Comparison between two frequency stabilization methods of external cavity diode laser for atom gravimeter

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Abstract: In order to choose appropriate frequency stabilization method in the atom gravimeter system under construction, Doppler-free dichroic atomic vapor laser lock (Doppler-free DAVLL) and saturated absorption spectroscopy (SAS) method were implemented on Rubidium atomic vapor. Basic principle and experimental details of both frequency stabilization methods were introduced. With restructuring of optical path and applying of self-made low noise photoelectric detector as well as digital lock module, excellent error signal was obtained. For each method, two sets of locking system were built and kept locked during 3 000 s′ data acquisition. A frequency fluctuation of 629 kHz when the laser is free running and of 16.2 kHz & 31.4 kHz after locked by SAS and Doppler-free DAVLL were calculated respectively, corresponding to a frequency stability of 1.64×10^{-9} in the condition of free running and 4.21×10^{-11} & 8.18×10^{-11} after locked for averaging time of 10 s. The strength and weakness of both methods were elaborated with the demand of system miniaturization. After compared with SAS, Doppler-free DAVLL isconsidered to be a promising choice for miniaturization and modularization in atom interferometry gravimeter.

Key words: frequency stabilization; external cavity diode laser; SAS; Doppler-free DAVLL; allan deviation

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用于原子重力仪的外腔半导体激光器的两种稳频方法比较

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摘 要: 在铷原子气室中实现了饱和吸收谱稳频法和消多普勒的双色谱稳频法,以给在建的原子干涉重力仪系统选择合适的激光稳频方法。介绍了两种稳频方法的基本原理及实验细节。通过调整光路设计、自制低噪声光电探测器以及应用数字锁定模块,获得了良好的鉴频误差信号。每种方法都搭建了两套稳频系统并在 3 000 s 采集时间内保持锁定。激光器在经饱和吸收法和消多普勒双色谱法锁定后,激光频率波动分别为 16.2 kHz 和 31.4 kHz,相应于在 10 s 采样时间下分别获得 4.21×10^{-11} 和 8.18×10^{-11} 的频率稳定度;相比之下,激光器自由运转时,频率波动和稳定度分别为 629 kHz 和 1.64×

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10^{-9}。在原子重力仪系统小型化的需求下详述了两种稳频方法的优缺点，比较而得消多普勒的双色谱稳频法在原子干涉重力仪的小型化模块化发展方向不失为具有潜力的一种选择。
关键词：稳频； 外腔半导体激光器； 饱和吸收谱； 消多普勒的双色谱； 阿伦方差

0 Introduction

Lasers with stable frequency and narrow linewidth are essential in frontier scientific research and metrology. With the most promising application prospect of all quantum technologies, quantum precision measurement has made remarkable achievements in areas of gravimeters[1-3]. The key point to realize a gravimeter based on atom interferometry[4-5], as shown in Fig.1, is the $\pi/2-\pi-\pi/2$ Raman pulses, after which the Raman laser phase and gravity are imprinted on the atomic wave packets, thus making atoms in same internal state from different quantum paths interfere with each other at point "D". The two employed lasers are supposed to be phase-locked to each other and achieve frequency stability of 100 kHz in order of magnitude so as to eliminate the influence of laser phase andcalculate the gravity acceleration. In addition, frequency-stabilized lasers are indispensable in applications of cooling laser, detecting laser and blowing-away laser.

A variety of active frequency stabilization methods are proposed such as SAS(saturated absorption spectroscopy), Doppler-free DAVLL (doppler-free dichroic atomic vapor laser lock), MTS (modulation transfer spectroscopy), FMS (frequency modulation spectroscopy), PDH(pound-drever-hall)and so on[6-10]. In the experiment of paper[11], the frequency of two DFB (distributed feedback) laser diodes of 1 560 nm is locked by the SAS method with a FWHM (full width at half maximum) of 1.5 MHz and the sensitivity of the gravity measurement can reach 1.0×10^{-8} within 200 s integration time. The paper[12] uses two-stage PDH frequency stabilization and ultra-high-finesse cavities in strontium optical lattice clock,achievinga frequency stability of about 2×10^{-15} for averaging time of 10 s. The paper[13] proposes a frequency stabilization method using a fiber electro-optic modulator and the analysis of the experimental results shows that the frequency-offset of the laser beam, with FWHM less than 1 MHz, can be tuned from 200 MHz to 10 GHz. A sub-kilohertz frequency stabilization of the ECDL (External Cavity Diode Laser) is achieved in this paper[14]. The linewidth of the laser is narrowed from 160 kHz to 400 Hz by laser frequency locking to a flank of a Fabry-Perot interferometer peak. However, these frequency stabilization methods are trades-off between each other from principles and practical methods in difficulty of operation, frequency capture range, cost, the position of zero crossing point compared to atomic lines reference and ability to hold the locking point against external interference.

