

Approximate formulas of temperature and stress distributions and thermal induced effects in a heat capacity slab laser

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Approximate formulas of transient temperature and stress distributions in the slab of a two-side pumped heat capacity laser are attained by solving the heat diffusion equation. Based on the formulas, the distributions of temperature and stress during the pump period of a two-sided symmetrically laser diode array (LDA) pumped Nd:GGG slab laser with 5-kW average output power are numerically simulated. The results show that the temperature in the slab will averagely increase by more than 70 °C after operating for 10 seconds; the stress and maximum of the temperature difference in the slab are about 30 MPa and 24 °C, respectively. The thermal induced effects are discussed also.

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Waste heat accumulated in the lasing medium is one of the main limits to output power of solid state lasers. Although the optical-to-optical efficiency has been improved with employing laser diode (LD) pumping instead of flash lamp pumping in the last decade, and the waste heat reduced greatly, the temperature gradients in the medium are still remarkable, because of simultaneous cooling at around the medium in the conventional steady operation mode. The temperature gradient brings thermal stress and beam distortion, which causes the beam quality degraded seriously. In order to solve this problem, Lawrence Livermore National Laboratory (LLNL) brought forward a new concept laser, named solid state heat capacity laser (SSHCL)^[1], in which the waste heat produced during the lasing periods is reserved in the medium without employing a simultaneous cooling, resulting in a low temperature gradient.

Research on SSHCL has been a hot subject in recent years^[2-7]. Especially in 2004, LLNL built a LD-pumped, 200-Hz repetition rate, Nd:GGG (neodymium doped gadolinium gallium garnet) heat capacity laser (HCL) with average output power of 30 kW, for a solid state laser (SSL) which was the highest output power of the day despite of only one second operation. SSHCLs in China have also been accomplished with kilowatts of output power^[8].

For some special applications, a SSHCL needs a period of 10-s operation and 30 s to 40 min process for cooling. Consequently, the thermal effect during the operation periods without cooling is of the key importance. In this paper, a model on a two-sided symmetrically laser diode array (LDA) pumped Nd:GGG slab laser is analyzed, and approximate formulas of the transient temperature and stress distributions in a slab are derived from the heat diffusion equation. Furthermore, as an example, the temperature and stress distributions in a slab laser

with 5-kW average output power are discussed under the condition of average irradiance of 1.160 MW/m² for each pump beam. The simulations and analyses will provide help for designing a SSHCL, especially for thermal management system in a SSHCL.

A two-sided symmetrically LDA-pumped slab SSHCL Nd:GGG laser is analyzed with a schematic structure, as shown in Fig. 1, where x and y axes are set along the slab length and width, and z -axis along its thickness; R1 and R2 are two cavity mirrors. There are 3 slabs of Nd:GGG crystal considered in this model, and each slab is two-sided symmetrically pumped by LDA.

Temperature distribution in the slab can be derived by solving a heat diffusion equation. For simplicity a uniform pumping in x - y plane and an adiabatic boundary condition at the slab surface are assumed, resulting in uniform temperature distribution, namely $\partial T/\partial X = \partial T/\partial Y = 0$. The thermal energy stored in the medium is considered as an internal heat source, which is uniform in x - y plane and non-uniform in z -direction. The heat source can be considered as a constant source during a square pump pulse. Under these assumptions the temperature in z -direction and its transient behavior are the investigated subjects by solving a heat diffusion equation written as

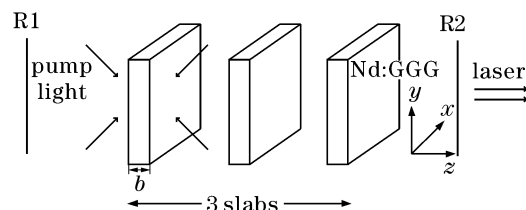


Fig. 1. Diagram of a two-sided symmetrically LDA-pumped slab laser.

$$\frac{\partial T(Z, t)}{\partial t} - \gamma \frac{\partial^2 T(Z, t)}{\partial Z^2} = \frac{\gamma}{k} Q, \quad (1)$$

where γ denotes the thermal diffusivity of Nd:GGG crystal (m^2/s), Q is the power density of the internal heat source (W/m^3), k denotes the thermal conductivity ($\text{W}/(\text{m}\cdot\text{K})$). Furthermore, for symmetrically two-sided LDA pumping, an exponential decayed absorbed energy $Q = Q(Z)$ in the slab thickness can be assumed as

$$Q(Z) = \eta\alpha I f(Z), \quad (2)$$

where I is irradiance (W/m^2) of each pump beam, η is the ratio of thermal irradiance (W/m^2) produced in the slabs to irradiance of the pump beam, α is absorption coefficient (m^{-1}), $f(Z) = \exp(-\alpha Z) + \exp[-\alpha(b - Z)]$, b is thickness of the slab (m).

