基于正交双向迭代的自由曲面激光光束整形设计 (特邀)

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摘　要：激光光束整形技术在现代激光工业领域具有重要作用，为了克服传统激光光束整形自由曲面构造中存在法向矢量偏差的局限性，文中提出一种正交双向迭代的自由曲面设计方法。该方法基于能量守恒和斯涅尔定律建立光源与目标面的能量映射关系，通过正交方向坐标点迭代获得自由曲面数据点，同时利用相邻坐标点的均值进行曲面法向矢量修正，从而提高自由曲面的构造精度。设计实例表明，准直入射的激光光束经自由曲面整形反射镜，在特定目标区域内可形成高均匀度光斑。进一步从自由曲面的连续性、拟合精度与面形误差三个方面，与传统设计方法进行了详细对比分析。结果表明，所提设计方法得到的激光光束整形自由曲面连续性与平滑度更好，拟合精度更高；且在加工误差允许范围内，目标面的辐照度均匀性更稳定。

关键词：激光光束整形；　自由曲面设计；　正交双向迭代；　法矢修正
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0　引　言

在激光加工、激光显示与照明等多个领域[1]，激光光束整形技术具有非常广泛的应用，并对光束形状、光斑均匀度与波前质量等方面都有不同的要求。例如，激光投影系统需要均匀性较高的照明光斑[2]，激光散斑检测需要能量均匀分布的激光光束以及获得不同形状的激光照明光斑[3] 等。目前典型的激光光束整形方式主要有衍射光学元件[4]、非球面透镜组[5]、微透镜阵列[6]、液晶空间光调制器[7] 以及自由曲面光学元件[8−10] 等。相比于其他光束整形技术，自由曲面技术具有更高的优化自由度[11−12]，能够获得结构紧凑、轻量化与高性能的激光光束整形系统[13−14]。

自由曲面光束整形设计方法主要有偏微分方程法[15−16]、支撑二次曲面法[17−18] 和光线映射法[19−24]。光线映射法相对简单，设计效率较高，可实现任意照度的分布。Wang 等人[19] 提出了一种变量可分离映射关系的自由曲面设计方法，在目标区域获得了“E”字形照明光斑。Feng 等人[20] 设计了输出光束波前可控并能实现特定均匀照度分布的自由曲面。人为划分法所确定的映射关系往往难以满足可积条件，使得自由曲面的连续性降低。进一步 Feng 等人[21] 对光源与目标面的能量映射关系进行了改进与优化，利用自适应网格法获取更为精准的初始能量映射关系，使得设计的自由曲面不仅能满足可积条件，而且能达到预定的目标辐照度要求。Bösel 等人[22] 通过含边界条件的线性平流方程构建光束整形自由曲面，进而改善激光光束的整形效果。Wei 等人[23] 运用最小二乘法求解蒙日安培方程，所获得的自由曲面能够同时控制辐照度分布和波前面形。此外，Pan 等人[24] 利用种子曲线扩展法进行整形自由曲面设计，获得了均匀辐照度分布光束与可控面形波前。在上述自由曲面迭代构造中仅考虑了单一方向的法向矢量，这使得曲面上各点法矢存在一定的偏差，导致自由曲面的构造精度降低，从而影响激光光束的整形效果。

文中利用正交方向相邻数据点的均值进行曲面法向矢量修正，逐点动态迭代构造自由曲面，获得了高性能激光光束整形自由曲面。准直激光光束经过

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自由曲面反射镜，在目标接收面上形成了高均匀度方形光斑。进一步，对自由曲面的连续性、拟合精度和面形误差等方面进行了深入对比分析，相比于现有单一方向自由曲面设计法，基于曲面法矢修正的正交双向迭代构造法所获得的激光光束整形自由曲面平滑度更好，拟合精度更高，目标面辐照度均匀性更稳定。

1 自由曲面激光光束整形原理

1.1 光源与目标面的映射关系

在激光光束整形技术中，光源与目标面的映射关系非常重要，文中采用可分离变量积分法获得光源与目标面的能量映射关系。入射准直激光光束的能量为：

$$\Phi_t = \int I_0 \exp\left(-2r^2 / w_0^2\right) \mathrm{d}r \tag{1}$$

式中：r 为到光轴的距离；w_0为束腰半径。入射激光在 X 与 Y 两个正交方向的强度分布均为高斯分布，因此在计算激光光束总能量时可进行变量分离，得到准直激光光束的总能量为：

$$\Phi_a = \int I(x)\mathrm{d}x \int I(y)\mathrm{d}y \tag{2}$$

将激光光束的束腰横截面 S 按等能量划分为 $N \times M$ 个矩形网格。激光光束沿 X 轴和 Y 轴的单位能量分别为：

$$\int_{x_{s,i}}^{x_{s,i+1}} I(x_s)\mathrm{d}x_s \int_{y_{s,1}}^{y_{s,M}} I(y_s)\mathrm{d}y_s = \frac{\Phi_a}{N} \tag{3}$$

$$\int_{x_{s,1}}^{x_{s,N}} I(x_s)\mathrm{d}x_s \int_{y_{s,j}}^{y_{s,j+1}} I(y_s)\mathrm{d}y_s = \frac{\Phi_a}{M} \tag{4}$$

联立公式 (2)、(3) 可得到光源所有能量网格点的横坐标$x_{s,i}$，类似地，联立公式 (2)、(4) 可求得网格点的纵坐标$y_{s,j}$。

设定目标接收面 T 为正方形平面，边长为 a，按等面积划分为 $N \times M$ 份，则目标接收面上的网格点横坐标为：

$$x'_{s,i} = \frac{a}{N}(i-1) \tag{5}$$

同样可求得目标面上网格点的纵坐标。已知入射光束横截面与目标面上每个网格点的坐标值，则入射光单位矢量$\boldsymbol{In}_{i,j}$和出射光单位矢量$\boldsymbol{Out}_{i,j}$可表示为$\boldsymbol{S}_{i,j}(x_{s,i}, y_{s,j}, z_{s,ij})$和$\boldsymbol{T}_{i,j}(x'_{s,i}, y'_{s,j}, z'_{s,ij})$，进而获得光源与目标面对应的能量映射关系。

