# Infrared and Laser Engineering

# 3D shape measurement method combining sinusoidal pulse width modulation fringe with phase coding fringe

Li Biao, Wu Haitao, Zhang Jiancheng, Fu Yanjun

(Key Laboratory of Nondestructive Testing (Ministry of Education), Nanchang Hangkong University, Nanchang 330063, China)

Abstract: A 3D shape measurement method combining sinusoidal pulse width modulation fringe with phase coding fringe was proposed. Two kinds of coding fringes were projected onto the object with defocus. Using a phase-shift algorithm, the wrapped phase was obtained by the sinusoidal pulse width modulation fringe, the fringe order was got by the phase coding fringe. Then, the three-dimensional shape of the measurement object can be restored. The experimental results demonstrate that the proposed method has two merits: (1) It can decrease the measurement error because of filtering the harmonics with slight defocus; (2) It can measure the different contrast object because it is based on phase code.

Key words: 3D shape measurement; defocusing; sinusoidal pulse width modulation fringe; phase coding fringe

**CLC number:** TB96 Document code: A **DOI:** 10.3788/IRLA201645.0617006

## 正弦脉冲宽度调制条纹结合相位编码条纹的三维测量方法

李 彪,吴海涛,张建成,伏燕军

(南昌航空大学 无损检测教育部重点实验室,江西 南昌 330063)

摘 要:提出了正弦脉宽调制和相位编码结合的一种三维形貌测量方法。这种方法离焦投影编码的 两种条纹到被测对象上,使用相移算法,由正弦脉冲宽度调制条纹得到截断相位,由相位编码条纹解 码得到条纹级次,从而恢复测量对象的三维形貌。实验结果证明了该方法有两个优点:(1) 轻度离焦 滤除了谐波从而能降低测量误差;(2)基于相位的编码方式能测量表面反射率不一物体。

关键词: 三维测量; 离焦; 正弦脉冲宽度调制条纹; 相位编码条纹

收稿日期:2015-10-05; 修订日期:2015-11-03

基金项目:国家自然科学基金(51365045,61462063);航空科学基金(2013ZE56013);上海航天科技创新基金(SAST2015046)

作者简介: 李彪(1988-), 男, 硕士生, 主要从事光电三维测量方面的研究。Email: jjudianzia0831@163.com

导师简介:伏燕军(1974-),男,教授,博士,主要从事光电成像、三维测量及计算机视觉等方面的研究。Email:fyjpkh@sina.com.cn

#### 0 Introduction

Three-dimensional (3D) shape measurement has been extensively studied for application in medical diagnosis, machine vision, on-line detection, reverse engineering, automatic control, 3D model of complex object, engineering design, and other fields [1-2]. Along with the evolution of digital image processing and computer technique, 3D shape measurement achieves great development [3-4].

The traditional digital fringe projection system utilizes the focusing projector to generate sinusoidal fringe patterns. Since it is usually 8-bit image, the measurement speed is typically limited to 120 Hz. As well as, the nonlinearity of the projector will induce measurement error. To conquer the two problems, the squared binary defocusing technique is proposed [5]. However, the deep defocusing based on squared binary fringe leads to a smaller depth measurement range. The pulse width modulation (PWM) technique was introduced to develop the binary defocusing technique. Ayubi et al. proposed a technique called sinusoidal pulse width modulation (SPWM) [6]. This SPWM fringe with slight defocus can easily eliminate the high-order harmonics. Wang et al. proposed optimal pulse width modulation (OPWM) to further improve the defocusing technique by selectively eliminating undesired harmonics<sup>[7]</sup>. Further more, Zuo et al. proposed a new sinusoidal fringe generation technique called TPWM and implemented it into dynamic scenes[8].

