is formed at $d_i = 7.4a$ away from the second interface. At the same time, a clear image appears inside the PhC slab, which is clear evidence of LHMs following the wave-beam negative refraction law. In Fig. 5 we change the object distance to $d_o = 2.5a$. The image moves to $d_i = 11.5a$ and the inner focus appears too. It can be found that the image distance varies according to the source distance and the summation of them is always equal to the thickness of the slab, which is required by Snell's law for a flat lens with $n_{\text{eff}} = -1$. The full widths at the half maximum (FWHM) of these two images are both 0.48λ . As shown in Fig. 6, the same phenomena are found in an $8a$ thick PhC slab. For the object distance $d_o = 2.5a$, the image is located at $d_i = 5.2a$.

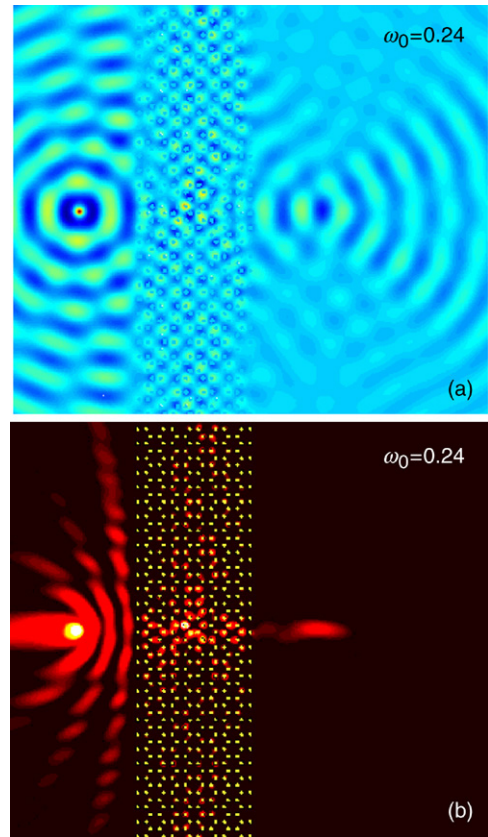


Fig. 4. The propagation maps for a $14a$ thick and $50a$ wide PhC slab scattered by a point source at normalized frequency $\omega_0 = 0.24$. The object distance is $d_o = 6.5a$.

As a contrast, we also check flat slab imaging for other normalized frequencies located in the second and third band respectively, i.e. $\omega_1 = 0.26$ and $\omega_2 = 0.38$. The field patterns are shown in Fig. 7(a) and (b). The PhC behaves as a left-handed optically thinner medium at these two frequencies and images are also formed behind the PhC slab. The discrepancies between Fig. 7(a) and (b) result from anisotropy, which is evident in Fig. 2(b).

4. Conclusion

In this paper, we have theoretically investigated a honeycomb-lattice PhC. Two left-handed frequency regions have been found in the second and third band. Different from the PhC with noncircular rods arranged in a triangular lattice, this PhC behaves with near isotropy at the optimal frequency for superlensing. Typical left-handed phenomena such as negative refraction and flat superlensing have been demonstrated by numerical simulations using the multiple scattering method. The dependences of the images on the object distance and slab thickness have also been discussed at the optimal frequency. As a comparison, flat slab focusing is also demonstrated at other frequencies in the two left-handed regions.

Acknowledgements

This work is partially supported by the Natural Science Foundation of China (Grant Nos. 10576012 and 60538010), the

