Scattering of spatially partially coherent light on quasi-homogeneous medium

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Abstract: Based on the accuracy of the first-order Born approximation, the scattering properties of a spatially partially coherent plane-wave light incident on a quasi-homogeneous random medium were investigated. The analytical expressions for the spectral density and the spectral degree of coherence of the scattered field in the far zone were derived. The influence of the spatial coherence of the incident field and the properties of the scattering on the spectral density and the spectral degree of coherence of the scattered field was examined. Compared with a fully coherent light, the spectral density and the spectral degree of coherence of the field generated by scattering of a partially coherent light were analyzed. The results indicate that the spatial coherence of the incident light has an important role in the behavior of the spectral density of scattered field. With an increase of the correlation length of the GSM beam, the effective angular width of the spectral density of the scattered field decreases. The spectral degree of coherence of the scattered field increases with decreasing beam width of the incident light or effective radius of the medium. The scattered field is fully coherent when the appropriate choices of the effective radius and the correlation length of the medium, the spatial correlation length of the incident light are made.

Key words: scattering;　spatially partially coherent light;　quasi-homogeneous medium;　spectral density;　spectral degree of coherence

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部分空间相干光经准均匀介质的散射

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摘　要: 基于一阶波恩近似,研究了部分空间相干平面光经准均匀介质的散射特性,得到了远场散射光谱强度和光谱相干度的解析表达式。讨论了入射光的空间相干性和介质特性对散射场光谱强度和光谱相干度的影响。比较了入射光是完全相干光和部分相干光时,散射场光谱强度和相干度的区别。研究结果表明:入射光的空间相干性对散射场光谱强度的分布有重要影响;随着入射光空间相干长度的增大,光谱宽度减小;光谱相干度随着入射光束腰宽度或介质有效半径的增大而减小。当准均匀介质的有效半径和相干长度、入射光的空间相干长度满足一定条件时,散射光是完全相干光。

关键词: 散射;　部分空间相干光;　准均匀介质;　光谱强度;　光谱相干度

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0 Introduction

Since Wolf and his collaborators found that the spectrum of light may change when it is scattered by an object[1], lots of work have been done on the subject of light scattering due to its potential applications, for example, in medical diagnostics, optical imaging and remote sensing of the atmosphere and ocean[2-6]. And the information about the properties of the scattering medium can be obtained from measurements of the statistical properties of the scattered field, such as the spectral density, spectral degree of coherence and spectral degree of polarization [7-9]. In the study of light scattering, it is generally assumed that the incident field is spatially fully coherent. However, the field is partially coherent under usual circumstances. In 1988, an ambient light with spatial coherence scattering by a Gaussian-correlated medium was studied[10]. The effect of spatial coherence of the incident light on the angular distribution of the scattered field was examined through an experiment[11]. Using the angular spectrum representation of fields, scattering of a wide class of beams of any state of coherence on a medium was analyzed[12-14]. More recently, Zhang et al. extended the analysis of weak scattering to Hermite-Gaussian incident beams and found some new features[15]. In this paper we will perform the scattering of a spatially partially coherent plane-wave light by a quasi-homogeneous random medium within the accuracy of the first-order Born approximation, where the influence of the spatial coherence of the incident field and the properties of the scatterer on the spectral density and the spectral degree of coherence of the scattered field will be investigated.

1 Theoretical formulation

It is considered that a spatially partially coherent plane-wave light incidents on a quasi-homogeneous random medium occupies a finite domain D, in a direction specified by a real unit vector s_0 (see Fig.1). The coherence properties of the incident field at any two points r_1' and r_2' can be characterized by the so-called cross-spectral density function, which can be expressed as[16]:

$$W^{(i)}(r_1',r_2',s_0;\omega)=W^{(i)}(r_1',r_2';\omega)\exp[iks_0\cdot(r_2'-r_1')] \quad (1)$$

where $W^{(i)}(r_1',r_2';\omega)$ represents the cross-spectral density of the incident light wave, with ω being the frequency, $k=\omega/c$, and c being speed of light in vacuum.