The combined 3D shape measurement method that increases additional fringes to assist the phase unwrapping includes Gray code plus phase-shifting method [9], dual-frequency pattern method [10], speckle-embedded method [11] and so on. However, the codeword of Gray code plus phase-shifting method is limited to 2<sup>m</sup> (here m is the number of Gray code patterns). In addition, the codeword identified by image intensity is less robust for measuring high contrast surfaces. As

the phase is better immune to surface contrast variations, ambient light, and camera noises comparing with intensity, Wang et al. proposed a novel absolute phase recovery technique with phase coding [12]. It not only produces more codewords but also receives more robust measurement result for different intensity fringes. Basing on the above foundation, Zheng et al. put forward a phase coding method for absolute phase recovery with a large number of codewords [13]. Zhou et al. proposed an improved stair phase encoding method for absolute phase retrieval [14].

In this paper, the 3D shape measurement method combining sinusoidal pulse width modulation fringe with phase coding fringe is presented. We carried out contrast experiments to verify performance of the method. The sinusoidal pulse proposed width modulation fringe is used to obtain the wrapped phase and the phase coding fringe is used to obtain the fringe order. Then, the phase information can be gained using the phase unwrapping algorithm. At last, the height information can be reconstructed using phase-height relationship after the measurement system parameters are calibrated<sup>[15]</sup>.

#### 1 Principle

#### 1.1 Measurement system

Measurement system is shown in Fig.1. A and B

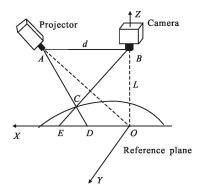


Fig.1 Measurement system

are the optic centers of projector and CCD camera, respectively. The distance between A and B is d. Projector obliquely projects fringe patterns and CCD

camera is perpendicular to reference plane. The optical axes of them intersect at point O. The distance between B and reference plane is L. The height of test object can be calculated as follows<sup>[16]</sup>:

$$h = \frac{L\Delta\phi}{2\pi f_0 d + \Delta\phi} \tag{1}$$

Here,  $f_0$  is the spatial frequency of fringe on reference plane,  $\Delta \phi$  is the continuous phase difference between test object and reference plane.

#### 1.2 Sinusoidal pulse width modulation fringe

Ayubi et al. first introduced the sinusoidal pulse width modulation technique in 3D shape measurement field. It evolves from the pulse width modulation which is well-known in electrical engineering. To design a good SPWM pattern, the frequency  $f_T$  of triangular wave we set is ten times than the frequency  $f_S$  of the sinusoidal wave, that is  $f_T = 10 \ f_S$ . The generating process can be depicted as follows:

(1) Code 10 periodical triangular waves based on one periodical sinusoidal wave, as the solid line shows in Fig.2(a).

$$F(1,j) = A + \operatorname{sawtooth}(10*j/T, A)/2 \tag{2}$$

Here, F(1,j) is the pixel intensity on line one column j, A is the amplitude of triangular wave, and T is the step length.

(2) Produce sinusoidal wave with four-step phase shifting, as dotted line shows in Fig.2(a).

 $Q_n(1,j)=A+A\sin(j/T+2\pi(n-1)/N)$ , n=1,2,3,4 (3) Here,  $Q_n$  is the intensity of sinusoidal wave, N is the number of total phase-shifting step, n is the current phase-shifting step, and T is the step length.

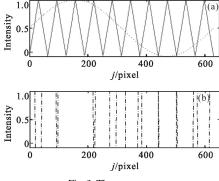


Fig.2 Three waves

(3) Comparing the two waves, as the dotted line

shows in Fig.2(b).

$$I_n(1,j) = \begin{cases} 1, & F < Q_n \\ 0, & F > Q_n \end{cases}$$
 (4)

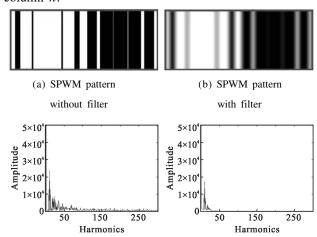
Here,  $I_n$  is the intensity of SPWM wave.