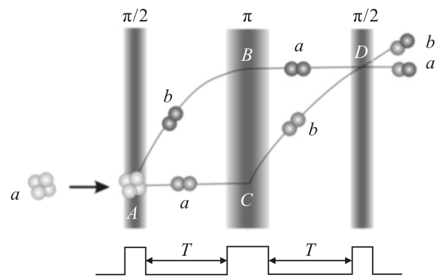


Fig.1 Atom interferometer in Raman-transition configuration

In a comprehensive comparison, we have investigatedon two basic frequency stabilization methods, namely, SAS and DAVLL both in theory and experiment. Line shapes of the SAS and DAVLL are obtained with the D2 transitions of ^{87}Rb atom. In order to maintain the lasers′ central frequency at a certain level, a digital feedback module is employed to apply a voltage to a PZT (piezoelectric transducer) and current controller ports. Through the frequency beat experiment, we record and analysie the data of frequency fluctuation via a data acquisition program written in LabVIEW language to evaluate whether the frequency stability meet the demand of gravimeter system.

1 Principle

1.1 Principle of SAS method

The main idea of SAS method derives from the interaction between laser and atoms as well as the Doppler shift of laser frequency experienced by atoms moving at a certain velocity. Depending on the deviation between laser frequency and transition energy, the transition of atoms from ground state to excited state absorbs laser thereby reducing its intensity. For counter propagating probe beam and pump beam, it consists six combinations of transition energy levels for D2 transition of ^{87}Rb atom that match the laser frequency in both directions experienced by the atoms.

With higher intensity, the pump laser excites adequate atoms, thus making there not enough atoms of ground state to interact with probe laser. A so-called saturated absorption peak is generated in the absorption spectrum of probe laser. The laser frequency can be locked to the slope of absorption spectrum through PID (proportion integration differentiation) feedbacks as its horizontal axis corresponds to frequency. However, owing to fluctuation of laser intensity, exact frequency can be only obtained by locking to the peak. The first derivative signal of absorption spectrum acts as error signal, which can be realized by small sinusoidal signal modulation and lock-in amplifier. The absorption spectrum (laser intensity) can be seen as a function of laser frequency ω: $F(\omega)$. A small increase in the ω is assumed as $f(t)$ with frequency ω_m and small equation A:

$$f(t)=A\sin\omega_m t \tag{1}$$

$$\omega(t)=\omega+A\sin\omega_m t=\omega+\Delta\omega \tag{2}$$

After modulation, saturated absorption signal, which is also a function of t, can be expressed as equation (3):

$$H(t)=F[\omega(t)]=F(\omega+A\sin\omega_m t) \tag{3}$$

$F\ (\omega)'$s first order Taylor expansion can be expressed as equation (4):

$$H(t)=F(\omega)+F'(\omega)A\sin\omega_m t \tag{4}$$

The lock-in amplifier is a phase discrimination process. After mixed, the modulation signal and absorption signal are processed in the low-pass filter, which can be expressed as equation (5):

$$e(t)=k\cdot LP[H(t)f(t)]=\frac{kA^2}{2}F'(\omega) \tag{5}$$

Where k is lock-in amplification factor and LP$[H(t)f(t)]$ is low-pass filtering function. After a small signal modulation and lock-in amplifier, the error signal is proportional to the first derivative of the saturated this paper absorption signal, which can be seen in this paper. The peak of the saturated absorption spectrum corresponds to the zero point of the error signal and can be used for PID feedback in frequency stabilization.