1.2 现有自由曲面构造法及其法矢偏差分析

已知光源与目标面的能量映射关系，结合斯涅尔定律的矢量形式，逐点迭代计算自由曲面的离散数据采样点。现有单一方向的自由曲面设计法[24] 及其法矢偏差示意图如图 1 所示。自由曲面的初始点为$\boldsymbol{P}_{1,1}$，准直激光光束初始点$\boldsymbol{S}_{1,1}$的光线经初始点$\boldsymbol{P}_{1,1}$反射后入射到目标面$\boldsymbol{T}_{1,1}$点，出射光线为$\boldsymbol{P}_{1,1}\boldsymbol{T}_{1,1}$。由斯涅尔定律可得$\boldsymbol{P}_{1,1}$点上的法向矢量为：

$$\boldsymbol{N}_{1,1} = \frac{n_2 \cdot \boldsymbol{Out}_{1,1} - n_1 \cdot \boldsymbol{In}_{1,1}}{\sqrt{n_2^2 + n_1^2 - 2n_1 n_2(\boldsymbol{Out}_{1,1} \cdot \boldsymbol{In}_{1,1})}} \tag{6}$$

式中：n_1, n_2 为折射率；$\boldsymbol{In}_{1,1}$为过初始点$\boldsymbol{S}_{1,1}$的入射光线；$\boldsymbol{Out}_{1,1}$为过自由曲面初始点$\boldsymbol{P}_{1,1}$的出射光线。由$\boldsymbol{P}_{1,1}$点上的法向矢量$\boldsymbol{N}_{1,1}$可求出过$\boldsymbol{P}_{1,1}$点的切平面方程。点$\boldsymbol{S}_{2,1}$的出射光线与过$\boldsymbol{P}_{1,1}$点的切平面相交于点$\boldsymbol{P}_{2,1}$，则有：

$$\begin{cases} (\boldsymbol{P}_{2,1} - \boldsymbol{P}_{1,1}) \cdot \boldsymbol{N}_{1,1} = 0 \\ \dfrac{\boldsymbol{P}_{2,1} - \boldsymbol{S}_{2,1}}{\left|\boldsymbol{P}_{2,1} - \boldsymbol{S}_{2,1}\right|} = \boldsymbol{In}_{2,1} \end{cases} \Rightarrow \boldsymbol{P}_{2,1} = \frac{\boldsymbol{P}_{1,1} \cdot \boldsymbol{N}_{1,1}}{\boldsymbol{In}_{2,1} \cdot \boldsymbol{N}_{1,1}} \boldsymbol{In}_{2,1} \tag{7}$$

同样地，根据斯涅尔定律求出数据采样点$\boldsymbol{P}_{2,1}$的法向矢量$\boldsymbol{N}_{2,1}$及点$\boldsymbol{P}_{2,1}$处的切平面方程，光线$\boldsymbol{S}_{3,1}\boldsymbol{P}_{3,1}$与$\boldsymbol{P}_{2,1}$点处的切平面相交于点$\boldsymbol{P}_{3,1}$。以此类推可得纵向第一列采样光线与自由曲面的交点$\boldsymbol{P}_{i+1,1}$为：

$$\boldsymbol{P}_{i+1,1} = \frac{\boldsymbol{P}_{i,1} \cdot \boldsymbol{N}_{i,1}}{\boldsymbol{In}_{i+1,1} \cdot \boldsymbol{N}_{i,1}} \boldsymbol{In}_{i+1,1} \tag{8}$$

进一步利用采样光线$\boldsymbol{S}_{1,2}\boldsymbol{P}_{1,2}$与$\boldsymbol{P}_{1,1}$点处切平面相

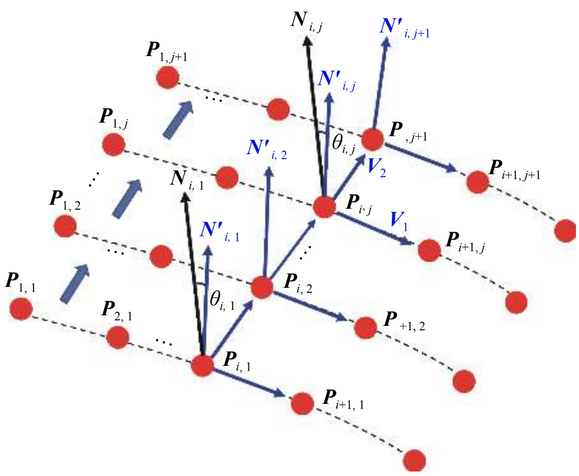

图 1 单一方向的自由曲面构造法及其法矢偏差示意图

Fig.1 Diagram of freeform surface construction in one direction together with its normal error

交得到交点$\boldsymbol{P}_{1,2}$，利用采样光线$\boldsymbol{S}_{2,2}\boldsymbol{P}_{2,2}$与过点$\boldsymbol{P}_{2,1}$处切平面的相交得到交点$\boldsymbol{P}_{2,2}$，同理可求得第二列的采样数据点$\boldsymbol{P}_{i+1,2}$。重复上述过程，可得自由曲面所有数据采样点的递推表达式为：

$$\begin{cases}(\boldsymbol{P}_{i,j+1}-\boldsymbol{P}_{i,j})\cdot \boldsymbol{N}_{i,j}=0\\ \dfrac{\boldsymbol{P}_{i,j+1}-\boldsymbol{S}_{i,j+1}}{\left|\boldsymbol{P}_{i,j+1}-\boldsymbol{S}_{i,j+1}\right|}=\boldsymbol{In}_{i,j+1}\end{cases}\Rightarrow \boldsymbol{P}_{i,j+1}=\frac{\boldsymbol{P}_{i,j}\cdot \boldsymbol{N}_{i,j}}{\boldsymbol{In}_{i,j+1}\cdot \boldsymbol{N}_{i,j}}\boldsymbol{In}_{i,j+1} \tag{9}$$

