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Focal shift effect of terahertz wide-aperture refractive lens

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Abstract: The focal shift effect of terahertz (THz) beam focusing when using a wide-aperture refractive lens has been investigated. The deviation of focus position caused by the focal shift effect can adversely affect the imaging or measurement quality of a THz system. In this study, reference values of relative focal shifts and combinations of different lens apertures, focal lengths, and working frequencies were analyzed and discussed through theoretical calculation and finite element analysis simulation. When using the commercial lens, the actual focus was determined based on the focal shift effect to ensure the working efficiency of a terahertz system. Concerning the customized lens design, the focal shift distance was compensated in the focal length optimization according to the working frequency. These two approaches guaranteed the good performance of a THz system.

Key words: terahertz lens, focal shift, finite element analysis, aspherics

太赫兹大孔径折射透镜的焦移特性研究

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摘要:研究了利用大孔径折射透镜对太赫兹波进行聚焦时产生的焦移效应。焦移效应所引起的焦点位置偏差会对太赫兹系统的成像或测量质量产生不利影响。通过理论计算和有限元分析仿真,研究并讨论了与透镜孔径、焦距和工作频率有关的焦移参考值。当使用商用透镜时,实际焦点位置需要通过焦移效应来确定,以保证太赫兹系统的工作效率。对于定制透镜的设计,焦移需要根据工作频率,在焦距的设计中进行补偿。这两个途径可以保障太赫兹系统的良好性能。

关 键 词:太赫兹透镜;焦移;有限元分析;非球面镜

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Introduction

The performance of a terahertz (THz) imaging or spectroscopy system is closely related to the quality of the THz beam[1-2]. Due to the diffractive divergence caused by the sub-millimeter scale wavelength, THz wave usually needs reshaping whether it is generated from electrical equipment or an optical system[3-4]. THz wide-aperture refractive lens (WARL) is a commonly used THz beam-shaping device with the advantages of broadband applicability, simple structure, high stability, and low cost. Sub-wavelength scale focusing spot and high spatial resolution THz image can be obtained using THz WARL with a high diameter to focal length ratio[5]. Most THz systems require an accurate focus position rather than a small spot because the focus is where test samples are usually

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placed and determines where to image or measure. One of the factors influencing the accuracy of the focus position is spherical aberration. In our previous work, the necessity of eliminating spherical aberration in the THz WARL design was discussed[6]. Another crucial and frequently overlooked factor is the focal shift effect, a phenomenon first known in Gauss optics. When a Gaussian beam is focused by a lens, with the beam waist in the plane of the lens and much smaller than the lens aperture, the maximum intensity is located at the waist of the focused beam that is closer to the lens rather than the geometric focus. The focal shift effect can be generalized to uniform pupil illumination in aberration-free systems as the incident beam waist becomes much larger than the lens diameter[7-8]. The focal shift effect can be explained by the diffraction theory and is related to the Fresnel number of a THz beam shaping device. For the same THz WARL, the lower the working frequency, the more severe the adverse effect of focal shift on a THz system[9]. Therefore, the focal shift effect should be fully considered and compensated with the design and use of THz WARLs. In this paper, the focal shift effect of the THz beam focused by THz WARLs was theoretically studied. Finally, the design and use of THz WARLs under different research or application requirements were proposed after the analysis of different apertures, focal lengths, and working wavelengths.

1 Theoretical calculation of focal shift effect

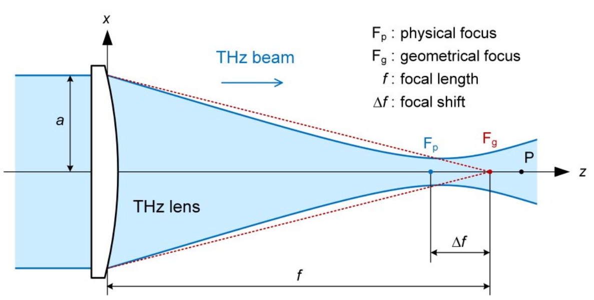

Fig. 1 The schematic of THz beam focusing and focal shift effect
图1 太赫兹波束聚焦及焦移效应示意图