1.2 Principle of Doppler-free DAVLL method

On the basis of SAS method, the Doppler-free DAVLL introduces magnetic field to generate the dispersion-like curve without modulation. According to Zeeman Effect, axial magnetic field causes driftand split of the atomic spectrum. For right-handed ($\sigma-$) circularly polarized, $\Delta m=-1$ and the frequency of atomic spectrum decreases; for left-handed ($\sigma+$) circularly polarized, $\Delta m=1$ and the frequency of atomic spectrum increases. Consequently, the absorption curves of two kinds of circularly polarized lasers move in opposite directions with equal frequency shift. In the configuration of counter propagating probe beam and pump beam similar to SAS, the linearly polarized probe beam can be decomposed into two circularly polarized beams with same intensity. When without magnetic field, both the right-handedand left-handed circularly polarized beams are absorbed with the same amount, thus making the differential signal of the two absorption curves zero for all frequencies. When under acertain magnetic field, the center frequency of the two absorption curves increases and decreases respectively. As a result, the differential signal behaves like dispersion-like curve, as shown in Fig.2. The zero point of the differential signal matchesthe center of the absorption curve, which is suitable for frequency locking. In addition, the laser frequency can be locked anywhere upon the dispersive signal through PID feedback for occasions that require frequency drift from the transition lines. This frequency stabilization method has a large capture range as well as steep slope around locking point. In other words, the system can tolerate a large frequency excursion and draw the frequency back again to the desired lock-point. It is less sensitive to the unexpected changes of laser frequency.

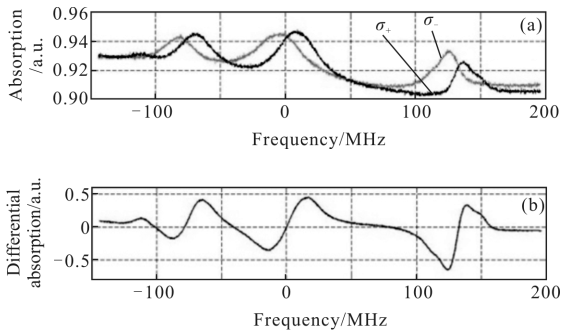


Fig.2 Generation of Doppler-free DAVLL signal(color online)

2 Experimental setup details

2.1 Dispersive curves generation experiments

For the interferometer system, the cold atom experiment requires laser linewidth in MHz magnitude or so. External cavities as well as grating feedback ECDLsare commonly introduced to increase the effective length of the cavity resonator, thus decreasing the laser linewidth in a large scale. Under this consideration, two Toptica DL Pro ECDLs at 780 nm with Littrow configuration are employed to serve as light source in our experiment as shown in Fig.3.

The module Digilock 110 is applied to provide feedback and lock the laser. By changing the voltage to drive the PZT blazed grating moving, the external cavity length is adjusted so as to change the wavelength and frequency longitudinal mode output. Frequency is also affected by the diode drive current and temperature. DL pro provides the temperature as well as current module to eliminate the impact of current temperature and current drift on the laser frequency. It provides three feedback interfaces, i.e. PZT, mod DC and mod AC, in order to control the laser frequency. Typically the laser frequency could be continuously tuned by a range of about 30 GHz without a mode hop.

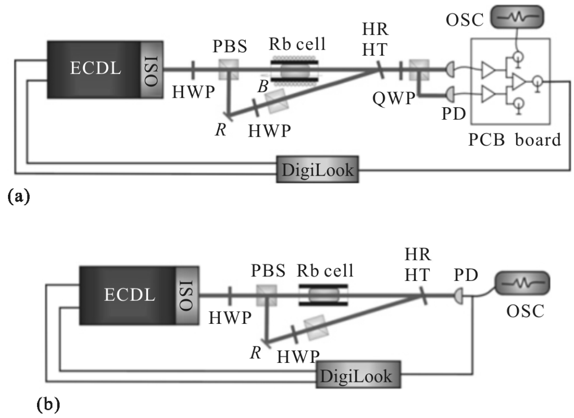


Fig.3 (a) Diagram of the optical design for the Doppler-free DAVLL setup. (HWP=half-wave plate, R=reflector, PBS=polarizing beamsplitter, HRHT=transflective mirror, PD=photodiode, OSC=oscilloscope)(b) Diagram of the optical design for the SAS setup

The Rb saturated hyperfine structure as known is employed to calibrated frequency. The laser frequency is supposed to be scanned around D2 transition lines in order to get the absorption curve. Two sets of frequency lock system are built, the laser is frequency-locked to the transitions $5^2S_{1/2}$, F_g=2→$5^2P_{3/2}$, F_g=1 & 3 (average of F_g=1 and F_g=3) and $5^2S_{1/2}$, F_g=2→$5^2P_{3/2}$, F_g=2 & 3 of ^{87}Rb atom separately with a frequency differential of 78.5 MHz (as shown in Chapter 4).