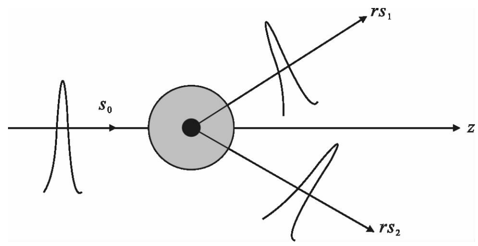

Fig.1 Illustrating the notation related to the scattering of a CGSM plane-wave pulse by a random medium

For a random medium, the scattering potential $F(r',\omega)$ is a random function of the position vector r'. Therefore, the properties of the medium should be described by a two-point correlation function of the scattering potential at a pair of points specified by the position vectors r_1' and r_2', which is defined by[6]:

$$G_F(r_1',r_2';\omega)=\langle F^*(r_1';\omega)F(r_2';\omega)\rangle_m \quad (2)$$

here the asterisk denotes the complex conjugate and the angular brackets, with subscript m, denote the average, taken over the ensemble of the random medium.

It is assumed that the scatterer is so weak that the scattering can be analyzed within the accuracy of the first-order Born approximation. In such a case, the cross-spectral density function of the scattered field specified at any two points with position vectors r_1 and r_2 is expressed as[16]:

$$W^{(s)}(r_1,r_2;\omega)=\int_D\int_D W^{(i)}(r_1',r_2',s_0;\omega)G_F(r_1',r_2';\omega)\times G^*(r_1,r_1';\omega)G(r_2,r_2';\omega)d^3r_1'd^3r_2' \quad (3)$$

where $G(r,r';\omega)$ is the outgoing free-space Green's function. In the far-zone, the Green's function has the well-know asymptotic approximation:

$$G(rs,r';\omega)\sim\frac{\exp(ikr)}{r}\exp[-iks\cdot r'] \tag{4}$$

Now let us suppose that the incident light of a spatially partially coherent beam takes the typical form of a Gaussian Schell-model(GSM) beam, whose cross-spectral density is[17]:

$$W^{(i)}(r_1',r_2';\omega)=A_0^2\exp\left[-\frac{r_1'^2+r_2'^2}{4\sigma_L^2}-\frac{(r_1'-r_2')^2}{4\delta_L^2}\right] \tag{5}$$

where σ_L and δ_L are the beam width and the spatial correlation length, respectively. In order for the incident field to be beamlike, the parameters σ_L and δ_L must satisfy the beam condition[17]:

$$\frac{1}{4\sigma_L^2}+\frac{1}{4\delta_L^2}\leqslant\frac{k^2}{2} \tag{6}$$

The correlation function of the scattering potential of a Gaussian-correlated, quasi-homogeneous medium has been given by the expression[2,16]

$$G_F(r_1',r_2';\omega)=C_0\exp\left[-\frac{|r_1'+r_2'|^2}{8\sigma_R^2}\right]\exp\left[-\frac{|r_1'-r_2'|^2}{2\delta_R^2}\right] \tag{7}$$

where C_0 is a positive constant. σ_R and δ_R represent the effective radius and the correlation length of the medium, respectively, satisfying inequality $\sigma_R\geqslant\sigma_r$.

Substituting Eqs.(4), (5) and (7) into Eq.(3), the cross-spectral density function of the scattered field in the far field is obtained as follow:

$$W^{(s)}(r_1s_1,r_2s_2;\omega)=\frac{1}{b}\exp[-\frac{\sigma_L^2\sigma_R^2(K_1+K_2)^2}{2(\sigma_L^2+\sigma_R^2)}-\frac{\delta^2\sigma_L^2(K_1-K_2)^2}{2(4\sigma_L^2+\delta^2)}] \tag{8}$$

$$b=\frac{A_0^2C_0(4\pi\sigma_L^2\sigma_R\delta)^3}{[(\sigma_R^2+\sigma_L^2)(\delta^2+4\sigma_L^2)]^{3/2}} \tag{9}$$

$$\frac{1}{\delta^2}=\frac{1}{\delta_R^2}+\frac{1}{\delta_L^2} \tag{10}$$

where $K_1=-k_1(s_1-s_0)$ and $K_2=k_2(s_2-s_0)$ are analogous to the momentum transfer vector of quantum mechanical theory of potential scattering[6].

The spectral density of the scattered field is obtained at once from Eq.(8) by setting $r_1=r_2$ and $s_1=s_2$. One finds that:

$$S^{(s)}(rs;\omega)=\frac{1}{b}\exp\left\{-\frac{2\delta^2\sigma_L^2K^2}{\delta^2+4\sigma_L^2}\right\}=$$

$$\frac{1}{b}\exp\left\{-\frac{8k^2\delta^2\sigma_L^2\sin^2(\theta/2)}{\delta^2+4\sigma_L^2}\right\} \tag{11}$$

where θ denotes the angle between the direction of incidence s_0 and the direction of scattering s. When the spatially fully coherent light incidents on a quasi-homogenous medium, the spectral density of the scattered field relates to the correlation length of the medium δ_R only through the reciprocity relations [13]. However, when the incident light is spatially partially coherent the properties of the incident light also affect the spectral density of the scattered field.

