Analysis of measurement error in the experiment of laser equation of state with impedance-match way and the Hugoniot data of Cu up to ~2.24 TPa with high precision

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This article has reported the detailed analysis about the error in the experimental measurement of laser equation of state. A kind of matrix method to calculate the uncertainty of state parameter was put forward and applied to the error estimation in the experiment with Al-Cu impedance-match target. The shock adiabatic data of Cu with the pressure up to ~2.24 TPa and the relative uncertainty of shock velocity of ~2% have been also presented. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2538097]

I. INTRODUCTION

The experimental data of equation of state (EOS) have the practical utility only when it has the enough high accuracy, thereby exactly estimating the error is very important in EOS experimental study. Actually, some published papers have treated the error problem, Ref. 1 discussed the experimental system error caused by the data uncertainty of the reference material and theoretic model, Ref. 2 studied the error transfer, Refs. 3 and 4 sketched out the expression of the measurement error, most of the papers gave the value of the error,[5–14] but do not expatiate on the source of the error and indicate how the error has been calculated. This article will give a detailed description of the experimental error.

In the experiment of equation of state by laser-driven shock wave, the shock velocity D can be measured easily,[2–17] its value that can displayed itself accuracy, can be expressed as follows:

$$D=\overline{D}\pm\delta D=\overline{D}\pm\sigma,\quad k=1,\quad p=68.3\%, \tag{1}$$

or

$$D=\overline{D}\pm 2\delta D=\overline{D}\pm 2\sigma,\quad k=2,\quad p=95.4\%, \tag{2}$$

or

$$D=\overline{D}\pm 3\delta D=\overline{D}\pm 3\sigma,\quad k=3,\quad p=99.7\%, \tag{3}$$

where $\overline{D}$ is the average shock velocity, p is the confidence level that corresponds to the different confidence factor (k), δD (or σ) is the combined standard uncertainty. Usually, the expanded uncertainty 2σ is employed as the error bar in the measurement, thus the relative uncertainty of D or its precision can be expressed

$$\eta=\frac{2\delta D}{\overline{D}}=\frac{2\sigma}{\overline{D}}. \tag{4}$$

Actually, the shock velocity D cannot be measured directly in the experiment, but obtained by

$$D=\frac{d}{t}, \tag{5}$$

where the sample step thickness d and the transit time t of the shock wave in the step (the shock wave must be stable in the step) can be measured directly and are independent of each other in measurement. Obviously, the uncertainty δD depends on the uncertainties δd and δt. Using the error transfer rule and Eq. (5), δD is given as

$$\delta D=\sqrt{\left(\frac{\partial D}{\partial d}\right)^2(\delta d)^2+\left(\frac{\partial D}{\partial t}\right)^2(\delta t)^2} = D\sqrt{\left(\frac{\delta d}{d}\right)^2+\left(\frac{\delta t}{t}\right)^2}. \tag{6}$$

In the impedance-match experiment,[15–17] we may classify the parameters into three types: the direct measured parameter, the standard parameter, and the indirect measured parameter (or say the calculated parameter). The classified parameters are listed in Table I.

For the direct measured parameter, its errors include both the statistic error brought by many times measurement (namely the type A uncertainty in the general error theory) and the error caused by the instrument precision (namely the type B uncertainty) and its combined standard uncertainty is the square root of the sum of the respective square of the types A and B uncertainty.

The standard parameter is the eigenvalue abstracted from the known EOS data of the reference material, its uncertainty can be obtained by the known data fitting, and attributed to the system error in the EOS measurement of the test material.

The indirect measured parameter, namely the calculated parameter, can be further divided into the direct and the indirect calculated parameter, the former is only related to the

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TABLE I. List of the classified parameters.

Sorts		Parameters		Reference material: S		Test material: T	
				Value	Uncertainty	Value	Uncertainty
Direct measured parameters		Step thickness: d		d_S	δd_S	d_T	δd_T
		Transit time of shock wave in the step: t		t_S	δt_S	t_T	δt_T
Standard parameters		Parameters of reference material	c_0	c_0	$\delta c_{0\ 0}$	⋯	⋯
			λ	λ	$\delta\lambda$	⋯	⋯
Calculated parameters	Direct	Shock velocity: D		D_S	δD_S	D_T	δD_T
		Initial density: ρ		ρ_{0S}	$\delta\rho_{0S}$	ρ_{0T}	$\delta\rho_{0T}$
	Indirect	Particle velocity: u		u_S	δu_S	u_T	δu_T
		Shock pressure: P		P_S	δP_S	P_T	δP_T

direct measured parameter, and the latter is not only related to the direct measured parameter, but also the standard parameter and the former. Their uncertainty is obtained by the relevant functional formula and the error transfer rule, and moreover, the error induced by the correlation between the parameters must be taken into account.

In Table I, the standard parameters c_0 and λ of the reference material meet the relationship

$$D_S = c_0 + \lambda u_S. \tag{7}$$

The initial densities ρ_{0S}, ρ_{0T} of the reference and test materials, and their uncertainties $\delta\rho_{0S}$, $\delta\rho_{0T}$ can be obtained from the measurement with the weight method, etc.

II. VARIOUS PARAMETERS AND THEIR UNCERTAINTIES IN THE EXPERIMENT WITH THE IMPEDANCE-MATCH WAY

Figure 1 shows the configuration of the impedance-match target, S and T are the reference and test materials, respectively. The step thickness d of the sample can be measured with a step-surface profiler or an optical interference profiler, it is a consecutive collection (namely the multipoint measurement of n), and the average height of the substrate and two steps can be given: $\overline{h_{S0}}=(\sum_{i=1}^{n} h_{S0i}/n)$, $\overline{h_S}=(\sum_{i=1}^{n} h_{Si}/n)$, and $\overline{h_T}=(\sum_{i=1}^{n} h_{Ti}/n)$. Their root mean square roughness, R_{S0q}, R_{Sq}, and R_{Tq}, also can be presented in the same time. The meaning of R_q just corresponds to the type A uncertainty described with the standard deviation (namely the error caused by the statistical calculation), so the type A uncertainties of the height measurements are: $\Delta h_{S0}^A = R_{S0q}$, $\Delta h_S^A = R_{Sq}$, and $\Delta h_T^A = R_{Tq}$, respectively.