In both experiments, a transflective mirror is applied to ensure the counter-propagating probe beam and pump beam, so as to guarantee that the atoms interact with both laser beam. In order to obtain significant sharper signal, the power of the probe and pump beam should be adjusted in accordance with a certain proportion which is accomplished by a set of half-wave plate and PBS. For actual operation, we set one of the ECDL′s laser (probe beam) power of (0.461±0.02)mW and the other (pump beam) (1.298±0.01)mW at the relative proportion about 1:3. With the same ratio, the other system′s probe power is (0.866±0.02)mW with pump power (2.576±0.01)mW. These values are chosen to optimize the DAVLL or SAS signal so as to provide a high resolution spectroscopy.

In SAS experiment, the probe beam signal detected by photo detector is sent to Digilock module for further processing described in Chapter 3 and frequency lock eventually. In Doppler free DAVLL experiment, the probe beam can be decomposed into two circularly polarized beams. After passing a quarter-wave plate, those circularly polarized beams change into two orthogonal linearly polarize beams and are detected respectively by self-made photodetectors. The error signal is obtained by photodetector which is made of photodiode and homemade differential circuit illustrated in Fig.4. By comparing with the commercial detector used in SAS experiment, we assemble a Si pin photodiode S5821 with differential amplifier circuit whose DC gain is designed at 54 dB. Coaxial cable is used after S5821 to improve signal′s anti-jamming performance.

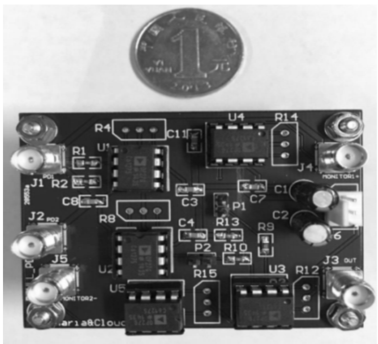


Fig.4 Printed differential amplifier circuit board of homemade photodiode

Typically photo sensitivity for spectral response at 780 nm is 0.52 A/W and a 1 mW beam generates an 8.09 V signal, thus making the peak to peak value of differential curve is sensitive to a weak laser power.

To generate the magnetic field for the Doppler-free DAVLL partand induce a dichroism related to the error signal, the solenoid unit is coiled up outside the room temperature vapor cell of rubidium. Its size is described as followed: 5 cm long and deep, 0.41 mm in diameter, 300 laps and 4 layers. The peak to peak amplitude and the slope of differential signal changes in different magnetic field related to the quality of dispersive curve. We experimentally optimize the current positive correlated to the magnetic field from a value of 0.1 A, 0.15 A and 0.2 A in order to achieve optimum signal. The intensity of the magnetic field is calculated as 118 Gauss, 178 Gauss and 236 Gauss separately. They cause different frequency stability which will be described in Chapter 4. The SAS and DAVLL dispersive curve get generated which can be observed on DigiLock front panel introduced in next paragraph with screenshot showed in Fig.5.