在自由曲面数据采样点的迭代过程中，实际数据采样点的法向矢量$\boldsymbol{N}'_{i,j}$与理论计算的法向矢量$\boldsymbol{N}_{i,j}$存在一定的偏差。当$\boldsymbol{P}_{i+1,j}$点利用光线过$\boldsymbol{P}_{i,j}$点的切平面交点进行计算时，$\boldsymbol{N}_{i,j}$和$\boldsymbol{N}'_{i,j}$均垂直于$\boldsymbol{v}_2$，但$\boldsymbol{P}_{i,j}$独立于同一曲线$\boldsymbol{P}_{i,j+1}$点，因此，相邻曲线切向量$\boldsymbol{v}_1$会引起$\boldsymbol{N}_{i,j}$和$\boldsymbol{N}'_{i,j}$之间的偏差。实际的法向矢量$\boldsymbol{N}'_{i,j}$可表示为$\boldsymbol{N}'_{i,j}=\boldsymbol{v}_1\times\boldsymbol{v}_2/|\boldsymbol{v}_1\times\boldsymbol{v}_2|$，其中 $\boldsymbol{v}_1=\boldsymbol{P}_{i,j+1}-\boldsymbol{P}_{i,j}$和$\boldsymbol{v}_2=\boldsymbol{P}_{i+1,j}-\boldsymbol{P}_{i,j}$，则$\boldsymbol{N}_{i,j}$与$\boldsymbol{N}'_{i,j}$的夹角偏差$\theta_{i,j}$可表示为：

$$\theta_{i,j}=\arccos\left(\frac{\boldsymbol{N}_{i,j}\cdot \boldsymbol{N}'_{i,j}}{\left|\boldsymbol{N}_{i,j}\right|\left|\boldsymbol{N}'_{i,j}\right|}\right) \tag{10}$$

1.3 基于曲面法矢修正的正交双向迭代自由曲面构造法

为了克服单一方向自由曲面构造法中存在的法矢偏差局限性，文中提出了基于曲面法矢修正的正交双向迭代自由曲面设计方法，如图 2 所示。首先确定起始点$\boldsymbol{P}_{1,1}$，获取纵向第一列$\boldsymbol{P}_{i+1,1}$和横向第一行$\boldsymbol{P}_{1,j+1}$的数据采样点，以此作为自由曲面构造的起始母线。剩余数据采样点采用正交方向迭代法进行扩展，即点$\boldsymbol{P}_{i,j+1}$坐标点由$\boldsymbol{P}_{i,j}$点和$\boldsymbol{P}_{i-1,j+1}$点共同决定。由斯涅尔定律求出$\boldsymbol{P}_{i,j}$点的法向矢量$\boldsymbol{N}_{i,j}$，光线$\boldsymbol{S}_{i,j+1}\boldsymbol{P}'_{i,j+1}$与过$\boldsymbol{P}_{i,j}$点切平面相交，交点即为$\boldsymbol{P}'_{i,j+1}$，其递推过程及表达式为：

$$\begin{cases}\left(\boldsymbol{P}'_{i,j+1}-\boldsymbol{P}_{i,j}\right)\cdot \boldsymbol{N}_{i,j}=0\\ \dfrac{\boldsymbol{P}'_{i,j+1}-\boldsymbol{S}_{i,j+1}}{\left|\boldsymbol{P}'i_{i,j+1}-\boldsymbol{S}_{i,j+1}\right|}=\boldsymbol{In}_{i,j+1}\end{cases}\Rightarrow \boldsymbol{P}'_{i,j+1}=\frac{\boldsymbol{P}_{i,j}\cdot \boldsymbol{N}_{i,j}}{\boldsymbol{In}_{i,j+1}\cdot \boldsymbol{N}_{i,j}}\boldsymbol{In}_{i,j+1} \tag{11}$$

同理求出$\boldsymbol{P}_{i-1,j+1}$点的法向矢量$\boldsymbol{N}_{i-1,j+1}$，光线$\boldsymbol{S}_{i,j+1}\boldsymbol{P}''_{i,j+1}$光线与过$\boldsymbol{P}_{i-1,j+1}$点切平面相交，交点即为$\boldsymbol{P}''_{i,j+1}$，其递推过程及表达式为：

$$\begin{cases}(\boldsymbol{P}''_{i,j+1}-\boldsymbol{P}_{i-1,j+1})\cdot \boldsymbol{N}_{i-1,j+1}=0\\ \dfrac{\boldsymbol{P}''_{i,j+1}-\boldsymbol{S}_{i,j+1}}{\left|\boldsymbol{P}''_{i,j+1}-\boldsymbol{S}_{i,j+1}\right|}=\boldsymbol{In}_{i,j+1}\end{cases}\Rightarrow \boldsymbol{P}''_{i,j+1}=\frac{\boldsymbol{P}_{i-1,j+1}\cdot \boldsymbol{N}_{i-1,j+1}}{\boldsymbol{In}_{i,j+1}\cdot \boldsymbol{N}_{i-1,j+1}}\boldsymbol{In}_{i,j+1} \tag{12}$$

通过两个方向分别获得的$\boldsymbol{P}'_{i,j+1}$和$\boldsymbol{P}''_{i,j+1}$坐标并求其平均值，即为实际$\boldsymbol{P}_{i,j+1}$点的坐标：

$$\boldsymbol{P}_{i,j+1}=\frac{(\boldsymbol{P}'_{i,j+1}+\boldsymbol{P}''_{i,j+1})}{2} \tag{13}$$

根据正交双向各自坐标值的递归关系，重复上述过程，即可得到自由曲面上的其他数据采样点。通过正交方向坐标点的均值来修正数据采样点的法向矢量，以降低自由曲面构造中法矢的累积偏差，提高自由曲面的构造精度，进而实现对激光光束的高性能整形。基于曲面法矢修正的正交双向动态迭代自由曲面构造法流程图如图 3 所示。