The focal shift theory was first derived from the Huygens-Fresnel principle by Li and Wolf[8]. Fig. 1 illustrates the schematic of THz beam focusing and focal shift effect. The field at any point P along the optical axis that is far away from the diffraction aperture[10] is expressed as:

$$U(\mathrm{P}) = -\frac{ik}{2\pi}\frac{A\exp(-ikf)}{f}\iint_S \frac{\exp(iks)}{s}\mathrm{d}S \quad . \qquad (1)$$

Eq. (1) can be simplified as:

$$U[\mathrm{P}(z)] = A\frac{\exp(ikz)}{z}\left[\exp\left(-\frac{1}{2}ik\frac{z}{f}\frac{a^2}{f+z}\right)-1\right], \qquad (2)$$

where a and f denote the semi-aperture and focal length of THz WARL, respectively. The intensity of point P is:

$$I(\mathrm{P}) = U(\mathrm{P})U^*(\mathrm{P}) = I_0\left(1-\frac{u_N}{2\pi N}\right)^2\left[\frac{\sin(u_N/4)}{u_N/4}\right]^2, \qquad (3a)$$

$$u_N = \frac{ka^2z}{z(f+z)} = 2\pi N\frac{z}{f+z} \quad , \qquad (3b)$$

where $I_0 = \left(\pi a^2|A|/\lambda f^2\right)^2$ indicates the intensity of geometrical focus, and $N = a^2/\lambda f$ represents the Fresnel number.

Fig. 2(a) exhibits the intensity distributions along the optical axis given by Eq. (3). The $z = 0$ on the horizontal axis is the position of geometrical focus, and the intensity is set to unity. As N decreases, the peak intensity increases, and the focus position moves towards the negative direction of the optical axis where the lens is located. Fig. 2(b) demonstrates the relative focal shift ($\Delta f/f$) and relative peak intensity (I/I_0) with N. As N increases, $\Delta f/f$ increases to zero, and I/I_0 approaches unity. $\Delta f/f$ is -1%, -10%, and -40% when $N = 11$, $N = 3$, and $N = 1$, respectively. The values of the focal shift are all negative, reflecting that the focus always moves toward the lens.

In the simplification of Eq. (2), the conditions of $a \gg \lambda$ and $(a/f)^2 \ll 1$ are used to neglect the high-order terms of the expansion. Regarding a THz WARL, the above two inequations cannot be satisfied in many cases. As the focal length decreases, the center thickness of the lens increases, and the thin lens approximation no longer holds. All of these factors may induce deviations between practical and theoretical focal shift values.

2 Simulation of focal shift effect of THz WARL

Electromagnetic simulation based on finite element analysis (FEA) was performed to analyze the intensity distribution of THz beam focusing, so as to explore the focal shift effect of THz WARLs. Considering broadband THz research and applications, the focusing characteristics of the same THz WARL at different working frequencies were first studied. The incident wave was chosen to be a Gaussian plane wave. The boundary was set to be an absorbing condition to simulate the actual infinite space. The specific methods for THz WARL design and optimization were analyzed in our previous work[6]. The refractive index of THz WARL material was set to 1.41, and the absorption factor was set to 0 since the weak absorption characteristic of polymer material was below 1 THz. The aperture and focal length of the selected THz WARL were both 20 mm, and the working frequencies were 0.1-0.6 THz with an interval of 0.1 THz. The collimated THz beam passed through the plano surface and then was refracted and focused by the aspherical convex surface, as illustrated in Fig. 1. Theoretically, the plano-convex surfaces can eliminate spherical aberration. Figures 3 (a) to 3(c) present the amplitude distributions obtained by FEA simulations at working frequencies of 0.1, 0.3, and 0.5 THz, respectively. In Fig. 3(a), the white sol-