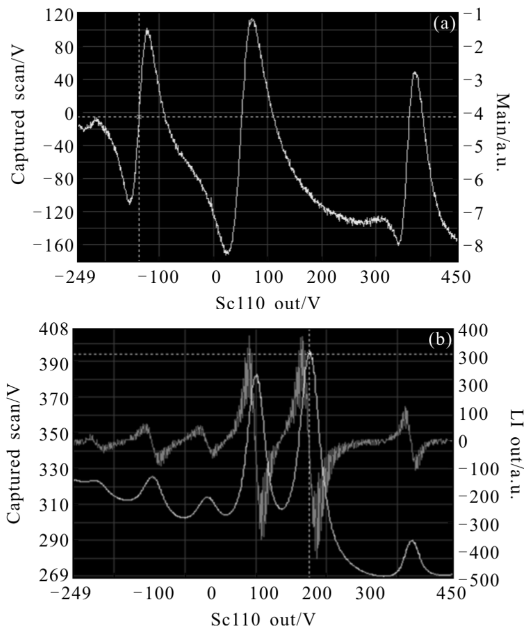
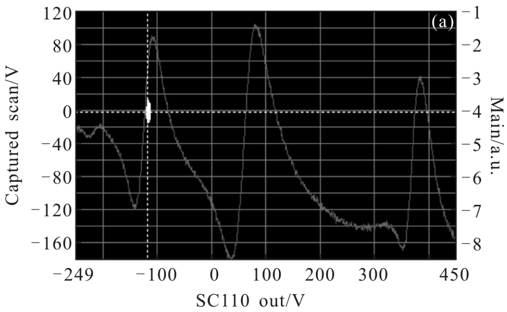


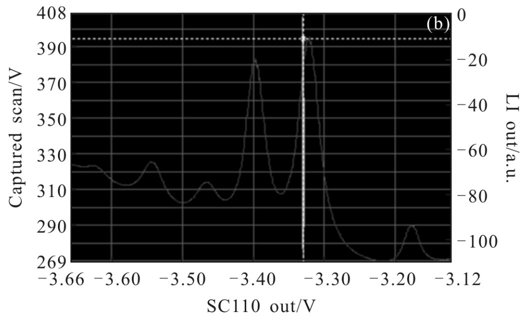
Fig.5 (a) curve is the differential signal with the zero-crossing in CO13(cross-over transition of upper hyperfine-level F'=1 and F'=3) and CO23(cross-over transition of upper hyperfine-level F'=2 and F'=3) in DAVLL experiment. (b) Lower shape is the saturated absorption spectroscopy in ^{87}Rb D2 transition and the upper one is the error signal

2.2 Frequency lock-in experiment details

For the lock-in part, the DigiLock module based on LabVIEW program plays a significant role in frequency stabilization. The core control circuit is digitized and the FPGA (field-programmable gate array) is adopted as the core processing unit connected with external IOs via ADC and DAC. FPGA integrates DDS (direct digital synthesizer) module, a scanning signal generation module, Lock-in amplifier, error signal generation module, Pound-Drever-Hall error signal generation module and two-way PID module. PID1 is designed for current feedback, providing fast jitter feedback. PID2 is for PZT feedback and suitable to realize a slow drift feedback control. DigiLock module consists of several parts to provide real time frequency feedback to the PZT and mod DC mentioned above. Feedback coefficient for PZT port is obtained at 415.8 MHz/V and for DC port is 319 MHz/V. It is superimposed the parameters set of amplitude and offset separately both in the digital software front panel and the controller for the laser. The PID parameters can be adjusted easily in the setting interface. Before setting GPID parameters, we simulate the differential curve using the simulation module on front panel to obtain adequate loop phase margin and stability. We get the parameters for PID1 with G20, P1000, I50, D0 and PID2 with G25, P8000, I50, D0. The differential curve′s peak to peak voltage is supposed to be less than 2 V to avoid damaging the DigiLock module. Locking to slope method can lock lasers′ frequency at different zero crossing points corresponding to the cross peaks of CO13 and CO23. Consequently, the frequency is calibrated against the known Rb hyperfine structure through locking the error signal which is graphically showed in Fig.6.



(a) Corresponding to Doppler-free DAVLL



(b) Corresponding to SAS

Fig.6 Error signal after locked. Curve is the latest position of dispersive signal and point is the feedback signal

After being locked, the yellow feedback curve changes around the locking point. It suggests that the laser frequency has been locked from the side.

3 Characterization of frequency stability

Our absolute gravimeter aims at reaching a resolution of 10 Gal(1 Gal=10^{-2} m/s^2=10^{-3} g). Therefore the relative frequency stability of $\Delta f/f$ has to be an order of magnitude smaller than it (10^{-9}). To evaluate frequency stability, two lasers are frequency locked to the same Doppler-broadened feature with different optical offsets, approximately 78.5 MHz apart adopted. In addition to the beam applied to stabilize laser frequency, other orthogonal polarized beams are combined at a PBS to fulfill the beat note experiment illustrated shown in Fig.7. The crossing angle of the reference pump and probe beams was (1±0.1)mrad.

