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Propagation characteristics of non-uniformly Sinc-correlated blue-green laser beam through oceanic turbulence

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Abstract: The propagation model of non-uniformly Sinc-correlated blue-green laser beam in oceanic turbulence was developed according to generalized Huygens-Fresnel principles. Based on the cross-spectral density, intensity variations in different propagation distances were discussed. When the oceanic turbulence parameters were varied, intensity and lateral shifted intensity maximum were numerically simulated. The results show that the propagation distance and ocean turbulence parameters have a certain influence on the intensity self-focusing effect of the non-uniformly Sinc-correlated blue-green laser beam. When the propagation distance is certain, the effect of the rate of dissipation of mean-square temperature on the intensity self-focusing is greater than the rate of dissipation of kinetic energy and the relative strength of temperature and salinity fluctuations.

Key words: oceanic turbulence; non-uniformly Sinc-correlated beam; intensity; lateral shifted intensity maximum

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非均匀辛格相关蓝绿激光波束通过海洋湍流的传输特性

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摘　要: 根据广义惠更斯-菲涅尔原理,建立了非均匀辛格相关蓝绿激光波束在海洋湍流中的传输模型。基于交叉谱密度函数,讨论了不同传播距离下波束光强变化。数值计算了波束光强和光强最大值横向偏移受海洋湍流参数的影响。结果表明,传播距离和海洋湍流参数对非均匀辛格相关蓝绿激光波束的光强自聚焦现象有一定影响。当传播距离一定时,温度均方耗散率对光强自聚焦的影响大于湍流动能耗散率和温度盐度波动相对强度。

关键词: 海洋湍流; 非均匀辛格相关波束; 光强; 光强最大值横向偏移

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0 Introduction

With the rapid development of technologies for underwater communication, imaging and sensing, it has been become ever more important to deeply understand how oceanic turbulence affect the propagation of laser beams[1]. In recent years, the effects of oceanic turbulence on the intensity[2], polarization[3], scintillation[4], beam spreading[5] and other characteristics of laser beams have been studied[6−7]. However, most of researches on laser beams are assumed that the field source with so-called Gaussian Schell-model (GSM) beams correlations, where the degree of coherence (DOC) is uniformly distributed and independent of the position of the two points[8]. Gori et al.[9] investigated the sufficient conditions for the cross-spectral density (CSD) matrix to satisfy the non-negative definition, and the result was shown that laser beams could expression some special non-uniformly correlation. Subsequently, scholars have been studied on the scattering and propagating characteristics of the non-uniformly correlated beams, the results demonstrated that non-uniformly correlated beams have the self-focusing effect[10], lateral shifted intensity maximum[11] and lower scintillation[12]. Lajunen and Saastamoinen[8] found that the partially coherent beams with spatially varying correlations have the lateral shifted intensity maximum during propagation. Tong and Korotkova[13]explored the phenomenon that the off-axis intensity maximum of the non-uniformly correlated scalar beams were inhibited in isotropic atmosphere turbulence. At the same time, some scholars had studied the generation and propagation characteristics of non-uniformly correlated laser beams by experiments[14−16]. Mei[17]proposed and studied uniformly Sinc-correlated beams, and the result was shown that it had flat profiles in the far field. In the next year, Mei and Korotkova et al.[18] proposed the concept of non-uniformly Sinc-correlated beams and found that this beam had the self-focusing and lateral shifted intensity maximum effects in free space. Studies have shown that blue-green lasers with wavelengths between 450-580 nm have strong penetrability and less propagation loss in seawater, so it is more suitable for propagation in seawater[19]. Therefore, the study of the propagation characteristics of the non-uniformly sinc-correlated blue-green laser beam in oceanic turbulence is of great significance.

In this paper, we provide the theoretical analysis in section 2 for the propagation model of non-uniformly sinc-correlated blue-green laser beams in oceanic turbulence. In Section 3, the influence of oceanic turbulence parameters and propagation distances on the intensity and lateral shifted intensity maximum of the non-uniformly sinc-correlated blue-green laser beam are numerically discussed. It gives the conclusions in Section 4.

