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Rapid phase retrieval using SLM based on transport of intensity equation

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Abstract: The transport of intensity equation(TIE) offers an experimentally simple technique for computing phase information directly from several defocused images. In this work we developed the traditional TIE intensity acquisition system. In order to avoid the shifting of CCD, a quadratic phase pattern was displayed on the SLM to provide a lens effect for realizing different defocus distance by varying the focal length of the lens. Two kinds of phase imaging experimental configurations guided by two different theories were designed. In most instances, since a camera is used to capture images in which the phase modulation of lens cannot be ignored, the former intensity acquisition system was designed based on the single lens optical propagation system, and the relationship between the focal length and the image distance in the lens law was used to facilitate the varying of the defocus distance by changing the phase pattern displayed on the SLM. The latter was designed by locating a SLM in the Fourier domain of the 4*f* setup, according to the Fourier transform property of the lens and the Fresnel diffraction theory, the relationship between the defocus distance and the focal length of the quadratic phase pattern can be derived. The experimental results verify that the proposed phase retrieval methods are reasonable and correct.

Key words: phase retrieval;　transport of intensity equation;　spatial light modulator

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基于光强传输方程与 SLM 的快速相位恢复

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摘　要: 基于光强传输方程(TIE)的相位恢复技术,可以通过测量一系列垂直于光轴分布的散焦强度图,直接求解得到光场的相位信息。传统的基于 TIE 的强度图像采集系统在获得散焦图像时需要机械移动。通过在空间光调制器(SLM)上加载透镜相位图,使其实现可编程透镜的功能,任意改变此透镜的焦距值,可以实现不同的散焦距离,从而避免采集图像过程中 CCD 的移动。根据不同的算法理论,设计了两种光强图像采集系统。由于日常生活中使用相机拍照时,透镜的相位调制作用不可忽略,使用 SLM 代替含透镜的光传播模型中透镜的作用,设计了第一种实验装置。此外,通过在 4*f* 系统的频率域位置放置 SLM,设计了第二种实验装置。真实实验的结果验证了这两种方法的有效性和正确性。

关键词: 相位恢复;　光强传输方程;　空间光调制器

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0 Introduction

Phase is an inherent characteristic of any wave field, statistics show that most of the information is encoded in the phase term rather than amplitude. Phase recovery is essential to 3D phase mapping as well as the object′s 3D shape[1]. Considerable efforts have been undertaken to devise proper algorithms for obtaining phase, one of which is the employment of transport of intensity equation (TIE). TIE states that the derivative of intensity respect to the optical axis is related to the phase[2]. Using several intensity images at different positions along the light propagation direction, the phase change due to a phase object can be recovered by TIE[3]. The TIE gains many unique advantages over interferometry techniques[4] such as no phase unwrapping, non-interferometry and does not requires a complicated system. However, this technique still has some challenges; it requires a set of intensity images at longitudinal planes, which is usually realized by translating the camera or the object manually or mechanically, slowing down the process, and the registration of the images requires careful alignment, as well as ensuring that no tilt is introduced between the images[5].

Therefore many researchers have introduced spatial light modulator (SLM) into phase retrieval and have achieved good experimental results. Yu et al. proposed a single-shot TIE method by loading phase pattern on SLM, which played two roles as wavefront division and defocus modulation, however, compensation should be employed to ensure the same intensity of the obtained multi-focal images before phase retrieval[6]. Zuo et al. presented a SQPM method, two laterally separated images from different focal planes could be obtained simultaneously by a single camera exposure, yet the accurate registration of the two experimental images was performed necessarily[7].

In this paper, we introduce the SLM based on LCOS technology[8] in our phase retrieval system. Programmed lens phase distribution with different focal length is loaded in this SLM to make it realize the function of tunable-lens. Two kinds of phase retrieval experimental configurations with the advantages of low-cost, fast-speed are designed. Different defocus distance can be realized by changing the lens phase distribution loaded in the SLM without the demand of adjusting the system, so the intensity images with different defocus distance can be recorded in same plane, avoiding the errors caused by the mechanical shift, the phase information is obtained by solving the TIE with a fast Fourier transform based Poisson solver.