(4) Convert the one-dimension wave to two-dimension picture, as shown in Fig.3(a).

$$I_n(i,j) = R(I_n(1,j), m, n)$$

$$\tag{5}$$

Here, R is the function of repeating  $I_n$  to line m and column n.



(c) Original frequency spectrum (d) Processed frequency spectrum

Fig.3 SPWM pattern in one period

As SPWM pattern can filter the harmonics in slight defocusing, we simulated SPWM pattern by the Gaussian filter, as shown in Fig.3(b). The size of the Gaussian filter is 29\*9. At the same time, we extracted the frequency spectrum of 16 fringe cycles in one cross. Figure 3(c) is the original frequency spectrum. Figure 3(d) is the processed frequency spectrum using filter.

#### 1.3 Phase coding fringe

As the phase is less sensitive to surface contrast variations, ambient light and camera noises, the phase information is more robust than the intensity information. Wang et al. proposed that the phase can be used as the codeword to retrieve the continuous phase. The method makes it possible to test object with different surface contrast. We depict the generating process of phase coding fringe with 16 cycles.

(1) Embed the codeword into the phase with a

stair phase form, as shown in Fig.4(a):

$$b(i,j) = -3.13 + [j/p] \cdot \frac{2*3.13}{N} \tag{6}$$

Here, [j/p] is the ideal fringe order obtained by rounding operation,  $\frac{2*3.14}{N}$  means the stair height,

N(=16) is the total number of fringe cycles, and p is the fringe pitch.

(2) Code the stair phase b(x,y) into four-step phase-shifting fringe intensity, as shown in Fig.4(b):

 $I_n(i,j)=A+A\sin(b(i,j)+2\pi(n-1)/N), n=1,2,3,4$  (7) Here, A is the amplitude of fringe intensity.

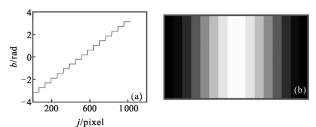


Fig.4 Codeword (a) and the phase coding fringe (b)

#### 1.4 Phase unwrapped algorithm

The sinusoidal pulse width modulation fringe is used to obtain the wrapped phase and the phase coding fringe is used to obtain the fringe order. Then, the phase information can be gained using the phase unwrapping algorithm. The process can be decomposed as:

(1) For the SPWM fringe and the phase coding fringe, the wrapped phases are solved as:

$$\varphi(i,j) = \arctan\left[\frac{I_4(i,j) - I_2(i,j)}{I_3(i,j) - I_1(i,j)}\right]$$
(8)

(2) For the phase coding fringe, the fringe order should be implemented:

$$k(i,j) = \operatorname{round} \left[ N \cdot \frac{\varphi(i,j) - \min[\varphi(i,j)]}{\max[\varphi(i,j)] - \min[\varphi(i,j)]} \right]$$
(9)

Here, round (•) means the integer operation, max

- $(\cdot)$  and min  $(\cdot)$  represent the maximum and minimum values of the  $\varphi(i,j)$ , respectively. N indicates the cycle number of phase coding fringe.
  - (3) The continuous phase can be expressed as:

$$\phi(i,j) = \varphi(i,j) + 2\pi \cdot k(i,j) \tag{10}$$

Here,  $\varphi$  is the wrapped phase of SPWM fringe, and k is the fringe order.

(4) The continuous phase difference is as follows:

$$\Delta \varphi(i,j) = \phi_1(i,j) - \phi_2(i,j) \tag{11}$$

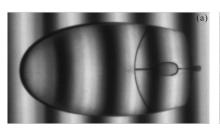
Here,  $\phi_1$  is the continuous phase with object, and  $\phi_2$  is the continuous phase with reference.