The spectral degree of coherence of the scattered field can also be derived from Eqs.(8) and (11) by use of the formula[6]:

$$\mu^{(s)}(r_1s_1,r_2s_2;\omega)=\frac{W^{(s)}(r_1s_1,r_2s_2;\omega)}{\sqrt{S^{(s)}(r_1s_1;\omega)S^{(s)}(r_2s_2;\omega)}}=$$

$$\exp\left\{-\frac{k^2\sigma_L^2(4\sigma_R^2-\delta^2)(s_1-s_2)^2}{2(\delta^2+4\sigma_L^2)(\sigma_R^2+\sigma_L^2)}\right\}\times\exp[ik(r_2-r_1)] \tag{12}$$

Without loss of generality we may take the z direction to be along the direction of incidence s_0 and the x direction along the victor s_1-s_2. If we restrict our attention to a pairs of observation points placed symmetrically along the direction of incidence, then $rs_1=r(s_x,0,s_z)$, and $rs_2=r(-s_x,0,s_z)$. Thus, Eq.(12) takes on the simple form:

$$\mu^{(s)}(rs_1,rs_2;\omega)=\exp\left\{-\frac{2k^2\sigma_L^2(4\sigma_R^2-\delta^2)\sin^2(\phi/2)}{(\delta^2+4\sigma_L^2)(\sigma_R^2+\sigma_L^2)}\right\} \tag{13}$$

where ϕ is the angle between s_1 and s_2. When the spatially fully coherent light incidents on a quasi-homogenous medium, the spectral degree of coherence of the scattered field relates to the effective radius of the quasi-homogeneous scatterer σ_R only through the reciprocity relations [11]. However, when the incident light is spatially partially coherent, not only the correlation length but also the effective radius of the scatterer affect the spectral degree of coherence of the scattered field. Furthermore, the scattered field is fully coherent at any angle ϕ when the following equation is satisfied:

$$\frac{1}{4\sigma_R^2}=\frac{1}{\delta_R^2}+\frac{1}{\delta_L^2} \tag{14}$$

2 Numerical results

According to Eqs.(11) and (13), we will show how the properties of the incident light and the scatterer influence the spectral density and the spectral degree of coherence of the scattered field. Figure 2 shows the behavior of the spectral density of the scattered field as a function of the scattering angle θ for different values of (a) the spatial correlation length of the GSM beam δ_L, (b) the beam width of the GSM beam σ_L. It is shown that the spatial coherence of the incident light has an important role in the behavior of the spectral density of the scattered field. With an increase of the correlation length of the GSM beam δ_L, the effective angular width of the spectral density of the scattered field decreases. However, the influence of beam width of the GSM beam σ_L on the spectral density of the scattered field is less evident.

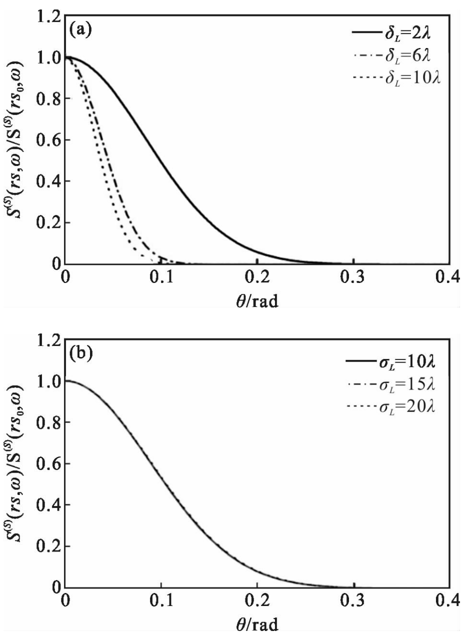

Fig.2 Normalized spectral density of the scattered field $S^{(s)}(rs,\omega)/S^{(s)}(rs_0,\omega)$ versus scattering angle θ for different values of (a) the spatial correlation length of the GSM beam δ_L, (b) the beam width of the GSM beam σ_L

The influence of the properties of the scatterer on the behavior of the spectral density of the scattered field is shown in Fig.3. From Fig.3 it can be seen that the correlation length of the medium δ_R has a great effect on the behavior of the spectral density of the far-zone field. The effective angular width of the spectral density of the scattered field decreases with increasing correlation length of the medium δ_R.