1 Theory and formulation

Assuming that ρ_1, ρ_2 is two spatial points at the source plane, the CSD can be described by[20]

$$W^{(0)}(\rho_1,\rho_2)=\langle E(\rho_1)E^*(\rho_2)\rangle \tag{1}$$

where $E(\rho)$ is electric field at the point ρ, * indicate complex conjugate, $\langle\rangle$ represent the ensemble averaging. As we all know, the correlation function for optical fields cannot be chosen at wish owing to the non-negative definiteness restrictions, and the non-negative definiteness restrictions refers that, for any $f(\rho)$, the CSD must satisfy the inequality[9]

$$\int\int \mathrm{d}^2\rho_1\mathrm{d}^2\rho_2 W^{(0)}(\rho_1,\rho_2)f(\rho_1)f(\rho_2)\geqslant 0 \tag{2}$$

$f(\rho)$ is an arbitrary function. At this time, the CSD of Eq. (2) can be expressed as[21]

$$W^{(0)}(\rho_1,\rho_2)=\int p(v)H_0^*(\rho_1,v)H_0(\rho_2,v)\mathrm{d}v \tag{3}$$

where the $p(v)$ is an arbitrary nonnegative weighting function, the $H_0(\rho,v)$ is an arbitrary kernel function. From Ref. [22], $H_0(\rho,v)$ has the form

$$H_0(\rho,v)=\tau(\rho)\exp[-if(\rho)v] \tag{4}$$

The $\tau(\rho)$ is a complex amplitude. When the kernel function $H_0(\rho,v)$ of Eq. (3) is known, different CSD can be obtained by selecting different weighting function[23]. The $p(v)$ and $H_0(\rho,v)$ is given by[18]

$$p(v)=\frac{1}{a}\cdot\mathrm{rect}\left(\frac{v}{a}\right)=\begin{cases}1/a, & |v|\leqslant a/2\\ 0, & |v|>a/2\end{cases} \tag{5}$$

$$H_0(\rho,v)=\tau(\rho)\exp\left[-2\pi i v(\rho-\rho_0)^2\right] \tag{6}$$

where $\tau(\rho)=\exp[-\rho^2/(2\sigma^2)]$, σ is the root-mean-square width. $\mathrm{rect}(x)$ is a rectangular function with width a, and a is a positive constant. Substituting Eqs. (5) and (6) in Eq. (3), it is obtained

$$W^{(0)}(\rho_1,\rho_2)=\exp\left[-\left(\rho_1^2+\rho_2^2\right)/\left(2\sigma^2\right)\right]\mu(\rho_1,\rho_2) \tag{7}$$

$\mu(\rho_1,\rho_2)$ is the DOC of the light field, usually defined as $\mu(\rho_1,\rho_2)=W(\rho_1,\rho_2)/\sqrt{I(\rho_1)I(\rho_2)}$, where $I(\rho)=W(\rho,\rho)$ is the spectral intensity. In this paper, the DOC of non-uniformly Sinc-correlated beams has the following form

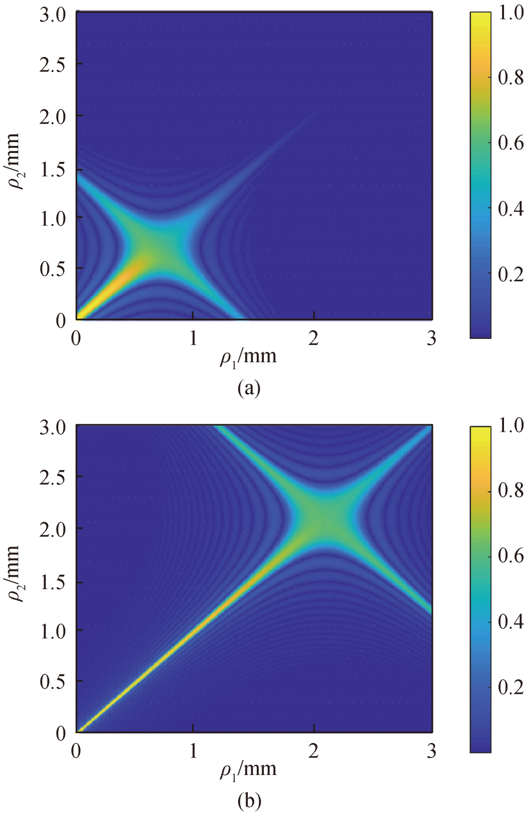
$$\mu(\rho_1,\rho_2)=\mathrm{sinc}\left\{c\left[(\rho_1-\rho_0)^2-(\rho_2-\rho_0)^2\right]\right\} \tag{8}$$