Then the step thicknesses of the reference and test materials are

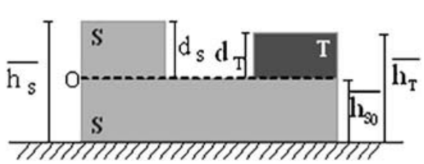

FIG. 1. Configuration of the impedance-match target.

$$d_S = |\overline{h_S} - \overline{h_{S0}}|, \tag{8}$$

$$d_T = |\overline{h_T} - \overline{h_{S0}}|. \tag{9}$$

Their type A uncertainties are

$$\Delta d_S^A = \sqrt{(\Delta h_{S0}^A)^2 + (\Delta h_S^A)^2}, \tag{10}$$

$$\Delta d_T^A = \sqrt{(\Delta h_{S0}^A)^2 + (\Delta h_T^A)^2}. \tag{11}$$

Further, their type B uncertainties (namely the errors caused by the instrument precision) depend on the relative measurement accuracy γ of the instrument. The meanings of γ is the standard deviation per unit thickness measurement, and the value of γ can be determined by the calibration. Thus, the type B uncertainties of d_S and d_T can be described as follows:

$$\Delta d_S^B = \gamma d_S, \tag{12}$$

$$\Delta d_T^B = \gamma d_T. \tag{13}$$

Obviously, the combined standard uncertainties of d_S and d_T are, respectively,

$$\delta d_S = \sqrt{(\Delta d_S^A)^2 + (\Delta d_S^B)^2}, \tag{14}$$

$$\delta d_T = \sqrt{(\Delta d_T^A)^2 + (\Delta d_T^B)^2}. \tag{15}$$

On the other hand, the shock luminescence from the rear surface of the impedance-match target can be recorded with the streak camera and shown in Fig. 2, the transit time t of the shock wave in the steps can be obtained from Fig. 2.

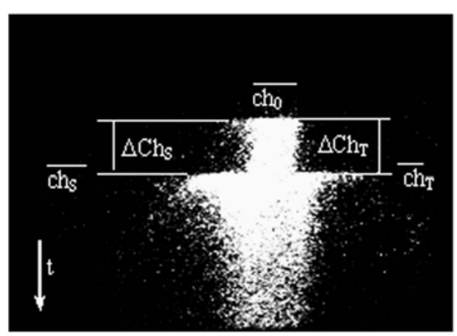

FIG. 2. Shock luminescence from the rear surfaces of the impedance-match target.

The channel number at the time of the shock breakout from the substrate and the two steps can be readout directly. After multipoint sampling and statistic calculation, the average channel numbers for the three surfaces can be given: $\overline{\mathrm{ch}_{S0}}=(\sum_{i=1}^{n}\mathrm{ch}_{S0i}/n)$, $\overline{\mathrm{ch}_S}=(\sum_{i=1}^{n}\mathrm{ch}_{Si}/n)$, and $\overline{\mathrm{ch}_T}=(\sum_{i=1}^{n}\mathrm{ch}_{Ti}/n)$. Their type A uncertainties: $\Delta\mathrm{ch}_{S0}^A$, $\Delta\mathrm{ch}_S^A$, and $\Delta\mathrm{ch}_T^A$ (namely the standard deviation caused by the statistical calculation), also can be calculated by the definition

$$\left[\sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(\mathrm{ch}_i-\overline{\mathrm{ch}})^2}\right]$$

of the standard deviation. Thus, the channel number differences between the substrate and the two steps are

$$\Delta\mathrm{Ch}_S=|\overline{\mathrm{ch}_S}-\overline{\mathrm{ch}_{S0}}|, \tag{16}$$

$$\Delta\mathrm{Ch}_T=|\overline{\mathrm{ch}_T}-\overline{\mathrm{ch}_{S0}}|. \tag{17}$$

Their type A uncertainties are

$$\Delta\mathrm{Ch}_S^A=\sqrt{(\Delta\mathrm{ch}_{S0}^A)^2+(\Delta\mathrm{ch}_S^A)^2}, \tag{18}$$

$$\Delta\mathrm{Ch}_T^A=\sqrt{(\Delta\mathrm{ch}_{S0}^A)^2+(\Delta\mathrm{ch}_T^A)^2}. \tag{19}$$

Then, the transit time of the shock wave in the two steps can be expressed as follows:

$$t_S=\Delta\mathrm{Ch}_S\times t_c, \tag{20}$$

$$t_T=\Delta\mathrm{Ch}_T\times t_c. \tag{21}$$

Their type A uncertainties just are

$$\Delta t_S^A=\Delta\mathrm{Ch}_S^A\times t_c, \tag{22}$$

$$\Delta t_T^A=\Delta\mathrm{Ch}_T^A\times t_c. \tag{23}$$

where t_c is the time duration per channel, its value can be obtained by the calibration and its unit is ns/ch.

