Star centroid location based on information entropy and minimum energy difference

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Abstract: Celestial navigation technology has entered a new stage of development with the development of electronic technology and computer technology. Otherwise it has been widely used in satellite, space shuttle, long-range missiles and other spacecraft. Star centroid location for celestial navigation is the key technologies. Due to the strong sky background, near-infrared stellar images which are taken during the day have the low signal noise ratio. First, this paper adopts the method of information entropy to make analysis the stellar images energy distribution. And then we proposed a method for star target centroid located based on multi-step minimum energy difference. This method uses the linear superposition to narrow the centroid area, using the symmetry of the stellar energy distribution to obtain the centroid when the minimum difference appears. We test the modal on natural pictures to verify the accuracy. Experiments show that it has good effect to calculate the centroid with low SNR conditions.

Key words: celestial navigation;   minimum energy difference;   information entropy;   centroid location

基于信息熵和最小能量差的星点质心定位

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摘 要： 天文导航技术已经进入一个新的发展阶段，随着电子技术和计算机技术的发展，它已广泛应用于卫星、航天飞机、远程导弹和其他航天器。星点质心位置是天文导航技术的关键。由于白天强烈天空背景的干扰，拍摄的近红外恒星图像信噪比很低。首先，采用信息熵的方法来进行分析恒星的图像能量分布。然后，提出了基于多级的最小能量差星点目标质心定位的方法。此方法使用线性叠加，以缩小质心区域范围，使用恒星能量分布的对称性出现的最小差值时，得到星点质心坐标。通过测试自然拍摄的图像来验证算法的准确性。实验表明：该方法具有良好的低信噪比条件下的质心定位精度。

关键词： 天文导航；   最小能量差；   信息熵；   质心定位

0　Introduction

　　The process of using the stars for celestial navigation can be divided into four steps: star image preprocessing, star centroid calculation, star recognition and attitude determination. The positioning accuracy is directly influenced by the star centroid calculation accuracy which has great significance for the algorithm of celestial navigation. Generally, star centroid localization[1] has two categories: one is based on the gray level and the other is based on the edge. The actual gray distribution of star target is approximately in accordance with the two−dimensional Gaussian Distribution[2]. It is appropriate to use the gray level method to centroid localization for the reason that the infrared images have shortcomings of low SNR and poor contrast[3], edge blurring and low frequency components which makes up the main background[4].

　　According to different processing methods, sub−pixel's centroid localization technology generally can be divided into interpolation technology and fitting technology[5]. Interpolation technique is easy to implement, but it has strong dependence on the weak signal of both sides of peak, and the method has poor ability to resist interference of noise; On the other side fitting technology has complicated process, but the precision is higher. In actual application, we usually use interpolation technology as the method of star centroid localization, literature [3] points out that positioning error of star centroid mainly depends on system and random error. It also pointed out that the choice of target area window has great influence for centroid localization accuracy. Literature[6] concluded that linear interpolation is better through analysis of the influence of different interpolation methods to the precision of centroid localization, and it pointed out that the centroid localization accuracy can be improved to some extent by interpolation 2−5 points, which means unlimited interpolation is unnecessary. In this paper, we put forward a location method with noise immunity for star target centroids based on multi−step minimum energy difference after the analysis of the star energy distribution.

1　Minimum energy difference

　　Star energy distribution satisfies the Gaussian distribution whether the star image is taken during the daytime or at night. Therefore, the basic idea of minimum energy difference theory is: the distribution of star energy is symmetrical about the center according to the point of view of energy analysis, the closer the distance from the center of mass, the greater the energy [7−8]. When we use infrared camera to get star images, the pixel gray value in image is proportional to the energy of objects. In determining the star center, we can assume a random point (x, y) is the center pixel point, and if the energy distribution of star about this point is not symmetrical, we will know this point is not the centroid position. And then there will energy difference in some direction (usually transverse or longitudinal), we can determine whether the pixel point is the centroid position by comparing the multi−step (to reduce noise interference) energy difference in that direction. When the energy difference reaches minimum, we get the centroid position. Embodied as following:

$$\Delta I_x = \left| \sum_{i=1/N}^{l_{left}} I(x-i,y) - \sum_{i=1/N}^{l_{right}} I(x+i,y) \right| \tag{1}$$

$$\Delta I_y = \left| \sum_{i=1/N}^{l_{up}} I(x,y+i) - \sum_{i=1/N}^{l_{down}} I(x,y-i) \right| \tag{2}$$

　　Where $I(x,y)$ is the energy value on the point (x, y), N is the number of interpolation points, ΔI_x is the L−step energy difference value about x pixel point of transverse direction when assume x pixel point as the centroid position, ΔI_y is the L−step energy difference value about y pixel point of longitudinal direction when assume y pixel point as the centroid position l_{up}、l_{down}、l_{left}、l_{right}、 respectively, is the step length of four direction: up, down, left and right, their values depend on the size of centroid regions and the number of interpolation points, and $|l_{up}| = |l_{down}|$, $|l_{left}| = |l_{right}|$. The determination of minimum ΔI_x ΔI_y can determinate centroid position (x, y).

