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Optik 116 (2005) 295–298
Optik Optics
www.elsevier.de/ijleo

Focal shifts in focused Hermite–cosh–Gaussian laser beams

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Received 1 September 2004; accepted 5 January 2005

Abstract

A closed-form propagation equation of Hermite–cosh–Gaussian beams passing through an unapertured thin lens is derived. Focal shifts are analyzed by means of two different methods according to the facts that the axial intensity of some focused Hermite–cosh–Gaussian beams are null and that of some others are not null but the principal maximum intensity may be located on the axis or off the axis. Optimal focusing for the beams is studied, and the condition of optimal focusing ensuring the smallest beam width is also given.

Keywords: Hermite–cosh–Gaussian beams; Focal shift; Optimal focusing

1. Introduction

Recently, a class of closed-form paraxial solutions of the Helmoltz equation is introduced by Casperson and Tovar [1], namely Hermite–sinusoidal–Gaussian (HShG) beams, which represent the more general beams such as Gaussian, Hermite–Gaussian, cosh and cosine beams, etc. Casperson and Tovar have also studied production and propagation of HShG beams [2]. Following them, several works have investigated some special cases of this family of the beams [3–5]. In these works, Belafhal and Ibnchaikh have studied the propagation properties of Hermite–cosh–Gaussian (HChG) beams [4], in which focusing of the beams by a thin lens is involved. However, they have only treated intensity distributions at the geometrical focal plane and shifts of focal plane are not considered. It is well known that when a beam is focused, the point of maximum intensity is not located at the geometrical focal plane but is somewhat closer to the lens, which is known as the focal shift [6]. This phenomenon has become a subject of interest [6–11] because of the requirement to determine the real focal plane accurately in practical applications.

This paper is devoted to studying focal shifts in focused HChG beams and finding the condition to achieve optimal focusing. In Section 2, a closed-form propagation equation of focused HChG beams is derived. By using different methods, focal shifts of focused TEM_{11}-mode and TEM_{22}-mode HChG beams are studied and the condition to achieve optimal focusing of focused TEM_{22}-mode HChG beams is treated in Section 3. An interesting phenomenon that the intensity on the propagation axis of focused TEM_{11}-mode HChG beams is null and that of focused TEM_{22}-mode HChG beams is not null is found, but its principal maximum intensity may be located on the axis or off the axis depending on the beam's parameter. This

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doi:10.1016/j.ijleo.2005.01.027

phenomenon raises a problem, namely, how to determine the real focal plane of the beam. Finally, the main results obtained in this paper are summarized in Section 4.

2. Closed-form propagation equation of focused HChG beams

Suppose that HChG beams are focused by an unapertured thin lens with focal length of $f (f > 0)$. We assume that the waist of the beam is located at the plane of the lens. The field distribution of HChG beams at the input plane $z = 0$ is characterized by [1]

$$E_0(x_0, y_0, 0) = A_0 H_m\left(\sqrt{2}\,\frac{x_0}{w_0}\right) H_n\left(\sqrt{2}\,\frac{y_0}{w_0}\right) \times \cosh(\Omega_0 x_0)\cosh(\Omega_0 y_0)\exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right), \quad (1)$$

where A_0, the amplitude of Gaussian beams at the central position of $x = y = z = 0$, is a constant, m and n are the mode indexes associated with the Hermite polynomial functions H_m and H_n, w_0 is the waist width of the Gaussian amplitude distribution and Ω_0 is the parameter associated with the cosh part.

According to the Huygens–Fresnel diffraction integral, the focused field at any position behind the lens is given by [12]

$$E(x, y, z) = -\frac{\mathrm{i}k}{2\pi z}\exp(\mathrm{i}kz)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_0(x_0, y_0, 0) \times \exp\left\{-\frac{\mathrm{i}k}{2z}\left[\left(1-\frac{z}{f}\right)(x_0^2 + y_0^2) - 2(xx_0 + yy_0) + (x^2 + y^2)\right]\right\}\mathrm{d}x_0\,\mathrm{d}y_0, \quad (2)$$

where $k = 2\pi/\lambda$ is the wave number, and λ is the wavelength. Substituting Eq. (1) into Eq. (2), we obtained the field distribution of the focused HChG beams as