From Eq. (8), we can see that the DOC include an extra shift by ρ_0, where $\rho_0=0.7\sigma$ is a real constant[24]. The parameter c is the amplification factor and can be used to control the beams profile of far field[17].

It can be seen from Eq. (8) that as $\mu(\rho_1,\rho_2)=\mathrm{sinc}[c(\rho_1-\rho_2)]$, we get the correlation function of uniformly sinc-correlated beam.

In Fig.1, we plot the CSD of the non-uniformly Sinc-correlated blue-green laser beams according to Eqs. (7) and (8) when the wavelength λ = 532 nm and root-mean-square width σ = 1 mm, 3 mm. From Ref. [16], we do not discuss the effect of amplification factor c on the CSD, we make c as a value of 8. It is indicated that the DOC of the non-uniformly Sinc-correlated laser beams are related to the lateral coordinate and takes the maximum value near the point ρ_0, which is different from the GSM beams[8].

Let us consider that a non-uniformly Sinc-correlated laser beams propagating close to z axis from the source plane $z=0$ to the half-space $z\geqslant 0$ in oceanic turbulence. With the help of the generalized Huygens-Fresnel principle, the CSD between two points (ρ'_1,z) and (ρ'_2,z) in any propagation plane are satisfied the following equation[25]



(a)

(b)

Fig.1 Cross-spectral density under different root-mean-square width

$$W(\rho'_1,\rho'_2,z)=\frac{k^2}{4\pi^2z^2}\iint W^{(0)}(\rho_1,\rho_2)\times \exp\left[-ik\frac{(\rho'_1-\rho_1)^2-(\rho'_2-\rho_2)^2}{2z}\right]\times \langle\exp[\phi(\rho'_1,\rho_1)+\phi^*(\rho'_2,\rho_2)]\rangle\,\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2 \tag{9}$$

where $W^{(0)}(\rho_1,\rho_2)$ is the CSD in the source plane. $k=2\pi/\lambda$ is wave number.

$\langle\exp[\phi(\rho'_1,\rho_1)+\phi^*(\rho'_2,\rho_2)]\rangle$ can be expressed as[23]

$$\langle\exp[\phi(\rho'_1,\rho_1)+\phi^*(\rho'_2,\rho_2)]\rangle=\exp\left\{-4\pi^2k^2z\int\int\kappa\Phi_n(\kappa)\{1-J_0[|(1-\gamma)\boldsymbol{u}+\gamma\boldsymbol{q}|\kappa]\}\mathrm{d}\kappa\mathrm{d}\gamma\right\} \tag{10}$$

In Eq. (10), $\boldsymbol{u}=\rho_1'-\rho_2'$, $\boldsymbol{q}=\rho_1-\rho_2$ and J_0 is the Bessel function of zero order. $\phi(\rho',\rho)$ is the complex phase perturbation. Eq. (10) can be calculated as

$$\langle\exp[\phi(\rho'_1,\rho_1)+\phi^*(\rho'_2,\rho_2)]\rangle=\exp\left\{-P\left[\boldsymbol{u}^2+\boldsymbol{uq}+\boldsymbol{q}^2\right]\right\} \tag{11}$$

In Eq. (11), $P=\dfrac{\pi^2k^2z}{3}\int_0^\infty\kappa^3\Phi_n(\kappa)\mathrm{d}\kappa$ means the turbulence strength. The $\Phi_n(\kappa)$ is the spatial power spectrum of the refractive index fluctuation in oceanic turbulence, and κ is the spatial angular frequency.