The initial condition and boundary condition for the considered situation can be set as

$$\begin{cases} T(Z, 0) = T_0 \\ \frac{\partial T}{\partial Z}|_{Z=0,b} = 0 \end{cases}, \quad (3)$$

here the heat dissipation by surface radiation and convection is neglected, resulting in a null temperature gradient at the surface. Equation (1) can be solved by using Fourier series expansion with a series form. The second order is reserved, an approximate solution is obtained as

$$\begin{aligned} T(z, t) = & \frac{2\gamma\eta I [1 - \exp(-\alpha b)]}{bk} t \\ & + \frac{1}{\pi^2} \frac{\eta I b [1 - \exp(-\alpha b)]}{k} \frac{\alpha^2 b^2}{\alpha^2 b^2 + 4\pi^2} \\ & \times [1 - \exp(-\gamma \frac{4\pi^2}{b^2} t)] \cos(\frac{2\pi}{b} z) + T_0. \end{aligned} \quad (4)$$

Equation (4) can be used to estimate the temperature increase of the SSHCL. In actual application situations the laser is operated with a typical repetition rate of 200 Hz and duty ratio of 10%. The thermal relaxation time τ can be evaluated to be about 24 s by using the formula $\tau = C_p \rho b^2 / 4k$ ^[9], where C_p is the specific heat capacity ($\text{J}/(\text{kg}\cdot\text{K})$), ρ is density of the lasing medium (kg/m^3), and b is thickness of the slab which is assumed to be 0.015 m. The pulse interval time of the pumping beam is negligibly small compared to the thermal relaxation time. Therefore, the profile of thermal distribution in the slab can be described by Eq. (4) as continuous pumping with an averaged intensity in spite of the pulse operation, and the thermal induced effects on laser performance depend only on the average input power in the heat capacity operation mode^[10].

Simulations were carried out by using Eq. (4) and with parameters of a Nd:GGG slab: pump wavelength of 808 nm with $\alpha = 300 \text{ m}^{-1}$; $\gamma = 2.39 \times 10^{-6} \text{ m}^2/\text{s}$; $k = 6.43 \text{ W}/(\text{m}\cdot\text{K})$; $T_0 = 293 \text{ K}$; the $x \times y \times z$ dimension of $6 \times 6 \times 1.5 \text{ (cm)}$; $\eta = 13.0\%$; $I = 1.16 \text{ MW}/\text{m}^2$.

It was shown from the simulation that the temperature in the slab would rise continuously during working period, and the temperature was higher at the slab surface than that in its central part because of no cooling during the period. After 10 s lasing, one can compute

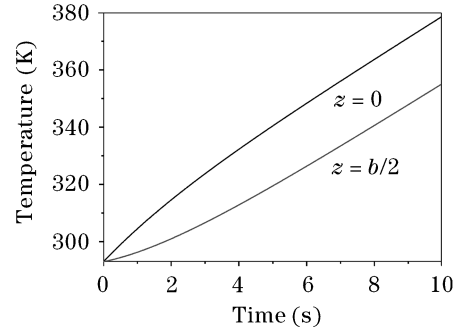


Fig. 2. Temperature versus time on surface ($z = 0$) and middle plane ($z = b/2$) of the slab.

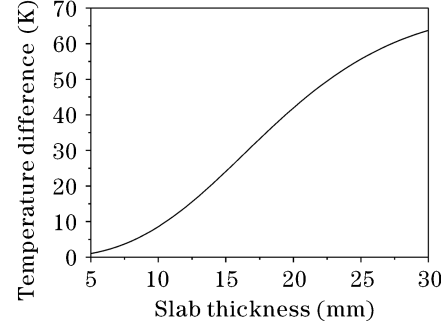


Fig. 3. Maximum of the temperature difference versus slab thickness after 10 s lasing.

that the temperature in the slab averagely increased 70°C and the maximum of temperature difference between the slab surface and centerline was about 24°C , as shown in Fig. 2. The maximum of temperature difference distribution in the slab with different slab thickness was shown in Fig. 3. The temperature difference was increased with thickness of the slab. So it could be better to enlarge the cross section of the slab rather than to thicken the slab for designing a high power SSHCL. Furthermore, for a time scale compared to or longer than the value of $b^2/\pi^2\gamma$, the temperature profile was almost independent of time.