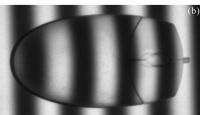
## 2 Experiments

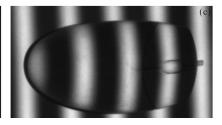
The measurement system is composed by a digital projector (Samsung) with a resolution of 800×600, a CCD camera (DH –HV1351UM) with a resolution of 1 280×1 024 and a computer. Before the experiment, measurement system should be conducted parallel calibration, perpendicular calibration and system parameters calibration. The measurements of a mouse and three standard step blocks were used to prove the proposed method. The two sets of fringes were projected onto the reference plane and the object, respectively.

# 2.1 Comparing SPWM plus phase coding method, binary defocusing method, sinusoidal method

The SPWM fringe and binary fringe were projected in a same slight defocus. The sinusoidal fringe was generated by a focused projector. Phase-shifting fringe patterns on the object using three methods are shown in Figs.5(a)-5(f). Figures 5(g)-5(i) show the corresponding 3D results. The results imply that the proposed SPWM plus phase coding method overmatches the others.







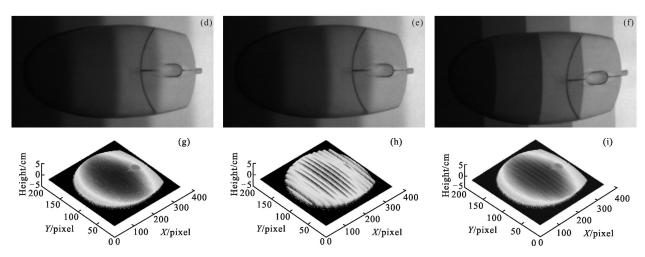


Fig. 5 One of four-step phase-shifting fringe patterns on the object using three methods: (a) SPWM fringe, (b) binary defocusing fringe, (c) sinusoidal fringe, and (d)-(f) the phase coding fringes; (g)-(i) the corresponding 3D results

#### 2.2 Measuring the different contrast object

In this paper, the fringe intensity was changed to simulate the different contrast object. Figures 6(a)-6(f) show the phase-shifting fringe patterns on the object

using three different intensities. The corresponding 3D results are shown in Figs. 6 (g) -6 (i). The results mean that there are no obvious differences among the three different intensities.

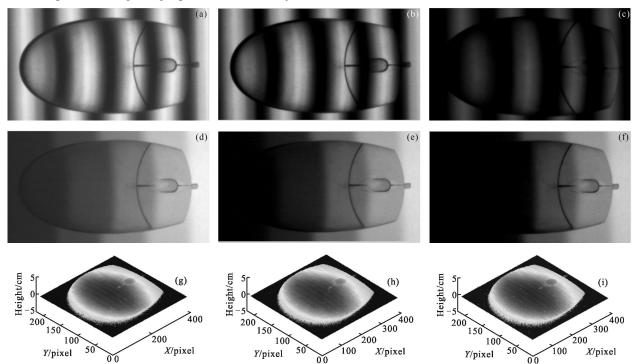


Fig.6 One of four-step phase-shifting fringe patterns on the object using three different intensities: (a) and (d) with high intensity, (b) and (e) with middle intensity, (c) and (f) with low intensity; (g)-(i) the corresponding 3D results

### 2.3 Measuring three standard step blocks

Three standard step blocks (the height of each step is 11 mm) was measured by the proposed method to verify the measurement accuracy. Figures 7(a) and 7 (b) are the blocks modulated by the SPWM fringe

and phase coding fringe, respectively. Figure 7 (c) is the reconstructed 3D shape and Figure 7(d) shows the step height distribution in the 175th row. Table 1 lists experimental results on the blocks. The maximum absolute error is 0.103 mm and the maximum standard

deviation is 0.085 mm.