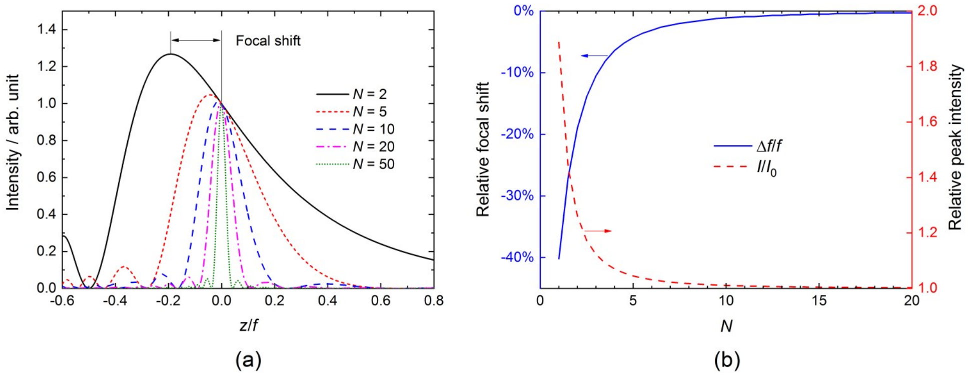

Fig. 2　Theoretical calculations of focal shift effect (a) intensity distributions along the optical axis at different Fresnel numbers, (b) relative focal shift and peak intensity with Fresnel number
图2　焦移效应的理论计算(a)不同菲涅尔数下沿光轴方向的强度分布曲线,(b)相对焦移和峰值强度与菲涅尔数的关系曲线

id line and dashed line indicate the positions of the physical focal plane and geometrical focal plane, respectively. Fig. 3(d) depicts the intensity distributions along the optical axis. The peak of each curve represents the focus and is marked by a solid circle. Fig. 3(e) exhibits $\Delta f/f$ and I/I_0 with the working frequency and N. $\Delta f/f$ is -11.1% at 0.2 THz and -1.4% at 0.5 THz. These figures verify that the focal shift effect is weaker at higher working frequencies. The differences between the simulation and calculation results in Fig. 3(e) are reasonable since the fixed lens is thick and does not meet $\left(a/f\right)^2 \ll 1$ and partly meets $a \gg \lambda$.

Similarly, FEA simulations were conducted using different apertures ($2a$), focal lengths (f), and working frequencies (ν), with N values ranging from 2 to 32. The results are rendered in Fig. 4, which are consistent with the theoretical calculation. The error was derived from two aspects: the neglect of high-order terms in the derivation of the theoretical equation; the invalidation of thin lens approximation. Nevertheless, the theoretical curve can still provide a good reference for the design and use of a THz WARL.

3　Minimize focal shift effect in design and use

According to the FEA simulations, the focus posi-

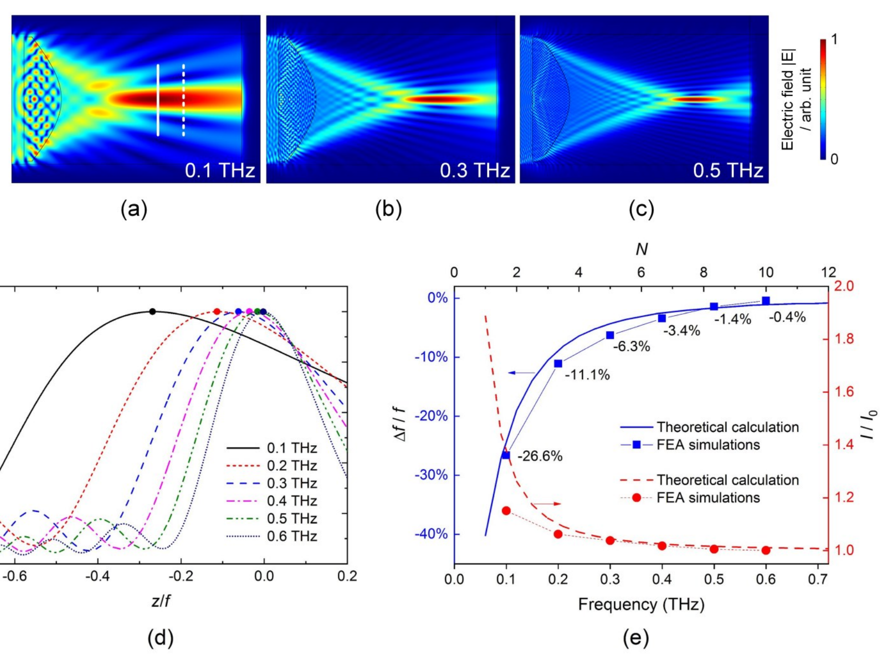

Fig. 3　Focal shift analysis of the THz WARL with a 20 mm aperture and focal length (a) to (c) FEA simulations at 0.1, 0.3, and 0.5 THz, respectively, (d) intensity distributions along the optical axis, (e) relative focal shift and peak intensity with working frequency and Fresnel number
图3　孔径和焦距均为20 mm的太赫兹WARL的焦移特性分析 (a)至(c)0.1、0.3和0.5 THz下的FEA仿真结果,(d)沿光轴方向的强度分布曲线,(e)相对焦移和峰值强度与工作频率和菲涅尔数的关系