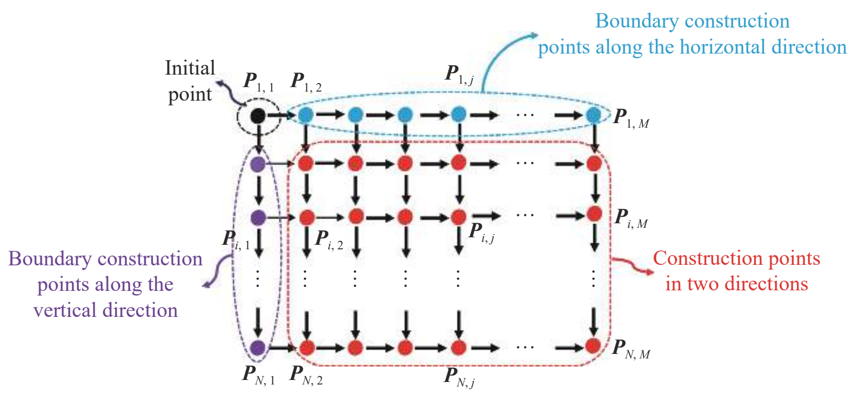

图 2 正交双向迭代自由曲面构造法原理示意图

Fig.2 Schematic of freeform surface construction method by iteration in two orthogonal directions

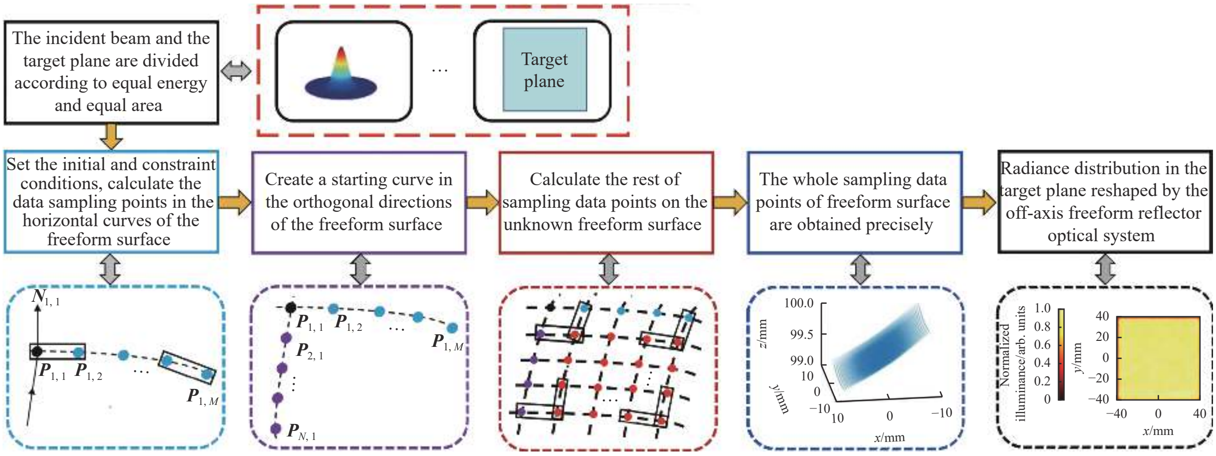

图 3 基于曲面法矢修正的正交双向动态迭代自由曲面构造法流程图

Fig.3 Flow diagram of the freeform surface construction method by iteration in two orthogonal directions together with surface normal correction

2 设计实例与结果分析

为了验证基于曲面法矢修正的正交双向迭代自由曲面设计法的有效性，构建了自由曲面反射式激光光束整形系统，如图 4(d) 所示。准直入射激光光束经自由曲面反射镜，在目标面上形成高均匀度的

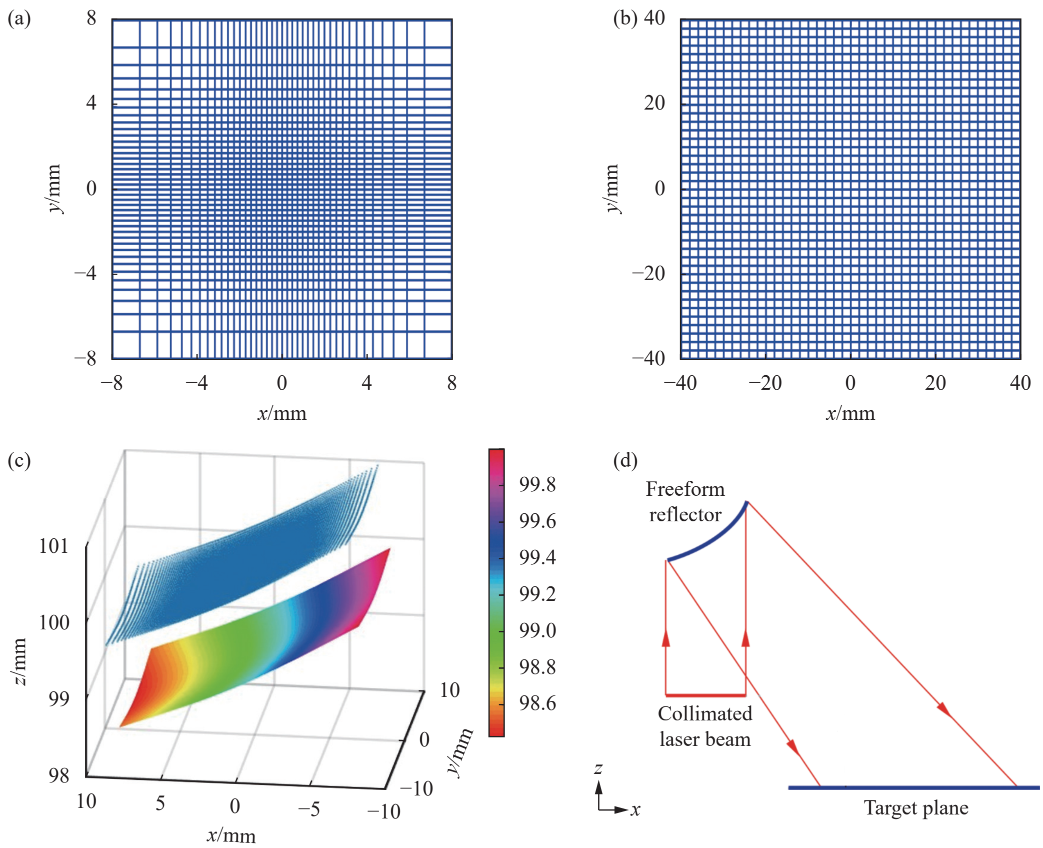

图 4 自由曲面反射式激光光束整形系统。(a) 光源网格划分；(b) 目标面网格划分；(c) 自由曲面数据采样点和三维自由曲面；(d) 系统光路结构示意图

Fig.4 Laser beam shaping optical system by reflective freeform surface. (a) Source grids; (b) Target plane grids; (c) Freeform surface sampling points and its three-dimensional surface; (d) Schematic of structure of beam shaping optical system