Introducing new variables $\boldsymbol{U} = (\rho'_{\mathbf{1}} + \rho'_{\mathbf{2}})/2$, $\boldsymbol{Q} = \rho'_{\mathbf{1}} - \rho'_{\mathbf{2}}$ and putting Eqs. (7), (8), (10) and (11) into Eq. (9), and exchange integral order, Eq. (9) can be simplified as

$$W(\rho'_{\mathbf{1}}, \rho'_{2}, z) = \frac{k^2}{4\pi^2 z^2} \int p(v) H^*(\rho'_{\mathbf{1}}, z, v) H(\rho'_{\mathbf{2}}, z, v) \mathrm{d}v \tag{12}$$

where

$$H^*(\rho'_{\mathbf{1}}, z, v) H(\rho'_{\mathbf{2}}, z, v) = \frac{\sigma}{\omega(z,v)} \exp\left[-\left(\frac{k\sigma}{2z}\right)^2 (\rho'_1 - \rho'_{\mathbf{2}})^2 - \frac{ik}{2z}\left(\rho'^{2}_{\mathbf{1}} - \rho'^{2}_{\mathbf{2}}\right)\right] \times \exp\left\{-\frac{1}{\omega^2(z,v)}\left[\frac{\rho'_{\mathbf{1}} + \rho'_{\mathbf{2}}}{2} - \frac{ik\sigma^2}{2z}(\rho'_{\mathbf{1}} - \rho'_{\mathbf{2}}) - \frac{4\pi v z \rho_0}{k}\right]^2\right\} \tag{13}$$

Now let $\rho'_{\mathbf{1}} = \rho'_{\mathbf{2}} = r$, Eq. (13) satisfies the following form

$$|H(r,v,z)|^2 = \frac{\sigma}{\omega(z,v)} \exp\left[-\frac{(r - 4\pi v z \rho_0 / k)^2}{\omega^2(z,v)}\right] \tag{14}$$

where

$$\omega(z,v) = \sqrt{\left(\frac{z}{k\sigma}\right)^2 + \sigma^2\left(1 - \frac{4\pi z v}{k}\right)^2 + \frac{\pi^2 k^2 z}{3}\int_0^\infty \kappa^3 \varPhi_n(\kappa) \mathrm{d}\kappa} \tag{15}$$

Now the spectral intensity I of the non-uniformly Sinc-correlated blue-green laser beams has the following form at the point (r, z)

$$I(r,z) = W(r,r,z) = \frac{k^2}{4\pi^2 z^2} \int p(v) |H(r,z,v)|^2 \mathrm{d}v \tag{16}$$

Oceanic turbulence is different from atmosphere turbulence, and the refractive index fluctuation of seawater is caused by the both change of temperature and salinity[2]. Assuming that the oceanic turbulence is isotropic and uniform, the absorption and scattering effects of seawater on laser beams are ignored, at this time the oceanic turbulence spectrum is given by[26−27]

$$\varPhi_n(\kappa) = 0.388 \times 10^{-8} \varepsilon^{-1/3} \kappa^{-11/3} \left[1 + 2.35(\kappa\eta)^{2/3}\right] f(\kappa, w, \lambda_T) \tag{17}$$

where $\eta = 10^{-3}$ m is the Kolmogorov internal scale and ε is the rate of dissipation of kinetic energy per unit mass of fluid ranging from $10^{-10}\ \mathrm{m}^2 \cdot \mathrm{s}^{-3}$ to $10^{-4}\ \mathrm{m}^2 \cdot \mathrm{s}^{-3}$, and $f(\kappa, w, \lambda_T)$ has the form

$$f(\kappa, w, \lambda_T) = \frac{\lambda_T}{w^2}\left(w^2 e^{-A_T\delta} + e^{-A_S\delta} - 2we^{-A_{TS}\delta}\right) \tag{18}$$