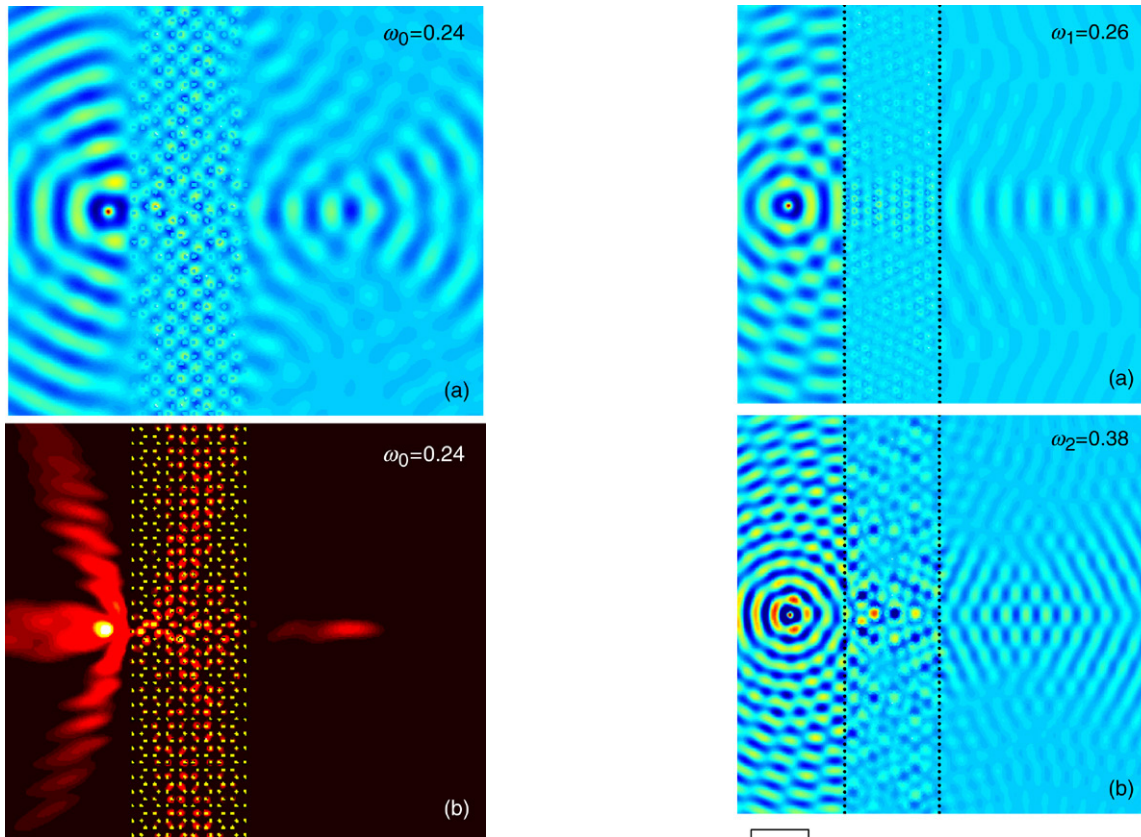


Fig. 5. The same as Fig. 4, except the object distance is $d_o = 2.5a$.

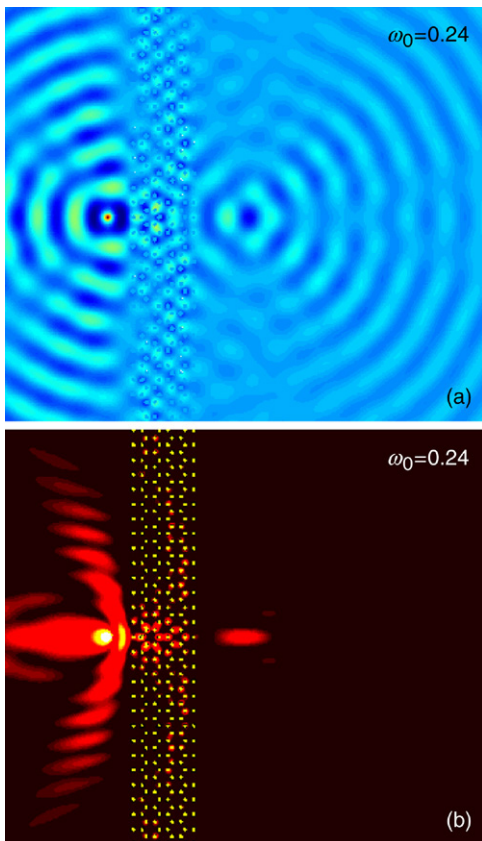


Fig. 6. The same as Fig. 5, except the thickness of the PhC slab is $8a$.

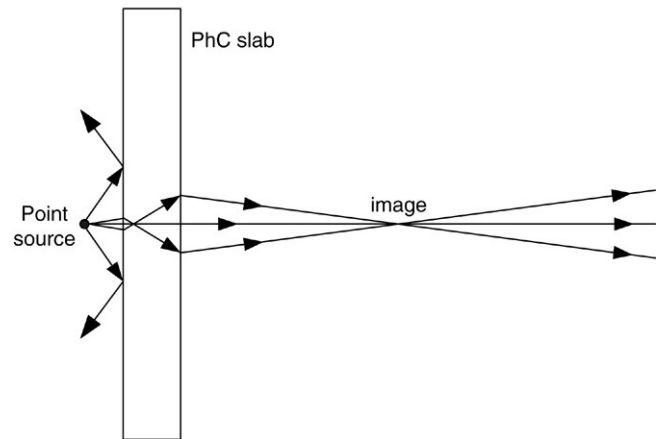


Fig. 7. Flat slab focusing for the PhC at two normalized frequencies. (a) $\omega_1 = 0.26$ and (b) $\omega_2 = 0.38$. (c) The conceptual layout of flat slab focusing by a left-handed optically thinner medium.

National High Technology Research and Development Program of China (Grant No. 2004AA84ts12).

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