方形光斑。以方形孔径准直激光光束作为光源$\Omega_s=\{(x,y)|\ |x|\leqslant 8\ \mathrm{mm},|y|\leqslant 8\ \mathrm{mm}\}$，目标接收面区域为$\Omega_t=\{(T_x,T_y)|\ |T_x|\leqslant 40\ \mathrm{mm},|T_y|\leqslant 40\ \mathrm{mm}\}$。光源与目标面的能量网格划分为 160×160，如图 4(a)、(b) 所示。根据前文所提构造法，求得光源与目标面的映射关系，进而获得自由曲面的离散数据采样点，如图 4(c) 所示。

自由曲面反射式激光光束整形系统如图 4(d) 所示，自由曲面到光源与目标面的距离分别设为 100 mm 和 200 mm。采用蒙特卡洛法进行光线追迹，利用辐照度分布均匀性 U 与均方根值 RMS 评价激光光束整形系统的性能，分别表示为：

$$U=\overline{E}\big/E_{\max} \tag{14}$$

$$\mathrm{RMS}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{E_t(i)-E_a(i)}{E_a(i)}\right)^2} \tag{15}$$

式中：$\overline{E}$与$E_{\max}$分别为目标面有效面积辐照度的平均值与最大值；N为采样点的数量；$E_t(i)$为第 i 个采样点的实际辐照度值；$E_a(i)$为其辐照度平均值。RMS 值越小，表示实际辐照度值与辐照度平均值的偏差越小，说明激光光束整形系统的性能越好。目标面全部区域和局部区域 (70 mm×70 mm) 内的辐照度用于分析辐照度均匀性，128×128 个数据采样点的辐照度用于评估辐照度分布的均方根值。

入射激光光束整形后在目标面的光斑归一化辐照度分布，以及沿 x=0 mm 和 y=0 mm 位置的辐照度分布曲线如图 5 所示。经自由曲面反射镜进行激光光束整形，在目标面全域内归一化辐照度均匀度为 88.70%，局部区域 (70 mm×70 mm) 的辐照度均匀度为 92%。沿 x=0 mm 和 y=0 mm 的辐照度均匀度分别为 U_x=88.18% 和 U_y=86.67%。另一方面，目标面全域的辐照度均方根值为 7.4×10^{-3}，局部区域 (70 mm×70 mm) 的辐照度均方根值为 9.662×10^{-5}。结合目标面全域与局域的辐照度均方根值，说明实际辐照度与辐照度均值的偏差很小，实现了高性能激光光束整形。

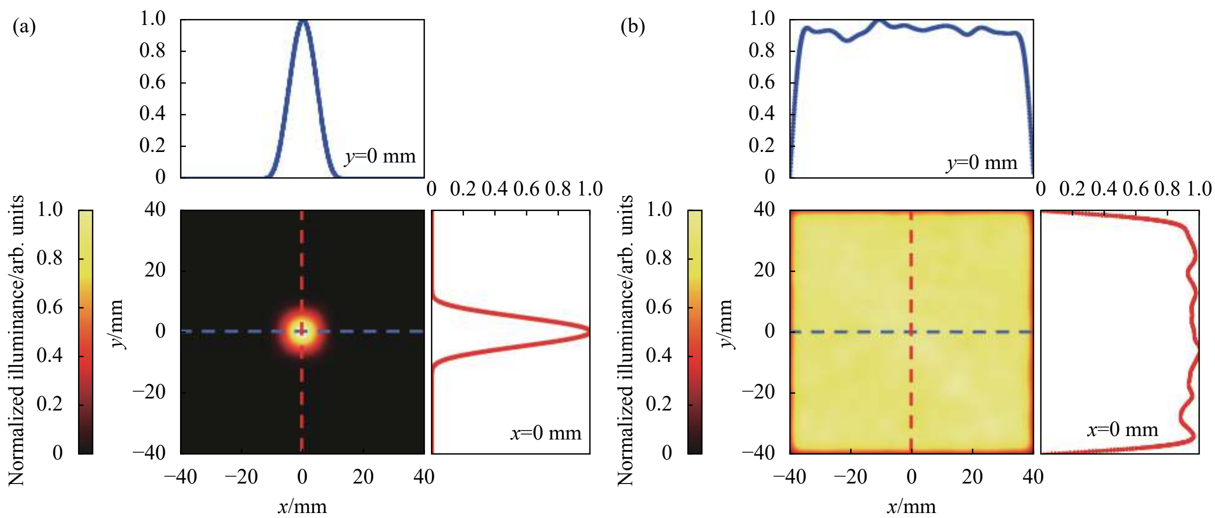

图 5 目标面光斑的归一化辐照度分布。(a) 整形前入射激光光束与沿 x=0 mm 及 y=0 mm 位置的辐照度分布；(b) 整形后目标面沿 x=0 mm 及 y=0 mm 位置的辐照度分布

Fig.5 Normalized irradiance distribution in the target surface. (a) Irradiance distribution of input laser beam and the target plane along the line x = 0 mm and y = 0 mm before beam shaping; (b) Irradiance distribution of target plane along the line x = 0 mm and y = 0 mm after beam shaping

除了将激光光束整形为正方形光斑，在激光工业应用中有时也需要整形为辐照度均匀分布的长方形光斑。入射光源不变，目标面区域为$\Omega_t=\{(T_x,T_y)||T_x|\leqslant 40\ \mathrm{mm},|T_y|\leqslant 20\ \mathrm{mm}\}$。经光线追迹后在目标接收面上的归一化辐照度分布如图 6(a) 所示，其辐照度均匀度为 94.30%，辐照度均方根值为 1.1×10^{-2}。沿 x=0 mm 和 y=0 mm 的辐照度分布如图 6(b) 所示，其均匀度分别为 U_x=92.96% 和 U_y=94.07%，同样实现了高性能激光光束整形。