λ_T is the rate of dissipation of mean-square temperature, which varies from oceanic surface to deep water layer is $10^{-10}\ \mathrm{K}^2 \cdot \mathrm{s}^{-1}$ to $10^{-2}\ \mathrm{K}^2 \cdot \mathrm{s}^{-1}$. w indicates the relative strength of temperature and salinity fluctuations, and it describes the contribution of both to the change in oceanic power spectrum. In the oceanic medium, the value of w ranges from −5 to 0, when $w = -5$, the oceanic turbulence is caused by temperature-induced, and $w = 0$ gives the oceanic turbulence of salinity-induced. The other parameters are evaluated as $A_T = 1.863 \times 10^{-2}$, $A_S = 1.9 \times 10^{-4}$, $A_{TS} = 9.41 \times 10^{-3}$, $\delta = 8.284(\kappa\eta)^{4/3} + 12.978(\kappa\eta)^2$.

2 Numerical results and analysis

We plot the intensity variations and the lateral shifted intensity maximum of non-uniformly sinc-correlated laser beams propagating through free space and oceanic turbulence according to Eqs. (12)–(18) as $\sigma = 1$ mm, $\lambda = 532$ nm. $r_{\max}$ is the x-coordinate corresponding to the peak of the curve.

Figure 2(a) shows the intensity variation of the non-uniformly sinc-correlated laser beam propagating through free space. Figure 2(b) illustrates evolution of intensity on the $r-z$plane. It can be seen that the non-uniformly Sinc-correlated blue-green laser beams has the self-focusing effect and lateral shifted intensity maximum, and the maximum intensity is not taken at the center of the laser beams. The intensity obeys a non-uniformly distribution and no longer satisfies the symmetry.

Figure 3(a) demonstrates the intensity of a non-uniformly Sinc-correlated blue-green laser beams propagating through oceanic turbulence. The intensity evolution in the $r-z$ plane is given in Fig. 3(b). It is shown that the oceanic turbulence significantly suppresses the intensity of the non-uniformly Sinc-correlated blue-green laser beams.

In Fig.4, we plot the intensity variation of non-

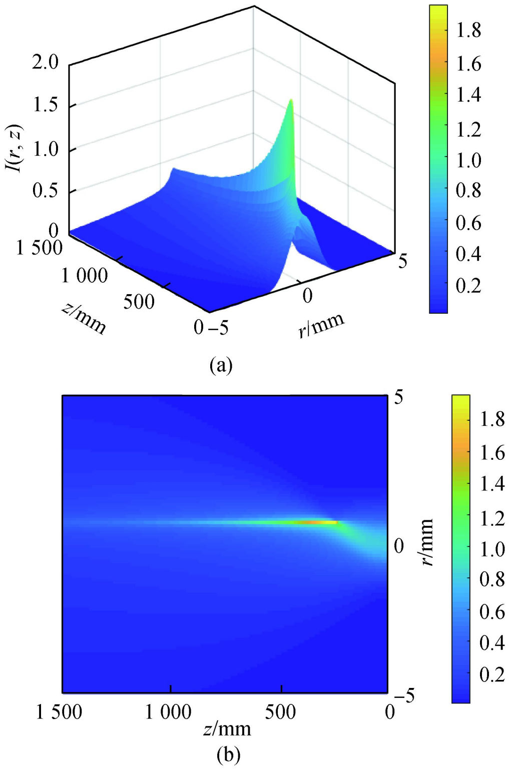


Fig.2 Intensity of non-uniformly Sinc-correlated blue-green laser beams through free space