The type B uncertainties of t_S and t_T are induced by two factors. One is the calibrating error Δt_c (ns/ch), it has brought $\Delta\mathrm{Ch}_S\times\Delta t_c$ and $\Delta\mathrm{Ch}_T\times\Delta t_c$ to the type B uncertainties of t_S and t_T, respectively. Another is the instrument precision, namely the time resolution Δt_q (ns) of the streak camera, Δt_q can be also obtained by the calibration (see the details in the Appendix). Because Δt_q is equivalent to the expanded uncertainty with the confidence factor $k=3$, it is just proper that $\Delta t_q/3$ serves as the standard uncertainty. Thus, the total type B uncertainties of t_S and t_T are

$$\Delta t_S^B=\sqrt{(\Delta\mathrm{Ch}_S\times\Delta t_c)^2+\left(\frac{\Delta t_q}{3}\right)^2}, \tag{24}$$

$$\Delta t_T^B=\sqrt{(\Delta\mathrm{Ch}_T\times\Delta t_c)^2+\left(\frac{\Delta t_q}{3}\right)^2}. \tag{25}$$

Obviously, the combined standard uncertainties of t_S and t_T are, respectively,

$$\delta t_S=\sqrt{(\Delta t_S^A)^2+(\Delta t_S^B)^2}, \tag{26}$$

$$\delta t_T=\sqrt{(\Delta t_T^A)^2+(\Delta t_T^B)^2}. \tag{27}$$

According to Eqs. (5) and (6), the shock velocities in the two steps, and their combined standard uncertainties can be expressed

$$D_S=\frac{d_S}{t_S}, \tag{28}$$

$$D_T=\frac{d_T}{t_T}, \tag{29}$$

$$\delta D_S=D_S\sqrt{\left(\frac{\delta d_S}{d_S}\right)^2+\left(\frac{\delta t_S}{t_S}\right)^2}, \tag{30}$$

$$\delta D_T=D_T\sqrt{\left(\frac{\delta d_T}{d_T}\right)^2+\left(\frac{\delta t_T}{t_T}\right)^2}. \tag{31}$$

Using Eq. (7), the standard parameters c_0 and λ can be obtained from the linear fitting for n pairs of the known data D_{Si} and u_{Si} $(i=1-n)$ of the reference material, their uncertainties δc_0 and $\delta\lambda$ are just the standard deviation of this linear fitting.

In addition, there is a covariance $\delta(c_0,\lambda)$ because c_0 and λ are interrelated with each other. If the calculation of the parameter concerns with c_0 and λ, its uncertainty must include the error caused by the covariance $\delta(c_0,\lambda)$. The covariance $\delta(c_0,\lambda)$ depends on the correlation between c_0 and λ and is defined as

$$\delta(c_0,\lambda)=r(c_0,\lambda)\,\delta c_0\,\delta\lambda, \tag{32}$$

where $r(c_0,\lambda)$ is the correlation coefficient and is expressed as follows:

$$r(c_0,\lambda)=\frac{\sum_{i=1}^{n}(c_{0i}-c_0)(\lambda_i-\lambda)}{(n-1)\,\delta c_0\,\delta\lambda}, \tag{33}$$

where c_{0i} and λ_i correspond to a certain pair of the known data D_{Si} and u_{Si}.

About the indirect calculated parameters in Table I, the particle velocity u_S and the shock pressure P_S of the reference material can be easily obtained by Eq. (7) and shock wave relationship

$$u_S=\frac{D_S-c_0}{\lambda}, \tag{34}$$

$$P_S=\rho_{0S}D_Su_S=\frac{\rho_{0S}D_S(D_S-c_0)}{\lambda}. \tag{35}$$

Then, using the error transfer rule and considering the correlation between c_0 and λ, their combined standard uncertainties can be expressed as

$$\delta u_S = \sqrt{\left(\frac{\partial u_S}{\partial D_S}\right)^2 (\delta D_S)^2 + \left(\frac{\partial u_S}{\partial c_0}\right)^2 (\delta c_0)^2 + \left(\frac{\partial u_S}{\partial \lambda}\right)^2 (\delta \lambda)^2 + 2\left(\frac{\partial u_S}{\partial c_0}\right)\left(\frac{\partial u_S}{\partial \lambda}\right)\delta(c_0, \lambda)}, \quad (36)$$

$$\delta P_S = \sqrt{\left(\frac{\partial P_S}{\partial \rho_{0S}}\right)^2 (\delta \rho_{0S})^2 + \left(\frac{\partial P_S}{\partial D_S}\right)^2 (\delta D_S)^2 + \left(\frac{\partial P_S}{\partial c_0}\right)^2 (\delta c_0)^2 + \left(\frac{\partial P_S}{\partial \lambda}\right)^2 (\delta \lambda)^2 + 2\left(\frac{\partial P_S}{\partial c_0}\right)\left(\frac{\partial P_S}{\partial \lambda}\right)\delta(c_0, \lambda)}. \quad (37)$$

Furthermore, using the shock wave relationships, the impedance-match principle and the approximate method of the mirror reflection, the particle velocity u_T and the shock pressure P_T of the test material can be expressed as follows:

$$u_T = \frac{\left(4D_S - 3c_0 + \frac{\rho_{0T}}{\rho_{0S}}D_T\right) + \sqrt{\left(4D_S - 3c_0 + \frac{\rho_{0T}}{\rho_{0S}}D_T\right)^2 - 8(c_0^2 - 3c_0 D_S + 2D_S^2)}}{2\lambda}, \quad (38)$$

$$P_T = \rho_{0T} D_T \frac{\left(4D_S - 3c_0 + \frac{\rho_{0T}}{\rho_{0S}}D_T\right) + \sqrt{\left(4D_S - 3c_0 + \frac{\rho_{0T}}{\rho_{0S}}D_T\right)^2 - 8(c_0^2 - 3c_0 D_S + 2D_S^2)}}{2\lambda}. \quad (39)$$