进一步以正方形光斑整形为例，对比分析了当目

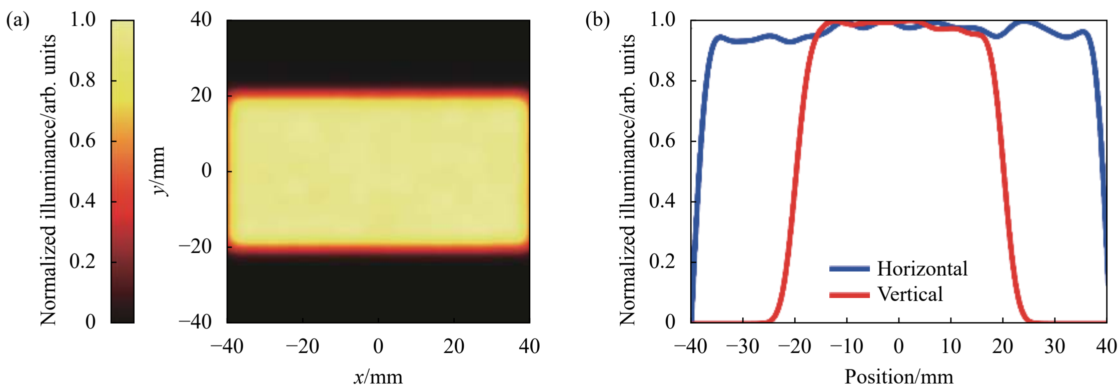

图 6 长方形光斑的归一化辐照度分布。(a) 整形后目标接收面的辐照度分布; (b) 整形后目标面沿 x=0 mm 及 y=0 mm 位置的辐照度分布

Fig.6 Normalized irradiance distribution of rectangular pattern. (a) Irradiance distribution in the target plane after beam shaping; (b) Irradiance distribution of target plane along the line x = 0 mm and y = 0 mm after beam shaping

标面处于不同位置时，目标面辐照度分布的变化情况。光源到目标面的距离分别设为 400、600、800、1 000 mm，相应的归一化辐照度分布如图 7 所示，其辐照度均匀度分别为 89.73%、89.72%、89.65% 与 89.19%，表明了在一定距离范围内目标接收面的辐照度分布总体稳定。另外，当目标面在不同位置时，分析了目标面全域与局部区域 (70 mm×70 mm) 的辐照度均匀度与均方根值的变化情况，如图 8 所示。无论是目标面全域还是局部区域，其辐照度均匀度与均方值都比较稳定，获得了高均匀度的激光光束整形效果。