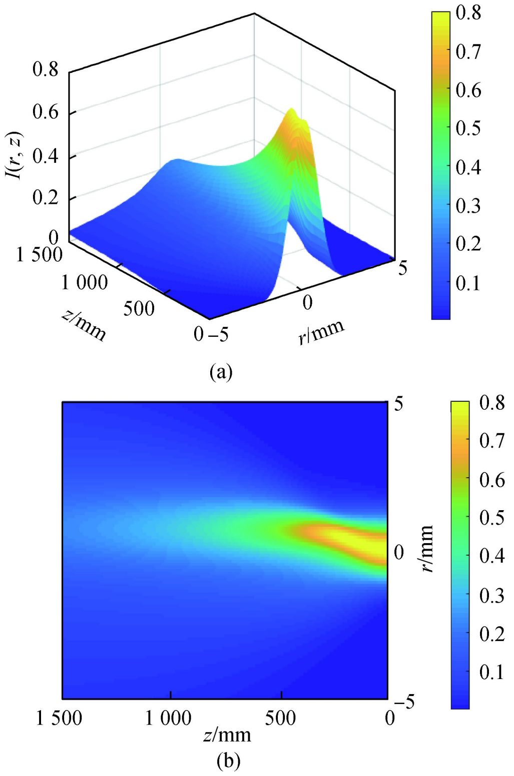


Fig.3 Intensity of non-uniformly Sinc-correlated blue-green laser beams through oceanic turbulence

uniformly Sinc-correlated laser beams in free space and oceanic turbulence with r when the propagation distances z = 0 m, 40 m, 80 m, 120 m. The corresponding parameters are as follows $\lambda_T = 10^{-8}\ \mathrm{K}^2\cdot\mathrm{s}^{-1}$, $\varepsilon = 10^{-7}\ \mathrm{m}^2\cdot\mathrm{s}^{-3}$, $w = -2.5$, $\eta = 10^{-3}$ m. In Fig. 4(b), when z = 0 m, 40 m, 80 m, 120 m, the corresponding $r_{\max}$ = 0 mm, 2.08 mm, 2.01 mm, 1.96 mm. It can be seen that the oceanic turbulence has suppression on intensity and the lateral shifted intensity maximum. When z = 0 m, the intensity distribution is still uniformly in both free space and oceanic turbulence. As the propagation distance increases, the uniformly distribution gradually degenerates into non-uniformly distribution. Specially, the oceanic turbulence accelerates this degradation. This characterizes is similar to the propagation of non-uniform partially coherent laser beams in free space[28].

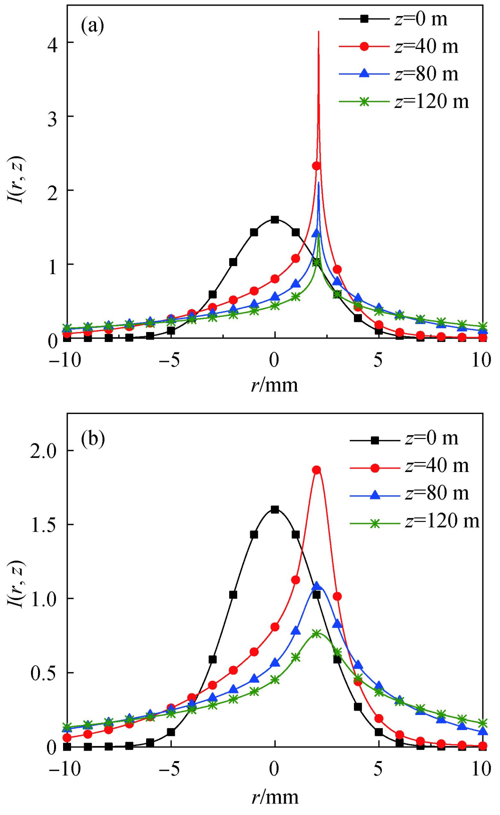


Fig.4 Intensity variation of non-uniformly Sinc-correlated blue-green laser beams at a certain propagation distance (a) free space; (b) oceanic turbulence

Figures. 5 and 6 show the λ_T and w on the intensity of the non-uniformly sinc-correlated blue-green laser beams at a certain propagation distance. The other parameters are taken as $\varepsilon = 10^{-7}\ \mathrm{m^2\cdot s^{-3}}$, $w = -2.5$, $\eta = 10^{-3}$ m in Fig. 5, and $\varepsilon = 10^{-7}\mathrm{m^2\cdot s^{-3}}$, $\lambda_T = 10^{-8}\mathrm{K^2\cdot s^{-1}}$, $\eta = 10^{-3}$m in Fig. 6. In Fig. 5, when $\lambda_T = 10^{-7}, 10^{-8}, 10^{-9}$, the corresponding (a) $r_{\max} = 0.26$ mm, 0.57 mm, 0.69 mm, (b) $r_{\max} = 0.43$ mm, 0.65 mm, 0.7 mm.