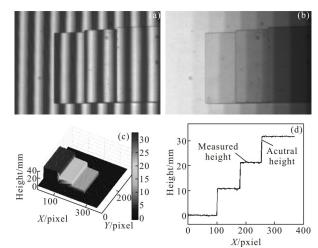


Fig. 7 Blocks modulated by (a) SPWM fringe and (b) phase coding fringe; (c) the reconstructed 3D shape;(d) step height distribution in the 175th row

Tab.1 Experimental results on standard step blocks (Unit: mm)

Actual step height	Absolute error	Standard deviation
11	0.103	0.085
22	0.098	0.079
33	0.092	0.076

#### 3 Conclusion

With regard to the high-order harmonics and the different contrast object, the 3D shape measurement method combining sinusoidal pulse width modulation fringe with phase coding fringe is proposed in this paper. The detailed explanations about the principle are given. The experiments certify that this method has two merits: (1) It can decrease the measurement error because of filtering the harmonics with slight defocus; (2) It can measure the different contrast object because it is based on phase code.

#### **References:**

- [1] Wang Ying, Zhang Rui. In-pipe surface circular structured light 3D vision inspection system [J]. *Infrared and Laser Engineering*, 2014, 43(3): 891–896.
- [2] Li Xudong, Sun Jian, Jiang Hongzhi, et al. 3D measurement for complex castings [J]. Optics and Precision Engineering,

2014, 22(4): 884-889.

- [3] Wang Jianli, Liu Xinyue. Concept and development of smart of smart optics [J]. *Chinese Optics*, 2013, 6(4): 437–448.
- [4] Wang Shuzhen, Xie Tiebang, Chang Suping. Combined profilometer for ultra-precision surface topography [J]. Optics and Precision Engineering, 2011, 19(4): 828-835.
- [5] Lei Shuangyan, Zhang Song. Flexible 3-D shape measurement using projector defocusing [J]. *Optics Letters*, 2009, 34(20): 3080-3082.
- [6] Gaston A Ayubi, Jaime A Ayubi, Matias Di Martino, et al. Pulse-width modulation in defocused three-dimensional fringe projection[J]. *Optics Letters*, 2010, 35(21): 3682–3684.
- [7] Wang Yajun, Zhang Song. Optimal pulse width modulation for sinusoidal fringe generation with projector defocusing [J]. Optics Letters, 2010, 35(24): 4121–4123.
- [8] Zuo Chao, Chen Qian, Gu Guohua, et al. High-speed threedimensional shape measurement for dynamic scenes using bifrequency tripolar pulse-width-modulation fringe projection [J]. Optics and Lasers in Engineering, 2013, 51(8): 953–960.
- [9] Wang Changbo, Xie Minghong. Binocular three-dimention reconstruction combined with gray coding and phase-shift [J]. Computer Engineering, 2013, 39(5): 178–182.
- [10] Liu Kai, Wang Yongchang, Lau Daniel L, et al. Dual-frequency pattern scheme for high-speed 3 -D shape measurement [J]. Optics Express, 2010, 18(5): 5229-5244.
- [11] Feng Shijie, Chen Qian, Zuo Chao. Graphics processing unit-assisted real-time three-dimensional measurement using speckle-embedded fringe [J]. *Applied Optics*, 2015, 54(22): 6865–6873.
- [12] Wang Yajun, Zhang Song. Novel phase-coding method for absolute phase retrieval [J]. *Optics Letters*, 2012, 37 (11): 2067–2069.
- [13] Zheng Dongliang, Da Feipeng. Phase coding method for absolute phase retrieval with a large number of codewords [J]. Optics Express, 2012, 20(22): 24139–24150.
- [14] Zhou Canlin, Liu Tongchuan, Si Shuchun, et al. An improved stair phase encoding method for absolute phase retrieval [J]. *Optics and Lasers in Engineering*, 2015, 66: 269–278.
- [15] An Dong, Chen Li, Ding Yifei, et al. Optical system model and calibration of grating projection phase method [J]. Chinese Optics, 2015, 8(2): 248–254.
- [16] Jia Tong, Zhou Zhongxuan, Gao Haihong, et al. Depth measurement based on infrared coded structured light [J]. *Infrared and Laser Engineering*, 2015, 44(5): 1628–1632.