Similar to Eqs. (36) and (37), their combined standard uncertainties can be expressed as follows:

$$\delta u_T = \sqrt{\left(\frac{\partial u_T}{\partial D_S}\right)^2 (\delta D_S)^2 + \left(\frac{\partial u_T}{\partial D_T}\right)^2 (\delta D_T)^2 + \left(\frac{\partial u_T}{\partial \rho_{0S}}\right)^2 (\delta \rho_{0S})^2 + \left(\frac{\partial u_T}{\partial \rho_{0T}}\right)^2 (\delta \rho_{0T})^2 + \left(\frac{\partial u_T}{\partial c_0}\right)^2 (\delta c_0)^2 + \left(\frac{\partial u_T}{\partial \lambda}\right)^2 (\delta \lambda)^2 + 2\left(\frac{\partial u_T}{\partial c_0}\right)\left(\frac{\partial u_T}{\partial \lambda}\right)\delta(c_0, \lambda)}, \quad (40)$$

$$\delta P_T = \sqrt{\left(\frac{\partial P_T}{\partial D_S}\right)^2 (\delta D_S)^2 + \left(\frac{\partial P_T}{\partial D_T}\right)^2 (\delta D_T)^2 + \left(\frac{\partial P_T}{\partial \rho_{0S}}\right)^2 (\delta \rho_{0S})^2 + \left(\frac{\partial P_T}{\partial \rho_{0T}}\right)^2 (\delta \rho_{0T})^2 + \left(\frac{\partial P_T}{\partial c_0}\right)^2 (\delta c_0)^2 + \left(\frac{\partial P_T}{\partial \lambda}\right)^2 (\delta \lambda)^2 + 2\left(\frac{\partial P_T}{\partial c_0}\right)\left(\frac{\partial P_T}{\partial \lambda}\right)\delta(c_0, \lambda)^2}. \quad (41)$$

Because the values of c_{0i} and λ_i cannot be given by a pair of the known data D_{Si} and u_{Si}, the values of the correlation coefficient $r(c_0, \lambda)$ in Eq. (33) and the covariance $\delta(c_0, \lambda)$ in Eq. (32) cannot be found yet, in the result, δu_S, δP_S, δu_T and δP_T expressed with Eqs. (36), (37), (40), and (41), also cannot be solved for their values. Therefore, we have advanced a new method on the basis of the matrix theory to resolve this problem.

III. A KIND OF MATRIX METHOD TO DETERMINE THE UNCERTAINTIES OF THE INDIRECT CALCULATED PARAMETERS

The following group of equations can be derived from Eq. (7), the shock wave relationships and the impedance-match principle

$$c_0 + \lambda u_S - D_S = 0, \quad (42)$$

$$\rho_{0S} D_S u_S - P_S = 0, \quad (43)$$

$$\rho_{0T} D_T u_T - P_T = 0, \quad (44)$$

$$\rho_{0T}(2u_S - u_T)[c_0 + \lambda(2u_S - u_T)] - P_T = 0. \quad (45)$$

In Eqs. (42) and (45), there are six known parameters of c_0, λ, ρ_{0S}, ρ_{0T}, D_S, and D_T, and four unknown parameters of u_S, u_T, P_S, and P_T, obviously, the latter can be solved easily. Now let us go to the solution of the uncertainties δu_S, δu_T, δP_S, and δP_T.

For the convenience of the calculation, each of the uncertainties δu_S, δu_T, δP_S, and δP_T may be separated into the system uncertainty and the measurement uncertainty, that expressed with δu_S^S, δu_T^S, δP_S^S, δP_T^S and δu_S^M, δu_T^M, δP_S^M, δP_T^M, respectively. The system uncertainties δu_S^S, δu_T^S, δP_S^S, δP_T^S are only related to the uncertainties δc_0, $\delta\lambda$ and the covariance $\delta(c_0, \lambda)$, and the measurement uncertainties δu_S^M, δu_T^M, δP_S^M, δP_T^M depend on the uncertainties $\delta\rho_{0S}$, $\delta\rho_{0T}$, δD_S and δD_T, they are irrelevant to each other. Thus, the uncertainties δu_S, δu_T, δP_S and δP_T can be expressed as follows:

$$\delta u_S = \sqrt{(\delta u_S^S)^2 + (\delta u_S^M)^2}, \quad (46)$$

$$\delta u_T = \sqrt{(\delta u_T^S)^2 + (\delta u_T^M)^2}, \quad (47)$$

$$\delta P_S = \sqrt{(\delta P_S^S)^2 + (\delta P_S^M)^2}, \quad (48)$$

$$\delta P_T = \sqrt{(\delta P_T^S)^2 + (\delta P_T^M)^2}. \tag{49}$$

Obviously, when we evaluate the system uncertainties δu_S^S, δu_T^S, δP_S^S, and δP_T^S with Eqs. (42) and (45), ρ_{0S}, ρ_{0T}, D_S, and D_T can be regarded as the invariants (namely the constants). After partial derivative with respect to the variables c_0 andλ, the group of partial differential equations is obtained, and can be expressed with the matrix form

$$f_1 \begin{bmatrix} \delta u_S^S \\ \delta P_S^S \\ \delta u_T^S \\ \delta P_T^S \end{bmatrix} = - f_2 \begin{bmatrix} \delta c_0 \\ \delta\lambda \end{bmatrix} \tag{50}$$

or

$$[\delta u_S^S \quad \delta P_S^S \quad \delta u_T^S \quad \delta P_T^S] f_1^T = -[\delta c_0 \quad \delta\lambda] f_2^T, \tag{51}$$

where

$$f_1 = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ \rho_{0S} D_S & -1 & 0 & 0 \\ 0 & 0 & \rho_{0T} D_T & -1 \\ 2\rho_{0S}\{\lambda(2u_S - u_T) + [c_0 + \lambda(2u_S - u_T)]\} & 0 & -\rho_{0S}\{\lambda(2u_S - u_T) + [c_0 + \lambda(2u_S - u_T)]\} & -1 \end{bmatrix}$$

and

$$f_2 = \begin{bmatrix} 1 & u_S \\ 0 & 0 \\ 0 & 0 \\ \rho_{0S}(2u_S - u_T) & \rho_{0S}(2u_S - u_T)^2 \end{bmatrix},$$

f_1^T and f_2^T are the transposed matrices of f_1 and f_2, respectively. Then, to multiply Eq. (50) by Eq. (51),

$$f_1 \begin{bmatrix} \delta u_S^S \\ \delta P_S^S \\ \delta u_T^S \\ \delta P_T^S \end{bmatrix} [\delta u_S^S \quad \delta P_S^S \quad \delta u_T^S \quad \delta P_T^S] f_1^T = f_2 \begin{bmatrix} \delta c_0 \\ \delta\lambda \end{bmatrix} [\delta c_0 \quad \delta\lambda] f_2^T \tag{52}$$