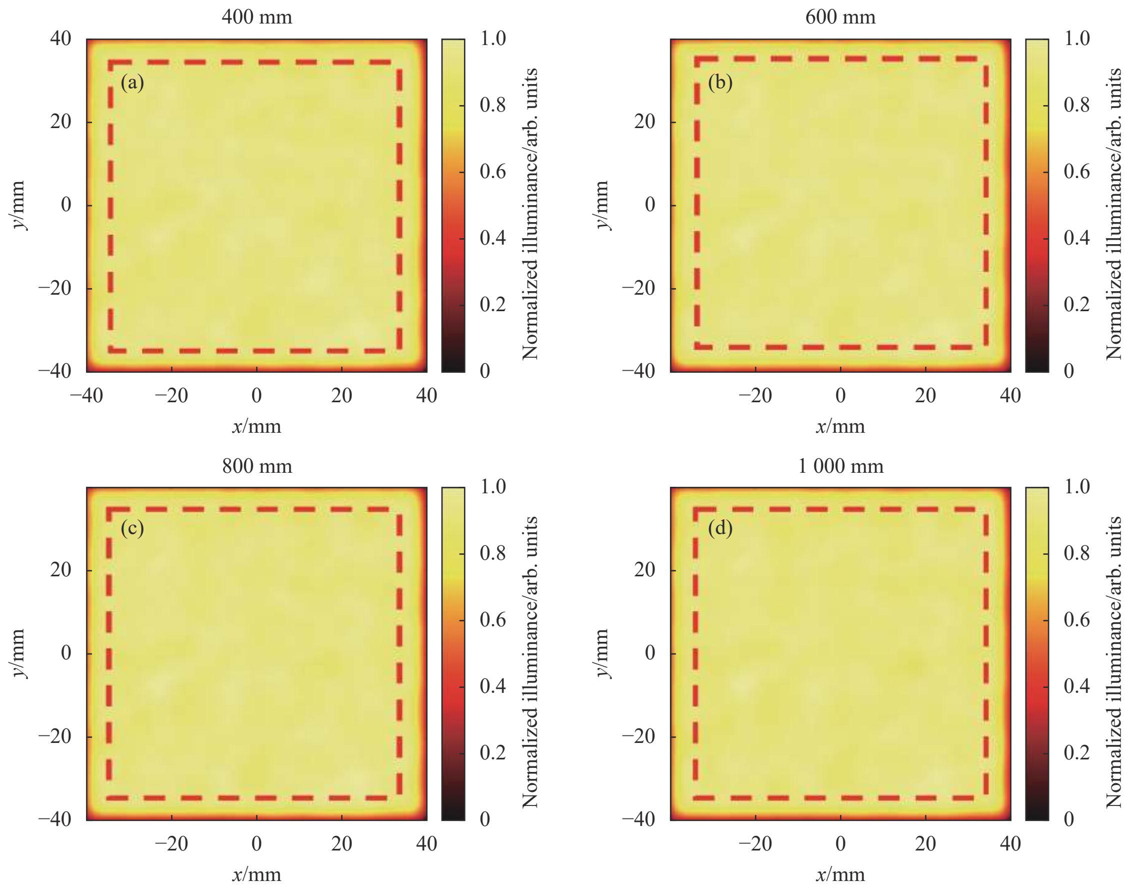

图 7 目标接收面在不同位置处的归一化辐照度分布

Fig.7 Normalized irradiance distribution of target plane at different distances

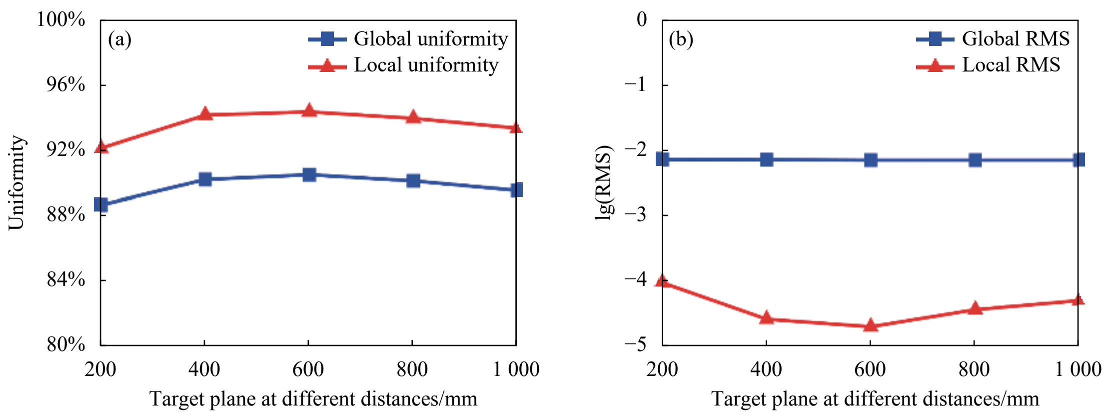

图 8 目标面在不同位置时全域与局部区域的辐照度均匀度与均方根值的变化情况。(a) 辐照度均匀度; (b) 辐照度均方根值

Fig.8 Variation of irradiance uniformity and RMS of global and local region in the target plane with different distances. (a) Irradiance uniformity; (b) The log (RMS) of irradiance

3 自由曲面面形特征对比分析

由前文设计结果可知,所提基于曲面法矢修正的正交双向迭代自由曲面构造法能够实现高均匀度激光光束整形。进一步从自由曲面的连续性、拟合精度与面形误差等方面与现有设计方法进行对比分析。如图 9 所示,为自由曲面在水平与竖直方向相邻数据采样点的法矢夹角,分别为θ_v与θ_h,通过分析相邻数据采样点的法矢夹角变化情况来评价自由曲面的平滑度。

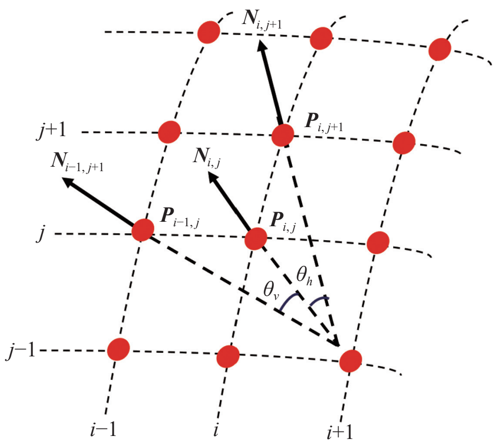

图 9 相邻数据采样点的法向矢量夹角

Fig.9 Normal vector angle of adjacent data sampling points

相比于现有设计法所构造的自由曲面,所提方法得到的同一曲线上相邻数据采样点的法矢夹角变化相对较小,如图 10 所示,自由曲面的平滑度更好。

根据所提方法获得自由曲面全部采样点数据,利用 *XY* 多项式进行自由曲面数据拟合,公式 (16) 的均

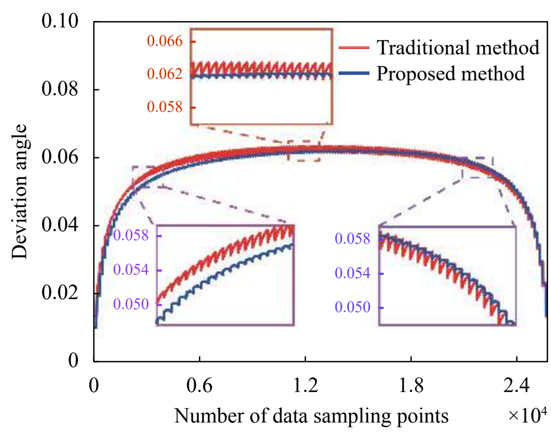

图 10 相邻采样数据点的法矢夹角变化情况对比

Fig.10 Comparison of variation of normal angle of adjacent data sampling points

方根误差 (RMSE) 表示自由曲面的拟合精度:

$$\mathrm{RMSE}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[f(x,y)-f'(x,y)\right]^2} \tag{16}$$

式中:N为采样点数量;$f(x,y)$为实际自由曲面数据;$f'(x,y)$为拟合的结果数据。RMSE 越小,说明自由曲面的拟合精度越高。

如图 11 所示,当目标接收面在不同位置时,相比于现有设计方法,所提方法构建的自由曲面均具有较小的拟合误差,其面形精度更高,结果与前文自由曲面的高连续性与平滑度相一致。

进一步,对比分析了自由曲面在一定加工误差范围内,激光光束经自由曲面整形后目标面的辐照度均匀度变化情况。采用变异系数 (C_V) 表征目标面辐照

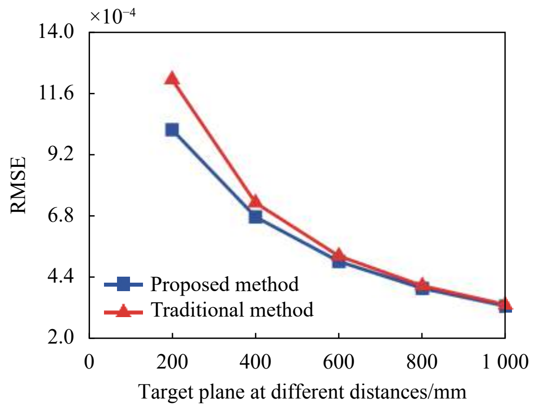

图 11 自由曲面拟合误差对比

Fig.11 Comparison of freeform surface fitting errors

度均匀度的稳定性，变异系数越小，辐照度均匀度的稳定性越高。T为模拟加工误差次数，x_i为样本数据值，$\overline{x_i}$为样本数据均值。假设自由曲面矢高的加工误差范围为−5~5 μm，模拟 10 次加工误差。现有设计法对应的 C_V 为 4.2×10^{-3}，而所提方法的 C_V 为 1.3×10^{-3}，其变异系数较小，辐照度均匀度的稳定性较高，如图 12 所示。

$$C_V = \frac{\sqrt{\dfrac{1}{T-1}\sum_{i=1}^{T}[x_i-\overline{x_i}]^2}}{\overline{x_i}} \tag{17}$$