1 Using SLM based TIE for phase retrieval

1.1 Theory

The TIE was originally derived by Teague from Helmholtz equation under paraxial approximation[2]. A light propagation model with SLM is illustrated in Fig.1, a quadratic phase pattern $p(x,y)=\exp(-i\pi(x^2+y^2)/\lambda f_{\rm SLM})$ is displayed on the SLM, providing a lens effect to realize different defocus distance through varying the focal length $f_{\rm SLM}$ of the lens phase. When the phase distribution with focal length $f_{\rm SLM}=f_1$ is displayed on SLM to modulate the input light field of SLM, we could capture an intensity image $I(x,y,v)$ by CCD at the imaging plane, that is, along the optical axis at the position v. In order to obtain the multi-intensity images of the symmetrical position before and after the imaging plane to estimate the intensity differential, we can set a defocus distance value Δz, according to

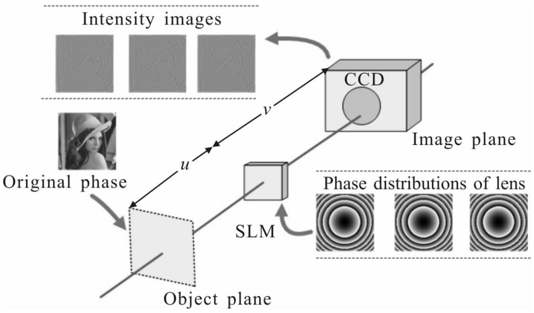

Fig.1 Schematic diagram of light intensity propagation with SLM

the lens imaging law $u^{-1}+v^{-1}=f^{-1}$, the two focal length belong to the lens phase that should be loaded on the SLM are $f_2=1/(1/(v+\Delta z)+1/u)$, $f_3=1/(1/(v-\Delta z)+1/u)$, where u is the distance between the object plane and the SLM, v is the distance between the SLM and the imaging plane. And the two captured intensity images at the same imaging plane corresponding to the focal length of f_2 and f_3 are recorded as $I(x,y,v')$ and $I(x,y,v'')$.

Ref.[9] illustrates that the TIE is available in the light propagation model based lens as follows:

$$-\nabla\cdot[I(x,y,v)\nabla\varphi(x,y,v)]\cong\frac{k_0}{2\Delta z}[I(x,y,v')-I(x,y,v'')] \quad (1)$$

where φ is the phase at imaging plane, k_0 is wave number, $k_0=2\pi/\lambda$, λ is wavelength, $\nabla=(\partial/\partial x+\partial/\partial y)$ denotes the two-dimensional gradient operator over the propagation direction z. Assuming the distribution of light intensity I at a certain transverse plane is uniform and constant, thus Eq.(1) could simplifies to

$$-k_0\cdot\frac{I(x,y,v')-I(x,y,v'')}{2I(x,y,v)\Delta z}\approx\nabla^2\varphi(x,y,v) \quad (2)$$

The Eq. (2) was solved using a fast Fourier transform (FFT) Poisson solver, where the boundary conditions were assumed to be zero

$$\varphi(x,y,v)=\pounds^{-1}\left\{\frac{-k_0}{k_x^2+k_y^2+a}\pounds\left[\frac{I(x,y,v')-I(x,y,v'')}{2I(x,y,v)\Delta z}\right]\right\} \quad (3)$$

where £ denotes the Fourier transform operator and $\pounds^{-1}$ represent, the inverse Fourier transform, k_x and k_y are the spatial frequencies in Fourier domain, and a is a regularization parameter.

1.2 Experiment

The intensity acquisition system is shown in Fig.2. A HeNe laser with wavelength of 532 nm is collimated through a collimating lens before illuminating the test sample. The test object is a micro-lens array, which is consist of some single lens made of silicone oil with refractive index of 1.579, the filling material surrounding the single lens is the PDMS with refractive index of 1.403, the maximal height of the lens is about 1.2 mm. The CCD (1 280 pixel×1 024 pixel, pixel size 7.3 μm×7.3 μm) along with the phase object and the phase-only refractive SLM (filling factor 87%, 1 280 pixel×1 024 pixel, pixel size 12 μm×12 μm,) based LCOS technique form the acquisition system.