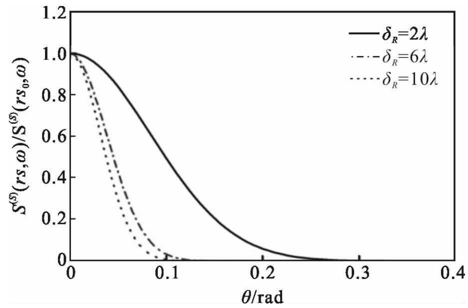

Fig.3 Normalized spectral density of the scattered field $S^{(s)}(rs,\omega)/S^{(s)}(rs_0,\omega)$ versus scattering angle θ for different values of the correlation length of the medium δ_R

Figure 4 gives the spectral degree of coherence of the scattered field $u^{(s)}(rs_1,rs_2;\omega)$ versus angle ϕ for different values of (a) the spatial correlation length of the GSM beam δ_L, (b) the beam width of the GSM beam σ_L. From Fig.4 it can be seen that with an increase of the

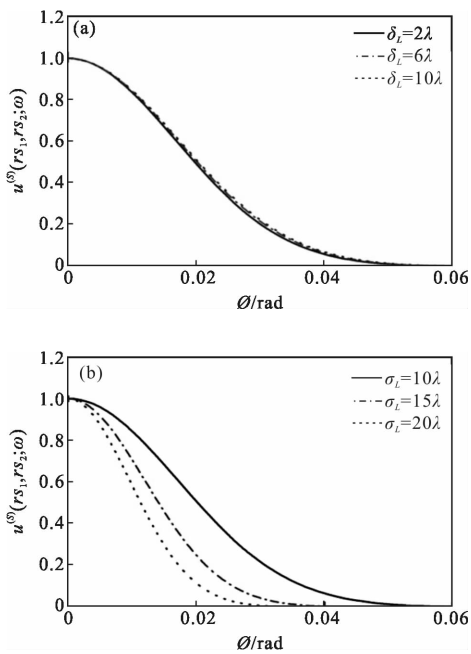

Fig.4 Spectral degree of coherence of the scattered field $u^{(s)}(rs_1, rs_2;\omega)$ versus angle ϕ for different values of (a) the spatial correlation length of the GSM beam δ_L, (b) the beam width of the GSM beam σ_L

spatial correlation length of the GSM beam δ_L, the spectral degree of coherence of the scattered field increases. The angular width of the spectral degree of coherence of the scattered field decreases when the beam width of the GSM beam σ_L increases.

The influence of the properties of the scatterer on the behavior of the spectral degree of coherence of the scattered field $u^{(s)}(rs_1,rs_2;\omega)$ is shown in Fig.5. As shown in Fig.5, with an increase of the correlation length of the medium δ_R, the spectral degree of coherence of the scattered field increases. The spectral degree of coherence of the scattered field decreases when the effective radius of the medium σ_R increases.

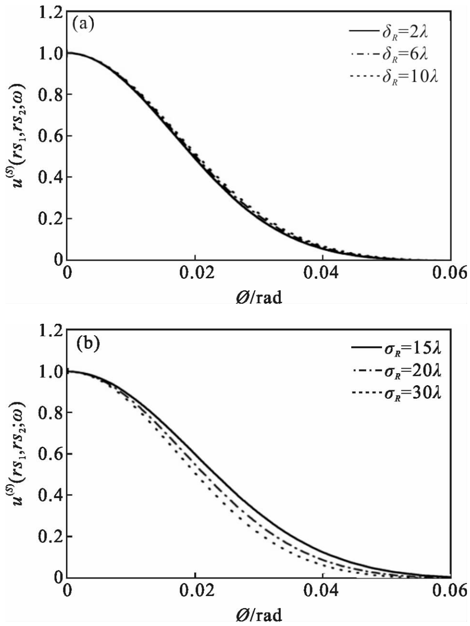

Fig.5 Spectral degree of coherence of the scattered field $u^{(s)}(rs_1, rs_2;\omega)$ versus angle ϕ for different values of (a) the correlation length of the medium δ_R, (b) the effective radius of the medium σ_R