$$\begin{aligned} E(\xi, \eta, z) = {} & \frac{A_0}{4}\frac{M}{P}\left(-\frac{Q}{P}\right)^{(m+n)/2} \times \exp\left[\frac{a^2 z/f}{2P} - \frac{2M(1-M)}{P}(\xi^2 + \eta^2)\right] \\ & \times \left[\exp\left(\frac{aM\xi}{P}\right) H_m\left(-\mathrm{i}\,\frac{2M\xi + az/f}{\sqrt{2PQ}}\right) + \exp\left(-\frac{aM\xi}{P}\right) H_m\left(-\mathrm{i}\,\frac{2M\xi - az/f}{\sqrt{2PQ}}\right)\right] \\ & \times \left[\exp\left(\frac{aM\eta}{P}\right) H_n\left(-\mathrm{i}\,\frac{2M\eta + az/f}{\sqrt{2PQ}}\right) + \exp\left(-\frac{aM\eta}{P}\right) H_n\left(-\mathrm{i}\,\frac{2M\eta - az/f}{\sqrt{2PQ}}\right)\right], \quad (3) \end{aligned}$$

where

$$M = \mathrm{i}\pi N_w, \quad (4)$$

$$P = z/f + (1 - z/f)M, \quad (5)$$

$$Q = z/f - (1 - z/f)M, \quad (6)$$

and $\xi = x/w_0$, $\eta = y/w_0$ are normalized coordinates, $N_w = w_0^2/\lambda f$ is the Fresnel number associated with the Gaussian beam and $a = \Omega_0 w_0$ is the decentered parameter. Then, intensity distribution at the focal region is expressed as

$$I(\xi, \eta, z) = \left|E(\xi, \eta, z)\right|^2. \quad (7)$$

3. Focal shifts in focused HChG beams

3.1. Focused TEM_{11}-mode HChG beam

Like some other focused beams [6–11], when the HChG beams are focused, focal shifts often occur. From Eq. (3), we know the on-axis intensity of focused TEM_{11}-mode HChG beam is null, which can also be seen from Fig. 5 in Ref. [4]. In order to treat the focal shifts in the beam, here we use the GH method, which determines z_{max} at which the smallest value of beam width w satisfies

$$\frac{\left(\int_{-w/w_0}^{w/w_0}\int_{-w/w_0}^{w/w_0} I(\xi, \eta, z_{max})\,\mathrm{d}\xi\,\mathrm{d}\eta\right)}{\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(\xi, \eta, z_{max})\,\mathrm{d}\xi\,\mathrm{d}\eta\right)} = 0.8. \quad (8)$$

From Eq. (8) the real focal plane $z = z_{max}$ can be obtained numerically, and the relative focal shift is given by

$$\Delta z_f = \frac{z_{max} - f}{f} \quad (9)$$

(for more details, see Ref. [7]).

The dependence of relative focal shift Δz_f in the focused TEM_{11}-mode HChG beam on the decentered parameter a and the Fresnel number N_w is given in Fig. 1, which shows that $|\Delta z_f|$ increases with decreasing a and N_w, and Δz_f tends to zero when N_w is large. In Fig. 1, the negative Δz_f means that the real focal plane is shifted toward the lens relative to the geometrical focal plane.

3.2. Focused TEM_{22}-mode HChG beam

The intensity distributions of focused TEM_{22}-mode HChG beam along z-axis are depicted in Fig. 2 ($\eta = 0$), from which we know the principal maximum intensity I_{maxp} of the beam may be located away from the axis (Fig. 2(a)) or on the axis (Fig. 2(b)), which depends on decentered parameters a. The locations of I_{maxp} can also

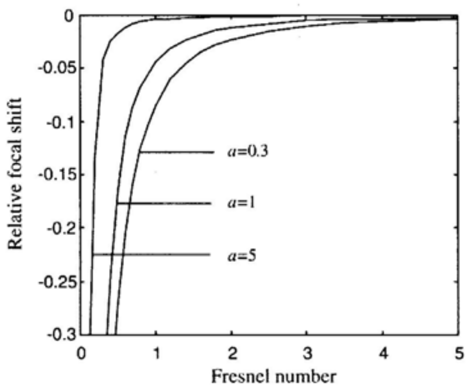

Fig. 1. Relative focal shift Δz_f of focused TEM_{11}-mode HChG beam versus Fresnel number N_w.