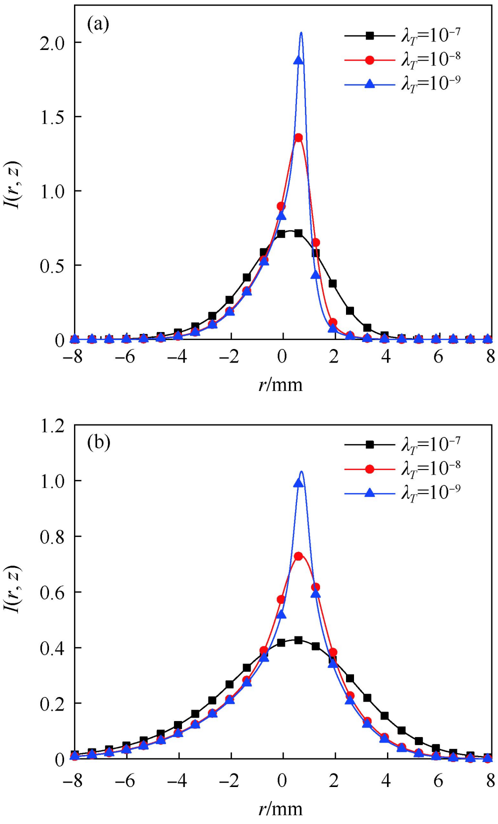


Fig.5 Effect of λ_T on the intensity of non-uniformly Sinc-correlated blue-green laser beams (a) $z = 35$ m; (b) $z = 85$ m

In Fig. 6, when $w = -1, -2.5, -4$, the corresponding (a) $r_{\max} = 0.4$ mm, 0.57 mm, 0.63 mm, (b) $r_{\max} = 0.57$ mm, 0.65 mm, 0.68 mm.

It can be concluded from Figs. 5 and 6 that when the propagation distance $z = 35$ m, with the increase of λ_T and w, the intensity is suppressed, and the lateral shifted intensity maximum is also reduced. This is similar to the propagation characteristics of a non-uniformly laser beams in oceanic turbulence[29]. When the propagation distance $z = 85$ m, the intensity is still suppressed with the increase of λ_{T} and w. In the same time, it is found that the suppression is stronger. The lateral shifted intensity maximum is less affected because the oceanic turbulence accelerates the degradation of the non-uniformly Sinc-correlated blue-green laser beams. When the propagation distance is large enough, the intensity distribution degenerates from a non-uniformly distribution to uniformly distribution.

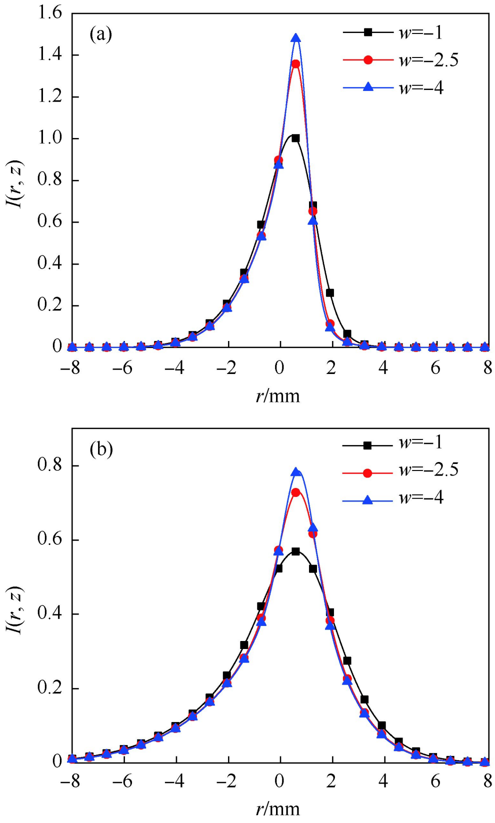


Fig.6 Effect of the w on the intensity of non-uniformly Sinc-correlated blue-green laser beams (a) $z = 35$ m; (b) $z = 85$ m