or

$$\begin{bmatrix} (\delta u_S^S)^2 & \delta u_S^S \delta P_S^S & \delta u_S^S \delta u_T^S & \delta u_S^S \delta P_T^S \\ \delta u_S^S \delta P_S^S & (\delta P_S^S)^2 & \delta P_S^S \delta u_T^S & \delta P_S^S \delta P_T^S \\ \delta u_S^S \delta P_T^S & \delta P_S^S \delta u_T^S & (\delta u_T^S)^2 & \delta u_T^S \delta P_T^S \\ \delta u_T^S \delta P_T^S & \delta P_S^S \delta P_T^S & \delta u_T^S \delta P_T^S & (\delta P_T^S)^2 \end{bmatrix}_T^S = f_1^{-1} f_2 \times \begin{bmatrix} (\delta c_0)^2 & (\delta c_0)\times(\delta\lambda) \\ (\delta c_0)\times(\delta\lambda) & (\delta\lambda)^2 \end{bmatrix} f_2^T f_1^{-T}, \tag{53}$$

where the matrices with superscript -1 are the inverse matrices,

$$\begin{bmatrix} (\delta c_0)^2 & (\delta c_0)\times(\delta\lambda) \\ (\delta c_0)\times(\delta\lambda) & (\delta\lambda)^2 \end{bmatrix}$$

is named the covariance matrix of c_0 and λ.

For the group of the known data D_{Si}, u_{Si} $(i=1-n)$ of the reference material S, supposing

$$X = \begin{bmatrix} 1 & u_{S1} \\ 1 & u_{S2} \\ \vdots & \vdots \\ 1 & u_{Si} \\ \vdots & \vdots \\ 1 & u_{Sn} \end{bmatrix}$$

and

$$Y = \begin{pmatrix} D_{S1} \\ D_{S2} \\ \vdots \\ D_{Si} \\ \vdots \\ D_{Sn} \end{pmatrix},$$

the linear fitting parameters c_0 and λ for D_{Si} and u_{Si} can be expressed with matrix form:

$$\beta = \begin{bmatrix} c_0 \\ \lambda \end{bmatrix} = (X^T X)^{-1} X^T Y. \tag{54}$$

The sum of the residual square of this linear fitting is

$$R_{SS} = Y^T Y - \beta^T X^T Y. \tag{55}$$

The residual mean square is

$$\sigma^2 = \frac{R_{SS}}{n-2}. \tag{56}$$

Upon that, the covariance matrix of c_0 and λ can be expressed as follows:

$$\begin{bmatrix} (\delta c_0)^2 & (\delta c_0)\times(\delta\lambda) \\ (\delta c_0)\times(\delta\lambda) & (\delta\lambda)^2 \end{bmatrix} = \sigma^2 (X^T X)^{-1}. \tag{57}$$

Actually, the meaning of $(\delta c_0)\times(\delta\lambda)$ in Eq. (57) is equal to the covariance $\delta(c_0, \lambda)$ in Eq. (32) after the operation

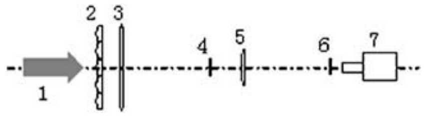

FIG. 3. Experimental setup: 1: driving laser, 2: lens-array, 3: focal lens, 4: target, 5: imaging magnifying optical system, 6: optical attenuating plate, and 7: streak camera.

from Eq. (54) to Eq. (56).[18] Thus, the system uncertainties δu_S^S, δu_T^S, δP_S^S, δP_T^S can be found after putting Eq. (57) into Eq. (53).

Similarly, the parameters c_0 and λ can be regarded as the constants when we evaluate the measurement uncertainties δu_S^M, δu_T^M, δP_S^M, δP_T^M with Eqs. (42) and (45). After partial derivative with respect to the variables ρ_{0S}, ρ_{0T}, D_S, and D_T the group of partial differential equations is obtained and can be expressed with the matrix form

$$f_1\begin{bmatrix}\delta u_S^M\\ \delta P_S^M\\ \delta u_T^M\\ \delta P_T^M\end{bmatrix} = -f_3\begin{bmatrix}\delta\rho_{0S}\\ \delta\rho_{0T}\\ \delta D_S\\ \delta D_T\end{bmatrix} \tag{58}$$

or

$$[\delta u_S^M \quad \delta P_S^M \quad \delta u_T^M \quad \delta P_T^M]f_1^T = -[\delta\rho_{0S} \quad \delta\rho_{0T} \quad \delta D_S \quad \delta D_T]f_3^T, \tag{59}$$

where f_1 is ditto and

$$f_3 = \begin{bmatrix} 0 & 0 & -1 & 0\\ u_S D_S & 0 & u_S\rho_{0S} & 0\\ 0 & u_T D_T & 0 & u_T\rho_{0T}\\ (2u_S-u_T)[c_0+\lambda(2u_S-u_T)] & 0 & 0 & -1\end{bmatrix}.$$

Then, to multiply Eq. (58) by Eq. (59),

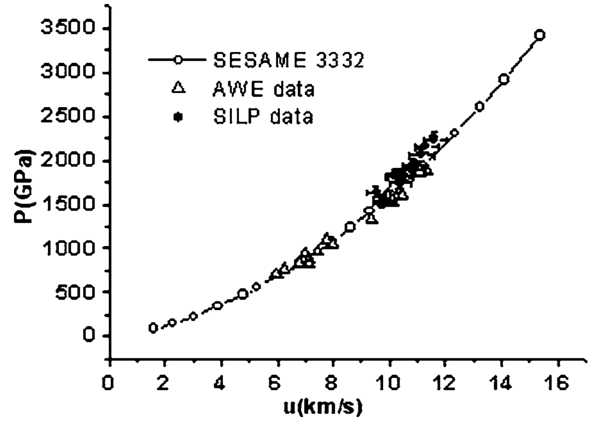

FIG. 4. Shock pressure vs particle velocity for Cu.