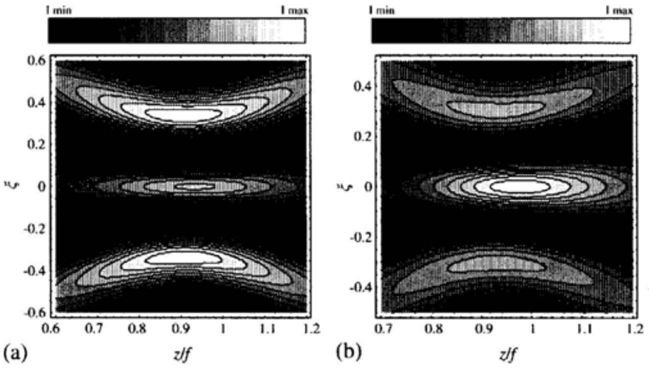

Fig. 2. Contour lines of the intensity of focused TEM_{22}-mode HChG beam along z-axis, $N_w = 1$. (a) $a = 0.3$; (b) $a = 0.8$.

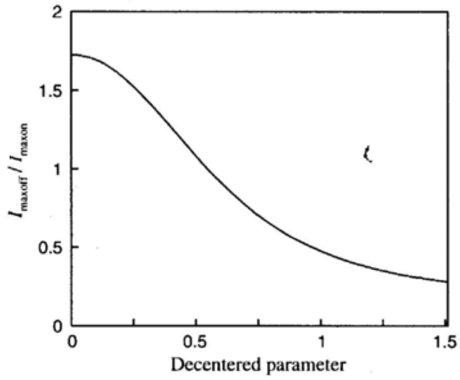

Fig. 3. The ratio of off-axis maximum intensity to that of on-axis versus decentered parameter a, $N_w = 1$.

be seen from Fig. 3, in which the ratio of the off-axis maximum intensity I_{maxoff} to the on-axis maximum intensity I_{maxon} is given with $N_w = 1$. In Fig. 3, the ratio is greater than unity means that I_{maxp} is located off the axis when $a<0.54$, and vice versa when $a>0.54$. When $a = 0.54$, the ratio is equal to unity and the beam has double I_{maxp} here. The ratio decreases with increasing a, which indicates that the larger a the greater the intensity concentrated on the axis.

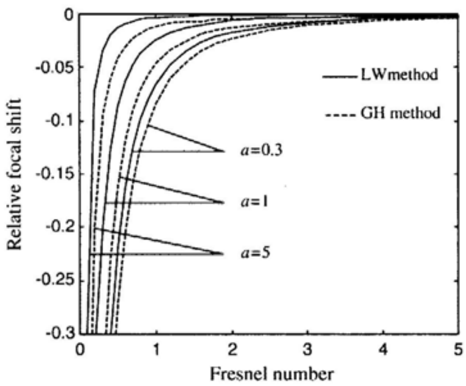

Fig. 4. Relative focal shift Δz_f of focused TEM_{22}-mode HChG beam versus Fresnel number N_w.

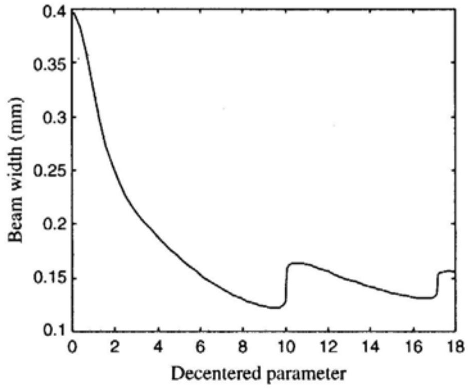

Fig. 5. Beam width w of focused TEM_{22}-mode HChG beam versus decentered parameter a, $N_w = 1$.

Because the on-axis intensity of focused TEM_{22}-mode HChG beam is not null, the traditional treatment of focal shift (LW method for short) [6] is applicable. According to LW method, the location $z_{\max}$ of on-axis maximum intensity is determined by $\mathrm{d}I(0,0,z)/\mathrm{d}z = 0$ and the relative focal shift is given by Eq. (9). Obviously, the GH method is also applicable in this case. The results obtained by using the two methods are shown in Fig. 4, from which we see the dependence of Δz_f on a and N_w is the same as that of focused TEM_{11}-mode HChG beam. However, there is quantitative difference between the results obtained from the two methods. For example, $|\Delta z_f|$ calculated by using of LW method is 0.099 and that of GH method is 0.126 when $a = 0.3$ and $N_w = 0.8$. From the intensity distributions in Fig. 2, there is an explanation for their difference that LW method considers only on-axis intensity but GH method considers most of intensity contains off-axis', and I_{maxoff} is closer to lens than I_{maxon}, which results in their difference. We also see the difference between them decreases when N_w increases in Fig. 4.