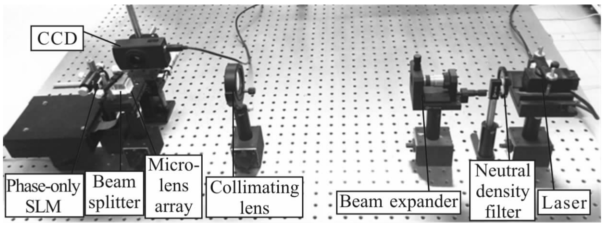

Fig.2 Practical intensity acquisition system

The focal length f=7 cm, and the defocus distance Δz=20 μm. The three intensity images corresponding to different focal length recorded by CCD are shown in Fig.3(a)–(c). Fig.3(d) illustrates the recovered phase of the test object and Fig.3(e) plotted the profile lines of recovered phase.

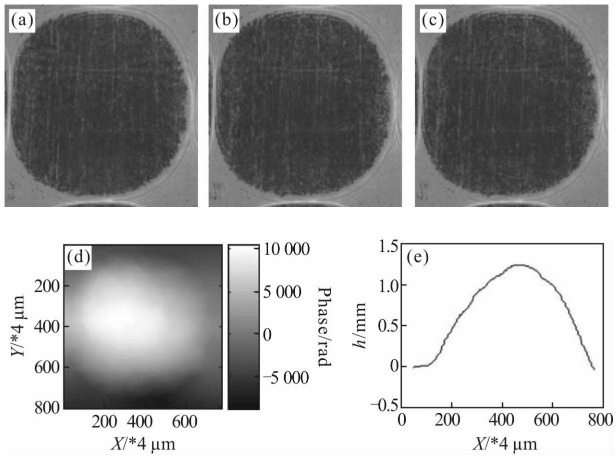

Fig.3 Experiment results (a)–(c) three intensity images; (d) retrieval phase of single micro-lens; (e) profile lines of the retrieved phase

Since the background noise in the calculation process and the diffraction effect caused by the pixel structure of the SLM are not removed very well, the obtained profile is not smooth enough compared to the ideal lens profile.

2 Using SLM and 4*f* setup based TIE for phase retrieval

2.1 Theory

Programmed lens phase distribution with different focal length is loaded in the SLM to make it realize the function of tunable-lens is presented in Sec.1. In this section, we will apply the same idea into a 4*f* setup to reconstruct the phase of an object. We located a SLM in the Fourier domain of the 4*f* setup as described in Fig.4, the same quadratic phase pattern is displayed in the SLM and the theory explanation are given as following.

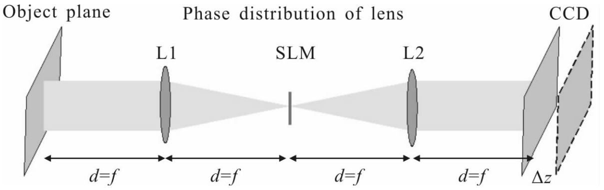

Fig.4 A 4*f* system with the SLM located at the Fourier plane for intensity acquisition

An object characterized by the complex transparency function $t(x,y)$ is placed in the front focal plane of lens L1. Considering the Fourier transform property of the lens L1, the complex field incident in the SLM at the back focal plane of lens L1 can be expressed as $T(k_x,k_y)=\pounds\{t(x,y)\}$, where $k_x=k_0x/f$, $k_y=k_0y/f$ is the focal length of the two Fourier lenses. So the complex field just behind the SLM can be denoted as

$$h(x,y)|_{\text{behind-SLM}}=T(k_x,k_y)\cdot p(x,y) \tag{4}$$

Then the complex field of the CCD plane can be written as the Fourier transform of the complex field behind SLM,

$$h(x,y)|_{\text{CCD}}=\pounds\{T(k_0x/f,k_0y/f)\}*P(k_x,k_y) \tag{5}$$

where $P(k_x,k_y)=\pounds(p(x,y))$. Eq.(5) can be further simplified as

$$h(x,y)|_{\text{CCD}}\propto t(-x,-y)*\exp\frac{-jk_0f_{\text{SLM}}(x^2+y^2)}{2f^2} \tag{6}$$