3 Conclusion

In conclusion, we study the scattering of a partially coherent GSM beam from a quasi-homogeneous medium within the accuracy of the first-order Born approximation. The analytical expressions for the spectral density and the spectral degree of coherence of the scattered field have been derived. The dependence of the spectral density and the spectral degree of coherence of the scattered field on the spatial coherence of the incident field and the properties of the scatterer is examined. Especially, the scattered field is fully coherent when the appropriate parameters are chosen. Numerical calculations illustrate how the spatial coherence of the GSM beam affects the spectral density and the spectral degree of coherence of the scattered field. With an increase of the correlation length of the GSM beam δ_L, the effective angular width of the spectral density of the scattered field decreases, however, the spectral degree of coherence of the scattered field increases. The influence of the properties of the scatterer on the spectral density and the spectral degree of coherence of the scattered field is also illustrated. With increasing correlation length of the medium δ_R, the effective angular width of the spectral density of the scattered field decreases. The spectral degree of coherence of the scattered field decreases when the effective radius of the medium σ_R increases. The results obtained in this paper will be useful in the application of partially coherent light scattering.

References:

[1] Wolf E, Foley J T. Scattering of electromagnetic fields of any state of coherence from space-time fluctuations[J]. ***Phys Rev A***, 1989, 40(2): 579-587.

[2] Du Xinyue, Zhao Daomu. Scattering of light by Gaussian-correlated quasi-homogeneous anisotropic media[J]. ***Opt Lett***, 2010, 35(3): 384-386.

[3] Xue Bin, Kong Xianggui, Wang Dan, et al. SERS effect of aggregation of silver nanoprisms induced by 785nm laser[J]. ***Chin Opt***, 2014, 7(1): 118-123. (in Chinese)

[4] Chen Huimin, Liu Xinyang. Backscattering polarization characteristic of pulsed laser fuze with coaxial optical system in water fog[J]. ***Optics Precis Eng***, 2015, 23(3): 626-631. (in Chinese)

[5] Yang Hong, Huang Yuanhui, Gong Changmei, et al. Advances on techniques of breaking diffraction limitation using scattering medium[J]. ***Chin Opt***, 2014, 7(1): 1-25. (in Chinese)

[6] Lahiri M, Wolf E, Fischer D G, et al. Determination of

correlation functions of scattering potentials of stochastic media from scattering experiments [J]. ***Phys Rev Lett***, 2009, 102(12): 123901.

[7] Li Zhilong, Wu Zhijun, Wang Xing, et al. Experiments on relationship between droplet radius and its lateral scattering luminous flux[J]. ***Optics Precis Eng***, 2013, 21(10): 2502-2507. (in Chinese)

[8] Tian Jing, Chen Hang, Tu Shijie. Numerical simulation and analysis on far-field scatter for micro-bubbles [J]. ***Infrared and Laser Engineering***, 2014, 43(8): 2443-2446.

[9] Shi Zhenhua, Lin Guanyu, Wang Shurong, et al. Numerical analysis of small particle measurement based on the theory of laser scattering [J]. ***Infrared and Laser Engineering***, 2015, 44(7):2189-2194. (in Chinese)

[10] Jannson J, Jannson T, Wolf E. Spatial coherence discrimination in scattering [J]. ***Opt Lett***, 1988, 13 (12): 1060-1062.

[11] Gori F, Palma C, Santarsiero M. A scattering experiment with partially coherent light [J]. ***Opt Commun***, 1990, 74(6): 353-356.

[12] Dijk T van, Fischer D G, Visser T D, et al. Effects of spatial coherence on the angular distribution of radiant intensity generated by scattering on a sphere [J]. ***Phys Rev Lett***, 2010, 104(17): 173902.

[13] Sahin S, Korotkova O. Scattering of scalar light fields from collections of particles[J]. ***Phys Rev A***, 2008, 78(6): 063815.

[14] Korotkova O, Wolf E. Scattering matrix theory for stochastic scalar fields[J]. ***Phys Rev E***, 2007, 75(5): 056609.

[15] Zhang Yuanyuan, Zhao Daomu. Scattering of Hermite-Gaussian beams on Gaussian Schell-model random media[J]. ***Opt Commun***, 2013, 300(15): 38-44.

[16] Wolf E. Introduction to the Theory of Coherence and Polarization of Light [M]. Cambridge: Cambridge University Press, 2007.

[17] Mandel L, Wolf E. Optical Coherence and Quantum Optics [M]. Cambridge: Cambridge University Press, 1995.