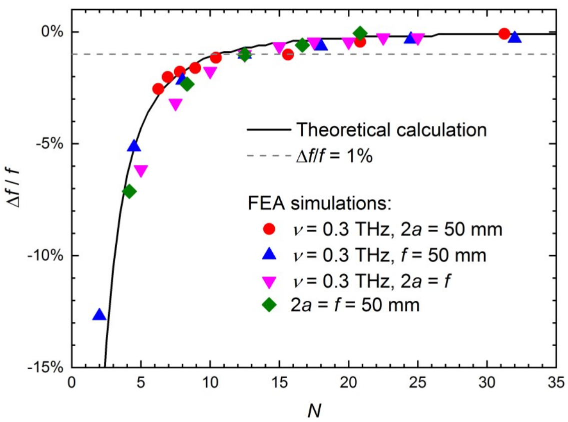

Fig. 4 FEA simulation results obtained using different lens apertures, focal lengths, and working frequencies
图4 不同透镜孔径、焦距和工作频率下的FEA仿真结果

tion of a THz WARL is closer to the theoretical position when N is large. With respect to a customized THz WARL, the aperture and focal length can be freely designed following the working frequency and desired N value. Although N is relatively small, the focal shift can be compensated by optimizing the surface parameters. For example, the THz WARL with a 20 mm aperture and the focal length was discussed in section 3. When the working frequency is 0. 1 THz, the THz WARL has an N of 1. 67 and $\Delta f/f$ of -23. 4%, and the physical focal length is calculated to be 15. 32 mm. If the physical focal length is set to 20 mm, the geometrical focal length is calculated to be 28. 75 mm. When the working frequency is 0. 1 THz, the physical focal length is 14. 7 mm, and the focal shift value is 5. 3 mm ($\Delta f/f$= -26. 5%), as demonstrated in Fig. 5(a). Fig. 5(b) renders the simulation with focal shift compensation. The central thickness of the THz WARL remained the same, and the curvature of the convex surface was modified based on the theoretical calculation in Fig. 2(b). The optimized physical focal length is 19. 0 mm, and the focal shift value is 1. 0 mm ($\Delta f/f$= -5. 0%), which is an improvement of more than 20%.

However, the choice of aperture and focal length is relatively limited for commercial THz WARLs. The commercial THz WARLs with plano-convex surfaces of three companies were investigated. The parameters and analyses of some commercial lenses are listed in Table 1. The focal length is greater than the aperture in most cases. Particularly, the real lens aperture should be smaller than the lens diameter since the THz WARL needs to be clamped when in use. Additionally, 90% of the diameter is taken as the aperture (2a) in the last column of Table 1.

Fig. 6 presents the focal shift ($\Delta f/f$) in relation to lens parameters (a^2/f) and working frequency. The area corresponding to the situation where commercial THz WARLs could be used is marked with a light-yellow background. With the curve of $\Delta f/f$ = 1% as an example, any combination of the a^2/f and working frequency above this curve satisfies $\Delta f/f < 1\%$. Considering that the convex surface of most lenses in Table 1 is spherical, the actual shift of focus position is larger due to spherical aberration. The spherical aberration causes the focal plane to form a diffuse spot, which is also approximated by a shift in the focus position. Thus, the actual focus position should be determined according to the focal shift distance and spherical aberration when using commercial THz WARLs to weaken the adverse effect of focal shift, such as reducing the accuracy of imaging or measurement. Regarding customized THz WARLs, the focal