Owing to the temperature increase, the slab expands thermally. According to the model discussed above, the temperature increase is non-uniform in z -direction, which produces non-uniform stress, that is, compressive stress near the surface and tensile stress in the middle plane of the slab. From Eq. (4) the stress distribution can be expressed as^[11]

$$\begin{aligned} \sigma = & \frac{\alpha' E}{(1-\nu)} (T_m - T) = -\frac{\alpha' E}{\pi^2(1-\nu)} \frac{\eta I b [1 - \exp(-\alpha b)]}{k} \\ & \times \frac{\alpha^2 b^2}{\alpha^2 b^2 + 4\pi^2} [1 - \exp(-\gamma \frac{4\pi^2}{b^2} t)] \cos(\frac{2\pi}{b} z), \end{aligned} \quad (5)$$

where σ represents the stress, α' is coefficient of thermal expansion, E is the elastic modulus, ν is Poisson ratio, and T_m is the average temperature of the slab. For GGG crystal, $E = 220 \times 10^9 \text{ Pa}$, $\alpha' = 80 \times 10^{-7}/\text{K}$, and $\nu = 0.3$ ^[3]. Simulations by using the same parameters mentioned above show that the maximum of tensile stress is 30 MPa at the middle plane of the slab, and the maximum of compressive stress at the surface is 30 MPa.

In practical, pumping is often non-uniform in x - y plane.

To take this situation, a Gaussian distribution of pump intensity I and relative difference $\Delta I/I_0 = 5\%$ as an example is considered in simulation. Although the approximate formula Eq. (4) is derived for uniform pumping, it is still useful to analyze the thermal characteristics in non-uniform pumping situation. Results calculated by using Eq. (4) and by inserting the intensity as a function of x - y were compared with the results of ANSYS simulation, showing that the error is not significant in smaller non-uniformity.

The thermal effects in the HCL slab come from several mechanisms. The first is the thermo-optic effect, that is, the index change with the temperature; the second is the elastic-optic effect caused by index change with the strain; and the third is the thickness change caused by thermal expansion. For most crystalline laser materials, the stress contribution to the refractive index is small, so we take only the thermo-optic effect and size distortion into consideration.

Taking the y -direction index variation as an example, the average index change can be described as

$$\begin{aligned} \Delta n(Y)_T &= [T(0) - T(Y)] \left(\frac{dn}{dT} \right) \\ &= \left[\frac{1}{b} \int_0^b \Delta T(0, Z) dZ - \frac{1}{b} \int_0^b \Delta T(Y, Z) dZ \right] \left(\frac{dn}{dT} \right), \quad (6) \end{aligned}$$

where $T(0)$ and $T(\pm \frac{L}{2})$ are the temperature at center and at the edge of the slab averaged over z -direction. $\frac{dn}{dT}$ is the thermo-optic coefficient, which is $1.75 \times 10^{-5} \text{ (K}^{-1}\text{)}$ for Nd:GGG crystal^[12]. Then the optical path difference ΔS can be estimated to be $\Delta n(Y)_T \times b$. Obviously Eq. (6) will give a value more than zero, therefore the thermal lens effect is positive.

Thermally induced distortion of the slab is caused by thermal expansion; the maximum thickness difference in y -direction can be calculated as

$$\Delta h = \alpha' b \left[T(0) - T\left(\frac{L}{2}\right) \right]. \quad (7)$$

The maximum distortions in the slab under various output power were calculated and summarized in Table 1. The relationship between $\Delta I/I_0$ and ΔS was shown in Fig. 4 in case of working period of 10 s.

Table 1. Maximum of the Distortions in the Slabs with $\Delta I/I_0 = 10\%$ in 10-s Lasing Time

P (kW)	I (MW/m ²)	$x \times y \times z$ (cm)	N	Δh (μm)
5	1.10	11 × 11 × 1	1	0.592
20	0.81	10 × 10 × 1.5	4	0.416
100	1.64	13 × 13 × 2	9	0.852
5	1.16	6 × 6 × 1.5	3	0.594

P : output power; I : average irradiance of each pump beam; $x \times y \times z$: physical dimension of a slab; N : number of slabs; Δh : maximum thickness difference in y -direction for single slab.

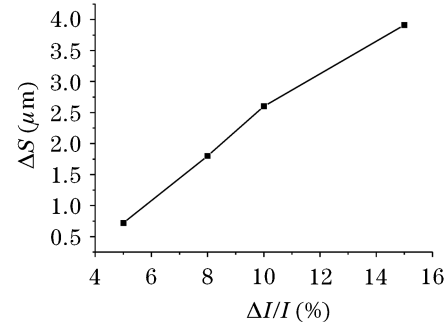


Fig. 4. Relationship between $\Delta I/I_0$ and ΔS with average output power of 5 kW.

In conclusion, temperature and stress distributions in a Nd:GGG slab HCL were analyzed by using a simplified model. Approximate formulas to describe the transient temperature and stress distributions in the slab were derived from the heat conduction equation. Thereby as an example considering a Nd:GGG slab with the dimension of $6 \times 6 \times 1.5$ (cm), it was calculated that at the end of working period of 10 s, temperature in the slab averagely increased 70°C , the maximum of temperature difference in the slab was about 24°C , the maximum of tensile stress was close to 30 MPa. We have also discussed the thermal induced effects, this would provide a guideline for designing a SSHCL with 10 kW or much higher output power and a cooling system in a SSHCL.

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