When propagation distances $z = 35$ m and $z = 85$ m, we plot the curve that the effect of ε on intensity of non-uniformly Sinc-correlated blue-green laser beams in Fig. 7. The other parameters are taken as $\lambda_T = 10^{-8}\ \mathrm{K^2\cdot s^{-1}}$, $w = -2.5$, $\eta = 10^{-3}$ m. When $\varepsilon = 10^{-5}, 10^{-7}, 10^{-9}$, the corresponding (a) $r_{\max} = 0.67$ mm, 0.57 mm, 0.39 mm, (b) $r_{\max}$ = 0.69 mm, 0.65 mm, 0.56 mm. When the propagation distance $z = 35$ m, it is indicated that the lateral shifted intensity maximum and intensity increases with the increase of ε. When the propagation distance $z = 85$ m, the intensity increases with the increase of ε. Due to the accelerated degradation of the non-uniformly sinc-correlated blue-green laser beams in the oceanic turbulence, the lateral shifted intensity maximum changes very little. It can be seen from Figs. 5, 6 and 7 that when the propagation distance increase, λ_T has greater influence on intensity than w and ε. As can be seen from Figs. 5, 6 and 7, when ocean turbulence parameters λ_T,w and ε are constant, $r_{\max}$ increases with the increase of propagation distance.

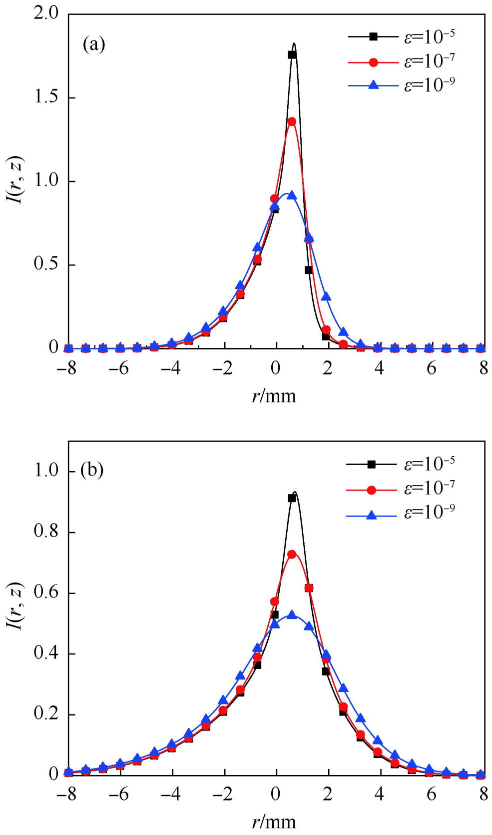


Fig.7 Effect of ε on the intensity of non-uniformly Sinc-correlated blue-green laser beams (a) z = 35 m; (b) z = 85 m

3 Conclusion

In summary, the propagation characteristics of non-uniformly Sinc-correlated blue-green laser beams are discussed in oceanic turbulence. When the root-mean-square width is different, we explore the cross-spectral density variations. The results are shown that the oceanic turbulence can suppress the lateral shifted intensity maximum and intensity of the non-uniformly Sinc-correlated blue-green laser beams, and accelerate the degradation of this beam, and affect the beam's intensity self-focusing. Considering the propagation distance z = 35 m, the lateral shifted intensity maximum and intensity decrease with the increase of λ_T and w, and increase with the increases of ε, the results of the propagation distance z = 85 m are similar to z = 35 m.

As the degradation of the non-uniformly Sinc-correlated laser beams is accelerated by the effect of oceanic turbulence, the influences of oceanic turbulence parameters λ_T, w and ε on the lateral shifted intensity maximum is relatively small when z = 85 m. In addition, with the propagation distance increases, the influence of λ_T on the intensity is greatest.

Our works provide a theoretical basis for the experimental research of the blue-green laser under the background of oceanic turbulence, and further study of the underwater communication and detection research.

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