相比于现有单一方向的自由曲面构造法，文中基于曲面法矢修正的正交双向自由曲面设计法获得的激光光束整形自由曲面具有高平滑度、曲面拟合误差小，且在一定加工误差范围内目标面辐照度均匀性高等优势。

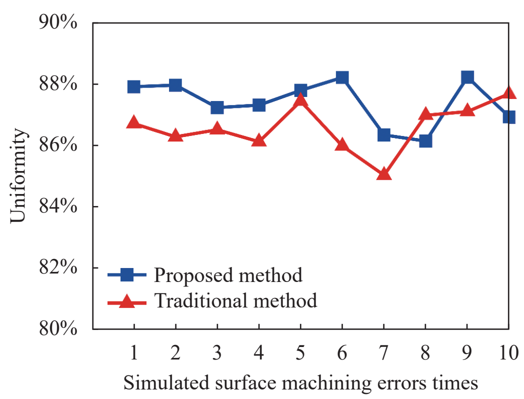

图 12 自由曲面加工误差对目标面辐照度均匀度的影响

Fig.12 Influence of irradiance uniformity on different freeform surface machining errors

4 结 论

文中提出了基于曲面法向矢量修正的正交双向迭代自由曲面设计法，用于构建激光光束整形系统，能够有效地将准直高斯型激光光束整形为高辐照度高均匀度的方形光斑。通过正交方向数据采样点坐标值的均值进行曲面法向矢量修正，降低自由曲面构造过程中的法矢偏差，提高自由曲面的构造精度，实现对激光光束分布的精准调控。结果表明，相比于现有单一方向的自由曲面整形技术，所提方法在自由曲面的连续性、拟合误差以及目标面辐照度稳定性等方面都具有优势。此外，所提自由曲面构造法可进一步应用于指定的几何辐照度分布照明或投影照明系统中。

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Freeform surface design of laser beam shaping by iteration in two orthogonal directions (*invited*)

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Abstract:

Objective In many laser industrial applications, laser beam shaping is an important process to redistribute the laser energy, which is highly essential to obtain uniform or prescribed spatial energy distribution with high efficiency. At present, there are different methods for laser beam shaping, including the grouped aspheric lenses, microlens arrays, diffractive optical elements, liquid crystal spatial light modulator and freeform optical technology. Compared with other laser beam shaping methods, the method using freeform surfaces is beneficial to make the shaping optical system more simplified and more compact. In the current freeform surface construction method for laser beam shaping, the seed curve extension algorithm in one direction has the non-negligible normal vector deviation during the generation of surface sampling points. Therefore, in this paper, a method is proposed to reduce the normal errors to improve the construction precision of freeform reflector for obtaining the highly uniform spatial energy distribution in the target plane.

Methods There are mainly three steps for constructing the freeform surface (Fig.2-3). At first, the incident beam and the target plane are divided in grids according to equal energy and equal area. The main purpose of this step is to obtain the one-to-one energy mapping between the light source and the target plane, which is based on the conservation of energy and Snell's law. Then, the initial constraint conditions are set according to the requirements, which are used to calculate the sampling data points of horizontal and vertical curves on the freeform surface. Finally, according to the results of previous two steps, the sample data points on the unknown freeform surface can be calculated by iteration together with normal error correction. The averaging approach of coordinates of adjacent sampling points in the orthogonal direction is applied to relieve the normal deviations, which is very useful for reducing the normal errors to obtain smooth freeform surface relatively.

Results and Discussions The proposed freeform surface construction method can effectively regulate a collimated Gaussian laser beam into the square or rectangular intensity distribution with high uniformity. On the target plane for a square pattern (Fig.5), the normalized irradiance uniformity is about 88.70% in the global region. Along the lines x=0 mm and y=0 mm on the target plane, the irradiance uniformity is about U_x=88.18% and U_y=86.67% respectively. Besides, the irradiance uniformity of local region (70 mm×70 mm) is about 92%. In the similar way, for a rectangular pattern on the target plane (Fig.6), the corresponding normalized irradiance uniformity is as high as about 94.30% as well as U_x=92.96% and U_y=94.07% along the lines x=0 mm and y=0 mm, which realize the laser beam shaping with high performance. On the other hand, the irradiance uniformity can also reach about 90% when the target plane has different distances for the square pattern (Fig.7-8). This indicates that the proposed method keeps robust elegantly. Further, in the aspect of surface smoothness, fitting precision and irradiance uniformity stability over a certain manufacturing error range (Fig.10-12), the freeform surface constructed by the proposed method shows great performance compared with that constructed by the traditional design method.

Conclusions The freeform surface design by iteration in two orthogonal directions with surface normal correction is proposed, which can effectively regulate a collimated Gaussian laser beam into the square or rectangular intensity distribution with high uniformity. The feature sampling points of the freeform reflector are calculated iteratively in two orthogonal directions based on energy conservation and Snell's law. Meanwhile, the averaging approach of coordinates of adjacent sampling points in the orthogonal direction can relieve the normal deviations effectively. Therefore, it is very helpful for constructing the freeform surface more precisely. The capabilities of the presented method are demonstrated and verified by examples. Moreover, for the target plane with different distances within a certain range, the energy uniformity maintains 90% well. At the same time, the freeform surface designed by the proposed method not only has high fitting accuracy, but also has a more stable irradiance uniformity on target plane within the allowable machining error range. It means that the proposed freeform surface construction method keeps robust elegantly, which is very necessary and critical for laser beam shaping.

Key words: laser beam shaping; freeform surface design; iteration in two orthogonal directions; normal vector correction

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