$$\begin{bmatrix}(\delta u_S^M)^2 & \delta u_S^M\delta P_S^M & \delta u_S^M\delta u_T^M & \delta u_S^M\delta P_T^M\\ \delta u_S^M\delta P_S^M & (\delta P_S^M)^2 & \delta P_S^M\delta u_T^M & \delta P_S^M\delta P_T^M\\ \delta u_S^M\delta P_T^M & \delta P_S^M\delta u_T^M & (\delta u_T^M)^2 & \delta u_T^M\delta P_T^M\\ \delta u_T^M\delta P_T^M & \delta P_S^M\delta P_T^M & \delta u_T^M\delta P_T^M & (\delta P_T^M)^2\end{bmatrix} = f_1^{-1}f_3\begin{bmatrix}(\delta\rho_{0S})^2 & 0 & 0 & 0\\ 0 & (\delta\rho_{0T})^2 & 0 & 0\\ 0 & 0 & (\delta D_S)^2 & 0\\ 0 & 0 & 0 & (\delta D_T)^2\end{bmatrix} f_3^T f_1^{-T}. \tag{60}$$

Thus, the measurement uncertainties δu_S^M, δu_T^M, δP_S^M, δP_T^M can be found with Eq. (60).

In this way, the combined uncertainties δu_S, δu_T, δP_S, δP_T can be obtained with Eqs. (46) and (49). Obviously, the matrix method can easily resolve the problem about the solution of the covariance $\delta(c_0, \lambda)$. In our practice calculation, a simple computer code edited by ourselves was used to treat with the earlier matrix operation.

In the same time, the values of δu_S, δu_T, δP_S, and δP_T also can be gotten by Eqs. (36) and (37) and Eqs. (40) and (41) after substituting $(\delta c_0)\times(\delta\lambda)$ in Eq. (57) for $\delta(c_0, \lambda)$. It is easy to be testified that the results from two methods are accordant.

TABLE II. Experimental results (SILP data) for Al-Cu impedance-match target.

	Al			Cu						
	D (km/s)			D (km/s)			u (km/s)		P (GPa)	
Exp. No.	D	$\sigma=\delta D$	$2\sigma/D$ (%)	D	$\sigma=\delta D$	$2\sigma/D$ (%)	u	$\sigma=\delta u$	P	$\sigma=\delta P$
1	24.867	0.22	1.769	19.985	0.201	2.012	10.205	0.182	1822.121	32.038
2	24.993	0.257	2.057	19.678	0.191	1.941	10.388	0.209	1826.262	36.506
3	25.771	0.25	1.940	20.01	0.208	2.079	10.9	0.208	1948.591	36.887
4	25.51	0.239	1.874	19.519	0.202	2.070	10.838	0.200	1889.954	34.686
5	25.637	0.217	1.693	19.705	0.193	1.959	10.883	0.186	1915.915	32.506
6	23.484	0.217	1.848	17.698	0.16	1.808	9.773	0.180	1545.267	28.281
7	23.358	0.199	1.704	17.43	0.158	1.813	9.752	0.167	1518.538	25.866
8	23.733	0.218	1.837	19.222	0.18	1.873	9.542	0.177	1638.583	30.027
9	27.266	0.243	1.782	21.602	0.218	2.018	11.614	0.205	2241.462	39.094
10	26.803	0.253	1.888	21.427	0.204	1.904	11.303	0.207	2163.651	39.323
11	26.337	0.264	2.005	20.723	0.201	1.940	11.139	0.216	2062.334	39.788
12	24.677	0.234	1.897	18.822	0.173	1.838	10.385	0.194	1746.287	32.392

TABLE III. Detailed calculation for the experiment No. 11 from Table II.