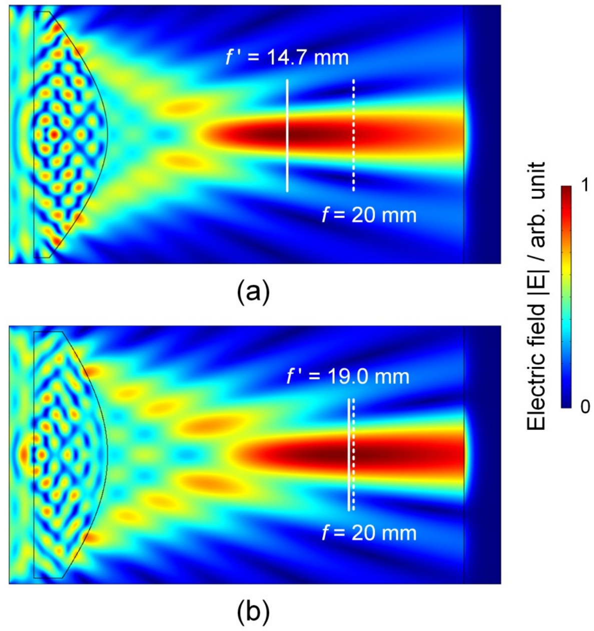

Fig. 5 FEA simulations at 0. 1 THz of the THz WARL with a 20 mm aperture and focal length, (a) without and (b) with focal shift compensation
图5 孔径和焦距均为20 mm的太赫兹WARL在0. 1 THz频率下的FEA仿真,(a)有焦移补偿,(b)无焦移补偿

Table 1 Commercial THz WARLs with plano-convex surfaces
表1 具有平凸表面的商用太赫兹WARL

Brand	Surface type	Diameter (mm)	Focal length (mm)	a^2/f (mm)
Batop	Aspherical	25. 4	10–67	1. 9–13. 0
		50. 8	35–100	5. 2–14. 9
	Spherical	25. 4	100–150	0. 9–1. 3
		50. 8	200–250	2. 1–2. 6
Thorlabs	Spherical	50. 8	75–500	1. 0–7. 0
		76. 2	115–150	7. 8–10. 2
		101. 6	151. 5–200	10. 5–13. 8
Tydex	Spherical	25. 4	50–250	0. 5–2. 6
		50. 8	75–2000	0. 3–7. 0
		76. 2	100–450	2. 6–11. 8
		101. 6	200–5 000	0. 4–10. 5

length should be increased following the working frequency in the design to compensate for the focal shift effect, if a specific aperture and focus position are required.

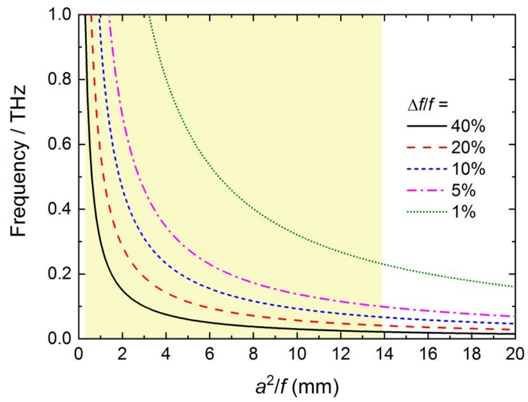

Fig. 6 Focal shift ($\Delta f/f$) in relation to lens parameters (a^2/f) and working frequency, the area corresponding to the situation where commercial THz WARLs could be used is marked with a light-yellow background
图6 焦移(Δf/f)与透镜参数(a2/f)和工作频率的关系,浅黄色背景标记了使用商用太赫兹WARL时涉及的参数所对应的区域

4 Conclusions

In conclusion, the focal shift effect of THz beam focusing when using THz WARLs was studied. As revealed by the calculation of the theoretical expression, the actual focus position always shifted towards the lens direction. The relative focal shift is inversely proportional to the Fresnel number of a THz WARL. FEA simulations of different lens apertures, focal lengths, and working frequencies were conducted. The focal shift values were consistent with the theoretical calculation. The inaccurate focus position will impact the working efficiency of the components of a THz system, leading to curtailed imaging or measurement quality. Hence, the focal shift effect should be minimized in the practical use of THz WARLs. Concerning commercial THz WARLs, especially the configurations of focal length greater than the aperture, the actual focus position should be determined under the focal shift effect. For customized THz WARLs, if there are specific requirements for the lens size and working frequency, the focal shift distance can be offset by increasing the theoretical focal length in the design. These two approaches can guarantee the good performance of a THz system, such as the high spatial resolution of a THz image and large dynamic range of the THz spectrum.

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