The dependence of beam width w of focused TEM_{22}-mode HChG beam on a is depicted in Fig. 5, where the calculation parameters are $w_0 = 1\,\mathrm{mm}$, $f = 1600\,\mathrm{mm}$, $\lambda = 632.8\,\mathrm{nm}$; it does not decrease monotonically but has a minimum beam width $w_{\min} = 0.122\,\mathrm{mm}$ when $a =$

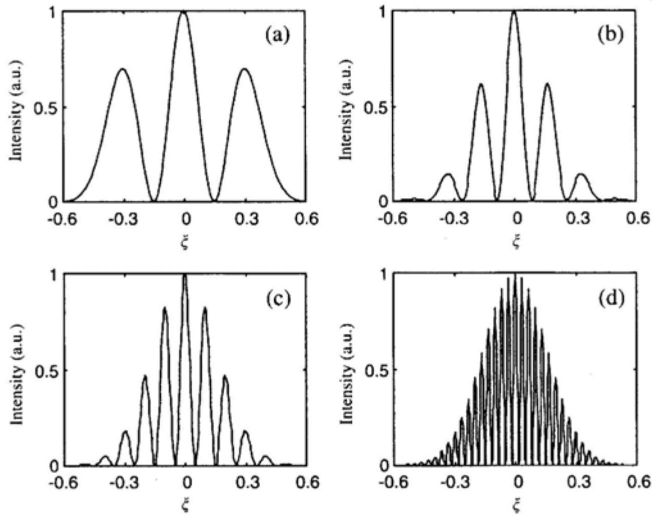

Fig. 6. Intensity distributions at the real focal plane (obtained by using GH method), $N_w = 1$. (a) $a = 1$; (b) $a = 5$; (c) $a = 9.5$; (d) $a = 30$.

9.5 and $N_w = 1$. The w oscillates with diminishing amplitudes when a increases. The oscillation of w can be explained from the intensity distributions at the real focal plane (obtained by using GH method) as shown in Fig. 6. The range of intensity distribution is large for relatively small a; when a increases, intensity concentrates on the central region, which results in the decreasing of w. However, at the same time, more and more secondary maxima emerge, and the associated power increases; when the ratio of the power contained within the integration limit in the numerator in the left-hand side of Eq. (8) to the total power is smaller than 80%, w will increase.

As shown in Fig. 6(d) ($a = 30$), the beam profile becomes the cosine curves with an oscillating frequency $2\pi N_w \Omega_0$ modulated by a Gaussian envelope for large value of a, which is similar to the propagation in free space [4].

4. Conclusions

In this paper, focal shifts in focused TEM_{11}-mode and TEM_{22}-mode HChG beams are studied. The numerical calculations demonstrate that the focal shifts depend on beams' mode index, decentered parameter and Fresnel number. When focal shifts in focused HChG beams are treated, method should be chosen appropriately according to intensity distribution of the beams. For focal shifts in focused TEM_{11}-mode HChG beam, only GH method is suitable because its on-axis intensity vanishes, but for focal shifts in focused TEM_{22}-mode HChG beam, both GH method and LW method are applicable. Differing from principal maximum intensity of beams that have been investigated in the previous works [6–11], which is located on or off axis all along, principal maximum intensity of focused TEM_{22}-mode HChG beam may be located on the axis or off the axis depending on decentered parameter. Therefore, compared with the results obtained in those works [6–11] where the focal shifts were considered, there are two different focal shifts of focused TEM_{22}-mode HChG beam by using the two methods. In this paper, the relative focal shift (absolute value) obtained by using of GH method is greater than that of LW method. Another interesting result in this paper is that the minimum beam width can be achieved when the decentered parameter takes some certain value, which results in the optimal focusing of the TEM_{22}-mode HChG beam. Since obtaining higher power density by minimizing the beam width is what we pursue in practical applications, it is worthwhile to obtain the condition of optimal focusing ensuring the smallest beam width.

Acknowledgments

This work was supported by the Foundations of the State Key Laboratory of Laser Technology and the National Hi-Tech Project of China.

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