Thickness	Al substrate surface	$\overline{h_{S0}}$ (μm)	19.99791
		$\Delta h_{S0}^A = R_{S0q}$ (μm)	0.00447
	Al step surface	$\overline{h_S}$ (μm)	34.56473
		$\Delta h_S^A = R_{Sq}$ (μm)	0.01107
	Cu step surface	$\overline{h_T}$ (μm)	31.70658
		$\Delta h_T^A = R_{Tq}$ (μm)	0.00406
	Al step	$d_S = \lvert \overline{h_S} - \overline{h_{S0}} \rvert$ (μm)	14.56682
		$\Delta d_S^A = \sqrt{(\Delta h_{S0}^A)^2 + (\Delta h_S^A)^2}$ (μm)	0.01194
		$\Delta d_S^B = \gamma d_S$ (μm)	0.05908
		$\delta d_S = \sqrt{(\Delta d_S^A)^2 + (\Delta d_S^B)^2}$ (μm)	0.06028
	Cu step	$d_T = \lvert \overline{h_T} - \overline{h_{S0}} \rvert$ (μm)	11.70867
		$\Delta d_T^A = \sqrt{(\Delta h_{S0}^A)^2 + (\Delta h_T^A)^2}$ (μm)	0.00604
		$\Delta d_T^B = \gamma d_T$ (μm)	0.04749
		$\delta d_T = \sqrt{(\Delta d_T^A)^2 + (\Delta d_T^B)^2}$ (μm)	0.04787
Time	Al substrate surface	$\overline{\mathrm{ch}_{S0}}$ (ch)	464.044
		$\Delta \mathrm{ch}_{S0}^A$ (ch)	0.821
	Al step surface	$\overline{\mathrm{ch}_S}$ (ch)	928.045
		$\Delta \mathrm{ch}_S^A$ (ch)	1.893
	Cu step surface	$\overline{\mathrm{ch}_T}$ (ch)	938.056
		$\Delta \mathrm{ch}_T^A$ (ch)	1.707
	Al step	$\Delta \mathrm{Ch}_S = \lvert \overline{\mathrm{ch}_S} - \overline{\mathrm{ch}_{S0}} \rvert$ (ch)	464.001
		$t_S = \Delta Ch_S \times t_c$ (ns)	0.553089
		$\Delta \mathrm{Ch}_S^A = \sqrt{(\Delta \mathrm{ch}_{S0}^A)^2 + (\Delta \mathrm{ch}_S^A)^2}$ (ch)	2.063
		$\Delta t_S^A = \Delta \mathrm{Ch}_S^A \times t_c$ (ns)	0.002459
		$\Delta t_S^B = \sqrt{(\Delta \mathrm{Ch}_S \times \Delta t_c)^2 + (\Delta t_q/3)^2}$ (ns)	0.004410
		$\delta t_S = \sqrt{(\Delta t_S^A)^2 + (\Delta t_S^B)^2}$ (ns)	0.00565
	Cu step	$\Delta \mathrm{Ch}_T = \lvert \overline{\mathrm{ch}_T} - \overline{\mathrm{ch}_{S0}} \rvert$ (ch)	474.012
		$t_T = \Delta \mathrm{Ch}_T \times t_c$ (ns)	0.565022
		$\Delta \mathrm{Ch}_T^A = \sqrt{(\Delta \mathrm{ch}_{S0}^A)^2 + (\Delta \mathrm{ch}_T^A)^2}$ (ch)	1.894173
		$\Delta t_T^A = \Delta \mathrm{Ch}_T^A \times t_c$ (ns)	0.002258
		$\Delta t_T^B = \sqrt{(\Delta \mathrm{Ch}_T \times \Delta t_c)^2 + (\Delta t_q/3)^2}$ (ns)	0.004427
		$\delta t_T = \sqrt{(\Delta t_T^A)^2 + (\Delta t_T^B)^2}$ (ns)	0.00497
Shock velocity	Al	$D_S = d_S / t_S$ (km/s)	26.337
		$1\sigma = \delta D_S = D_S \sqrt{(\delta d_S / d_S)^2 + (\delta t_S / t_S)^2}$ (km/s)	0.206
		$\eta = 2\delta D_S / D_S = 2\sigma / D_S$	2.005%
	Cu	$D_T = d_T / t_T$ (km/s)	20.723
		$1\sigma = \delta D_T = D_T \sqrt{(\delta d_T / d_T)^2 + (\delta t_T / t_T)^2}$ (km/s)	0.201
		$\eta = 2\delta D_T / D_T = 2\sigma / D_T$	1.940%

IV. THE HUGONIOT DATA OF COPPER UP TO ~2.24 TPA WITH AL-CU IMPEDANCE-MATCH TARGET

One double frequency beam (wavelength of 0.53 μm) from Shenguang-II laser facility with eight beams was employed in the experiment of Al-Cu impedance-match target. The maximum output energy and the trapezoid pulse width are 350 J and ~1 ns, respectively. The uniform focal spot of ~900 μm diam was formed by the beam smooth technique of the small lens-array.[6,19–21] Figure 3 shows the experimental setup.

The Al-Cu impedance-match target is shown in Fig. 1, S and T represent the reference material Al and the test material Cu, respectively. The Al and Cu steps foils have been overlaid on the Al substrate without any glue in the target manufacture, the coupling between step and substrate only depended on Van der Waals molecular force, and the whole target was sustained by the auxiliary clamp that was made of two pieces of the stainless steel with a center aperture of 2 mm diameter. The thicknesses of Al substrate, Al and Cu steps are ~20, ~15, and ~11 μm, respectively. It must be

TABLE IV. δu and δP of Cu reckoned with and without the covariance $\delta(c_0, \lambda)$ for the experiment No. 11.

Exp. No. 11	δu (km/s)	δP (GPa)
With $\delta(c_0, \lambda)$	0.216	39.788
Without $\delta(c_0, \lambda)$	0.227	41.720

indicated that the propagation of the shock wave in the above steps is steady; actually it has been confirmed by both experiment and theory.[6,21–24]

Using the optical imaging system with the magnification of 17 times and the spatial resolution of 1.5 μm, the shock luminescence from the rear surface of the Al-Cu impedance-match target was recorded by the streak camera with ~2 ns scan range and the time resolution Δt_q of ~12 ps and is shown in Fig. 2.

The 12 Hugoniot data (SILP data) of Cu have been obtained from the experiment, the shock velocity D, the particle velocity u, and the shock pressure P; also their uncertainties are listed in Table II. The initial densities and their uncertainties of Al and Cu are $2.707 \pm 0.005\ \mathrm{g/cm^3}$ and $8.934 \pm 0.011\ \mathrm{g/cm^3}$. Al standard parameters with $c_0 = 5.826$ km/s and $\lambda = 1.208$, and their uncertainties with $\delta c_0 = 0.094$ km/s and $\delta\lambda = 0.005$, that obtained from the data fitting for SESAME 3715,[3,25] have been employed in the calculation. The matrix method was applied to calculate the δu and δP.

The $P-u$ diagram of the test material Cu is showed in Fig. 4, in which the error bars of u and P correspond to 2σ ($2\delta u$ and $2\delta P$, respectively). Also, Cu data from SESAME 3332 (Ref. 25) and the AWE experimental results[3] have been given in Fig. 4.

The results in Table II show that the relative uncertainty of the shock velocity D is about ~2% for the confidence factor of $k=2$. Taking the experiment No. 11 in Table II as the example, Table III gives the detailed calculation, and tells how the uncertainty of the shock velocity D has been gotten.

In Table III, γ is 0.4056% (the step-surface profiler was verified by the standard step from VLSI Standards Inc., and the height of the standard step is certified by National Institute of Standards and Technology of U.S.); t_c is 1.192 ps/ch, its uncertainty Δt_c is 0.004 ps/ch, they are all obtained from the calibration. The time resolution Δt_q of the streak camera is 12 ps in the condition of ~2 ns scan range; it will be described in the Appendix in detail.