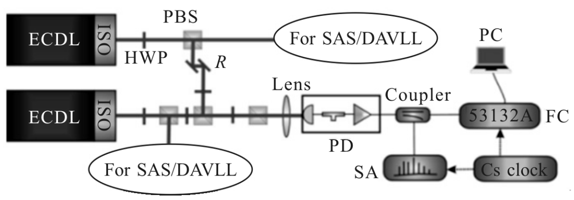


Fig.7 Scheme of the optical design for the beat note frequency experiment (SA=spectrum analyzer, FC=frequency counter)

The beat photodiode consists of ultra-fast photodiode (G4176, 0−11.3 GHz), bias-tee (ZX85−12 G+) and power amplifier (ZFL−500LN+). Corresponding to the difference between the two laser frequencies, the beat signal would be fed into a high speed counter (agilent 53132A) for further data processing. The data acquisition program is written in LabVIEW language. A GPIB convertor associates the program with the counter. Through enabling NI-VISA passport in NI-MAX, the third-party GPIB convertor is able to access NI-VISA. Afterwards, the data acquisition program record the beat note frequency with given gate times shown on the counter synchronously and continuously. It acquires one frequency point every second and 3 000 data is collected for data analysis. Figure 8 shows the beat frequency gathered by counter with the lasers free run as well as locked to F_g=2→F_g=1 & 3 and F_g=2→F_g= 2 & 3 of D2 transition of ^{87}Rb through SAS method. As seen from the chart, the frequency jitter in 3 000 s reduces sharply from 12 MHz to several hundred kHz, which means the laser has obtained high frequency stability.

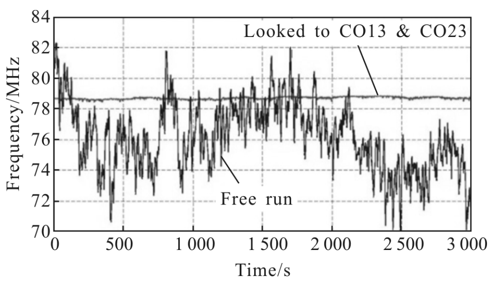


Fig.8 Frequency comparison between free running and locked state (color online)

By changing the coil current from 0.1 A to 0.2 A, we set the magnetic field intensity from 118 Gauss to 236 Gauss equivalently which is illustrated in Fig.9. We find that slope of the dispersive curve relates to the effect of frequency stabilization. With the choice of SAS method, the frequency difference of two lasers changes within the range of 1 MHz. However, the Doppler-free DAVLL method chooses to lock the laser frequency to a certain point of the slope, whose voltage is susceptible to laser intensity jitter, thus making it harder to lock to the exact transition frequency. It does better in short-term jitter when the magnetic field intensity is 178 Gauss. In our SAS experiment, the locking point corresponds to the peak of saturated absorption spectrum strictly.

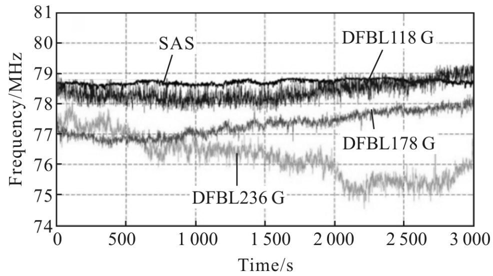


Fig.9 Change of frequency when the laser is locked through different methods(color online)

Further, we calculate the Allan deviation of the data collected above for accurate evaluation. According to the composition of the uncertainty, the relative stability of the beat frequency signal and the relative stability of the two laser beams satisfy the equation(6):

$$\sigma_{l_1}=\sigma_{l_2}=\frac{\sqrt{2}}{2}\sigma_{\text{beat}} \tag{6}$$

Figure 10 shows the results of the frequency stability of different methods. With restructuring of optical path and excellent error signal, we obtain a frequency stability of 4.21×10^{-11} within a 10 s averaging time, which is a relatively excellent result to our knowledge. Of the three configurations of Doppler-free DAVLL, the 178 Gauss one gets the best performance as well as mentioned above, whose frequency stability reaches 8.18×10^{-11} within a 10 s averaging time.