Considering the SLM does not place in the 4*f* system, the complex field of the back focal plane of lens L2 can be expressed as $u(x,y)\propto t(-x,-y)$, according to the Fresnel diffraction theory, the complex field away from the back focal plane of lens L2 with a distance of Δz at CCD plane can be shown as

$$u(x,y)|_{\text{CCD}}\propto t(-x,-y)*\exp\frac{-jk_0(x^2+y^2)}{2\Delta z} \tag{7}$$

Comparing the Eq.(6) and Eq.(7), we can draw such a conclusion

$$h(x,y)|_{\text{CCD}}\propto u(x,y)|_{\text{CCD}},\ \text{if}\ \Delta z=f^2/f_{\text{SLM}} \tag{8}$$

Thus the shift of CCD with a distance of Δz can be substituted by changing the focal length f_{SLM} of the programmed phase pattern displayed on the SLM.

The employment of multi-plane intensities can improve the quality of retrieval phase, the intensity differential which is estimated by five intensity images can be expressed as following

$$\frac{\partial I(x,y,z)}{\partial z}\Big|_{z=0}\approx\frac{I_{-2}-8I_{-1}+8I_1-I_2}{12\Delta z} \tag{9}$$

where Δz is the defocus distance, and the subscript of the intensity I represents the position of the defocus images compared to the focus one.

2.2 Experiment

The intensity acquisition system with 4*f* setup is shown in Fig.5. The test sample is a resolution test chart (1951 USAF resolution test chart). The 4*f* setup is formed with two lenses L1 and L2 with the focal length of 300 mm. A transmission-type SLM (filling factor 57%, 1 024 pixel×768 pixel, 18 μm×18 μm pixel size,) is located at the Fourier plane of the 4*f* setup. And the CCD and light source are same as in Sec.1.2.

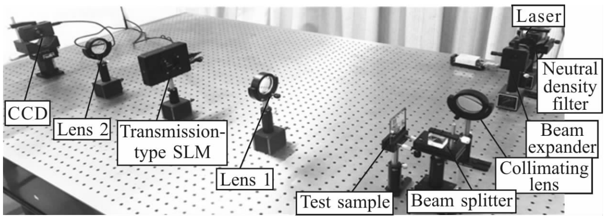

Fig.5 Practical intensity acquisition system

The defocus distance $\Delta z=500$ μm. The five intensity images corresponding to different focal length recorded by the acquisition system are shown in Fig.6 (a)-(e). Figure 6 (f) illustrates the recovery phase results of the test object.

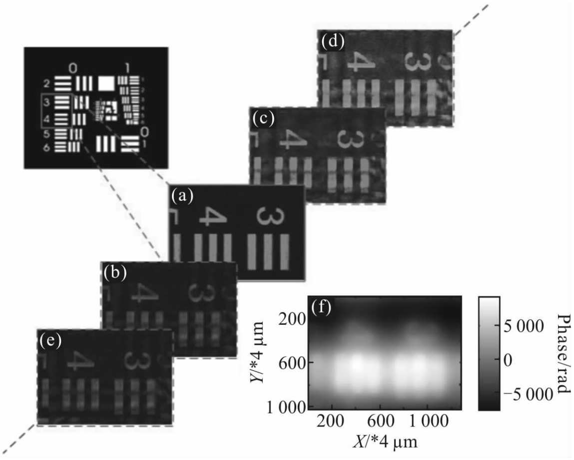

Fig.6 Results of real experiment

The experimental results indicate that the SLM has an influence on the experimental results, especially the transmission-type SLM. A better result can be obtained when the diffraction effects of the SLM are removed properly.

3 Conclusion

In this paper, we have given two different methods and configurations for phase retrieval. The average time of capture and save per intensity image takes about 1 s(manual operation is needed during image collection, if the system is integrated and synchronized, it will take less time), the process of phase retrieval using MATLAB 2014a takes about 0.8 s. The experimental results obtained by the designed intensity acquisition system verify that the proposed methods are reasonable and correct.

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