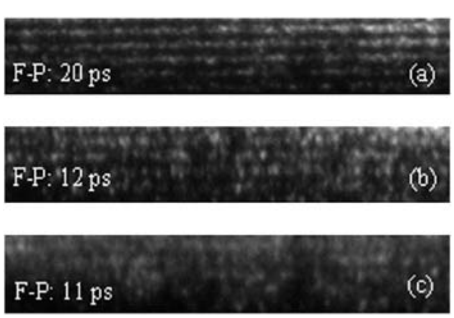

FIG. 5. Calibrating results for the time resolution of the streak camera.

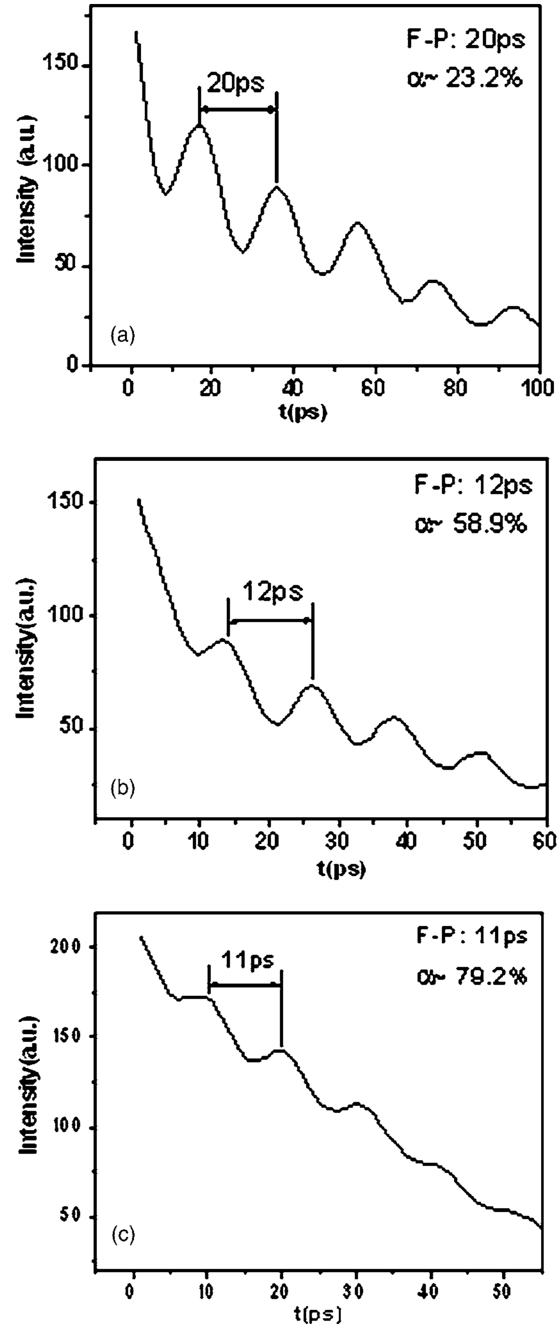

FIG. 6. Characteristics of intensity vs time in allusion to Fig. 5.

V. COMPENDIOUS DISCUSSION AND CONCLUSIONS

It is easy to understand that, the error brought by the target surface roughness has not only contributed to the uncertainty δd of the sample step thickness (d), but also to the uncertainty δt of the transit time (t) of the shock wave in the step. This accords with the munificent principia of the error calculation.

Although the approximate method of the mirror reflection will introduce some errors to the values of u and P, but it has hardly any influences to the uncertainties δu and δP of u and P. In the practice calculation, we may adopt the exact second shock method to calculate the values of u and P.

On the other hand, the covariance $\delta(c_0, \lambda)$ will affect the values of $\delta u$and δP to a certain extent. Taking the experiment No. 11 as the example, Table IV gives the uncertainties δu and δP of Cu which correspond to reckon with and without the covariance $\delta(c_0, \lambda)$, it is seen that the influence brought by $\delta(c_0, \lambda)$ is more notable. In fact, because there is a minus correlation between c and λ, the correlation coefficient $r(c_0, \lambda)$ is −0.8278 for Al data from SESAME 3715. Obviously, the uncertainties δu and δP have been reduced after considering the covariance $\delta(c_0, \lambda)$.

In conclusion, this article has placed emphasis on the uncertainty analysis to the experiment of the laser equation of state, a variety of uncertainties have been sorted, discussed and estimated, it has the important guidance to purposefully improve the precision of the experimental data. A matrix method that can expediently determine the uncertainties of u and P, has been brought forward and described in detail.

Also, the experimental results with Al-Cu impedance-match target have been reported, the relative uncertainty of the shock velocity almost keeps the level of ~2% (k=2). A series of Cu Hugoniot data with the maximal pressure of ~2.24 TPa have been given too.

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APPENDIX: TIME RESOLUTION OF THE STREAK CAMERA

The time resolution Δt_q of the streak camera can be calibrated by the Fabry–Pérot (F) etalon and the laser with the wavelength of 0.53 μm and pulse width of 1 ps. Rayleigh criterion is regarded as the judgment standard, namely two adjacent signals cannot be distinguished when the superposition intensity between them reaches 73.5% (α) of the peak intensity of the lower intensity one of them. In the calibrating experiment, the space distance of the FP etalon was decreased continually until two adjacent signals cannot be distinguished, the double pass duration of the FP etalon is just the time resolution Δt_q.

For the streak camera with 2 ns scan range and the slit width of 30 μm, the calibrating results are shown in Figs. 5(a)–5(c), which correspond to the different double pass duration of the FP etalon with 20, 12, and 11 ps. Figures 6(a)–6(c) show the characteristics of intensity versus time, α is 23.2%, 58.9%, and 79.2%, respectively. Obviously, the time resolution Δt_q is ~12 ps.

It will affect the calibrating result if the slit of the streak camera is too wide. But the calibrating results indicated that, Δt_q is almost invariable while the slit width is changed from 20 to 50 μm. In fact, it is no problem as long as the slit width keeps a fixed value between the calibration and the EOS measurement.

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