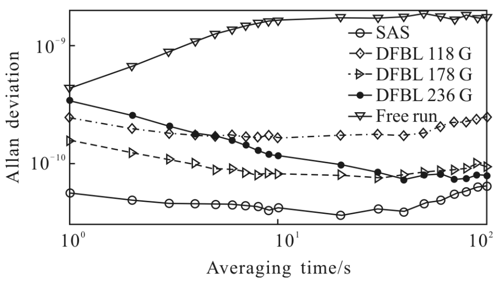


Fig.10 Frequency stability of different lock methods (color online)

The lineshape of the beat note signal can be observed with spectrum analyzer and coupler, from which the linewidth can also be measured. For SAS method, the modulation signal adds extra frequency jitter The lineshape of the beat note signal can be observed with spectrum analyzer and coupler, from which the linewidth can also be measured. For SAS method, the modulation signal adds extra frequency jitter to the laser. Free of modulation, the DAVLL provides narrower linewidth than SAS. From our measurement results, itshow that the linewidth of Doppler-free DAVLL method is 6 times better than SAS, of which linewidth are 0.8 MHz and 5.4 MHz respectively.

4 A comparison between SAS and Doppler-free DAVLL

In this chapter, a comprehensive comparison between the traditional SAS and Doppler-free DAVLL frequency stabilization methods is presented.

In a longitudinal comparison, the Doppler-free DAVLL is built on the basis of the saturated absorption spectroscopy. In the Doppler-free DAVLL method, the magnetic field is introduced into the system to derive the error signals as well as to avoid modulation in SAS method.

In a horizontal comparison, the principle of SAS is in use of the peak-locking method. The frequency of the locked peak will not be affected even if there is a power fluctuation in laser beam. The zero crossing point is exactly corresponding to the desire of the transition frequency, that′s why the beat note frequency is approximately at 78.5 MHz in SAS experiment. By contrast, there exists a deviation from the ideal frequency in DAVLL measurement. However, SAS provide a higher signal-to-noise rate at the cost of direct modulation and incremental linewidth.

The side-locking on a saturation spectroscopy signal is applied in the Doppler-free DAVLL. The Doppler-free DAVLL background is subtracted from the two Doppler-free DAVLL signals. During the locking process, the differential curve shakes up and down in a small scale obviously. The fluctuations of the two photodetectors′ beam intensity can cause a zero point off the center of the error curve. It can result in changes of the relative locking point position between ideal and practical. With its intrinsic disadvantage, side-locking method is sensitive to the fluctuation of the signal such as a current jitter during the process of generating the magnetic field. However, side-locking is able of locking the laser at any frequency around theresonance frequency, which is significant in application such as cooling laser that needs to be about 15 MHz negative detuning to theresonance frequency. What′s more, the DAVLL has a large re-capture range of the size of the Doppler-broadened linewidth. It offers an excellent signal with a sharp slope of dispersive signal for more transitions and it is sensitive to weak lines. The frequency measurement is permitted even with weak laser sources. Furthermore, Doppler-free DAVLL requires fewer optical and electronics components. It is easily compact and inexpensive which meets the integration demand of our gravimeter.

5 Conclusion

In summary, we demonstrate two active frequency stabilization methods both in theory and experiment. Doppler-free DAVLL can be considered as derivative of saturated absorption spectroscopy while shows greater benefits than the former method. Digital frequency-feedback module is introduced to lock the laser frequency to the Doppler-broadened D2 transition of ^{87}Rb and digital data acquisition is programmed to evaluate the frequency stability. After 3000 s acquisition, a frequency fluctuation of 16.2 kHz and 31.4 kHz after locked by SAS and Doppler-free DAVLL were calculated respectively, corresponding to a frequency stability of 4.21×10^{-11} and 8.18×10^{-11} for averaging time of 10 s. Both of the frequency stabilization methods are able to meet the requirements in atom interferometer gravimeter. Alternatively, the Doppler-free DAVLL is free of inducing modulation and generally imply less optical and electric component. Further improvements in the DAVLL scale can be achieved using compact and smaller components which are perfectly in tune with the miniaturization and modularization of the optical path and circuits for our interferometer system.

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