2　Sub−pixel centroid calculation

　　In ideal optical system conditions, the stellar imaging

point is less than one pixel, so centroid position can not accurately be calculated to the sub−pixel level through algorithm. As a result of many factors such as optical difference, the influence of atmospheric transmission, star image is generally a dispersion spot whose gray distribution obeys the Gaussian distribution. We can make a linear interpolation in the simulated star image. In the process of calculating the centroid position, firstly, if we can get the area of centroid, the calculation will reduce. Secondly, we can calculate the energy difference in that area to get the centroid position of minimum difference. Specific steps are as follows.

The first step: determination of the area of centroid. First of all, add each column pixels' value of the simulated image, take out the biggest one column, and then add each line of pixels' value, take out the biggest one line, take the intersection pixel of biggest one column and line as a center to get a 3*3 size area, in this area we can make the sub−pixel structure which is considered as a core region.

The second step: sub−pixel centroid localization. We can get the centroid in the core region through multi−step minimum energy difference method. Firstly, to determine the sub−pixel centroid position, each pixel point of image should be artificially amplified. Secondly, we can assume that all the pixels take a certain amount of area, and then the amplified pixels will be subdivided according to the demand of actual location accuracy, which means each pixel point can be composed by several smaller uniform points. Here we can divide each point evenly into 4 smaller points[7], the gray value $f(m,n)$ after interpolation at (m,n) is calculated as following[5]:

$$f(m,n)=f(i,j)(1-\alpha)(1-\beta)+f(i+1,j)\alpha(1-\beta)+$$
$$f(i,j+1)(1-\alpha)\beta+f(i+1,j+1)\alpha\beta \quad (3)$$

Where $\alpha=\dfrac{x_m-x_i}{x_{i+1}-x_i}$, $\beta=\dfrac{y_n-y_j}{y_{j+1}-y_j}$,

The calculation as shown in Figure 1, hypothesizes that P_0 (x_0,y_0) as the centroid, l_{right} as the right step length, calculating the sum of L−step pixels' gray scale the same as energy sum on the right side (the left side use the same

calculation method), So we can determine the centroid position by comparing the difference of energy sum in symmetrical direction.
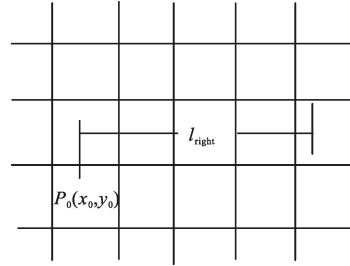

Fig.1 Multi−step sum value of gray

3 Simulation and result analysis

For the real image, the positioning accuracy can't be directly evaluated. In order to validate the effectiveness and accuracy of the proposed method, we can use the simulated image under the condition of different noise to analyze the positioning precision and stability of different centroid localization methods. Using Gaussian distribution to simulate star image, and the size is[20, 20] pixels, with Gaussian white noise, as shown in Figure 2.
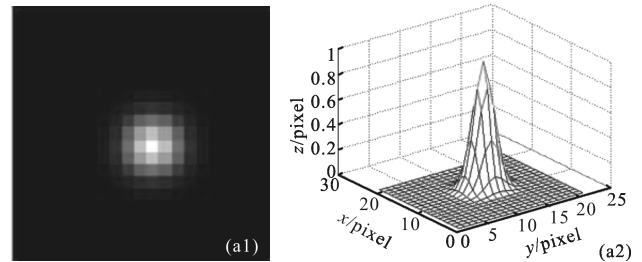
(a) Simulation of single star image
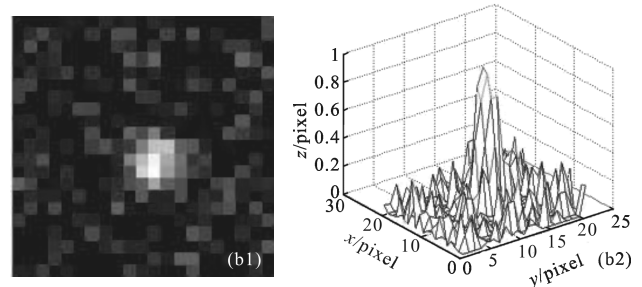
(b) Simulation of single star image with Gaussian noise

Fig.2 Gray distribution of orginal image and Gaussian noise

Take the top left corner as the origin of coordinate system, star center coordinates (11, 11), interpolation

points is 5[9], under different SNR conditions , we respectively use 4 methods to calculate positioning accuracy, they are first moment centroid localization method, the square weighted centroid localization method, Gaussian surface fitting method and the proposed method. We use the mean positioning error of 40 times as the estimated error .The result is shown in Table 1. Obviously, Gaussian fitting has the highest accuracy in high SNR condition, and the method proposed by this paper is suitable for the condition of low SNR.

Tab.1 Calculations of positioning precision with different SNRs

Method	SNR	
	SNR=3	SNR=9
A moment mass center method	0.171	0.041
Square weighted mass center method	0.124	0.092
Gaussian surface fitting method	0.149	0.018
Multi−step minimum energy difference method	0.125	0.025

4　Calculate the sub−pixel centroid for real image

Using Monte Carlo Method simulating star map with the function of Gaussian gray scale diffusion, as shown in Figure 3. The upper left corner is chosen as the pixel origin of coordinate and SAO for the basic catalog. In the simulated star map we added to the gray Gaussian noise with $N(0,12)$ and $N(0, 52)$. Using different algorithms to determine the location of centroid to the star map, as shown in Table 2.
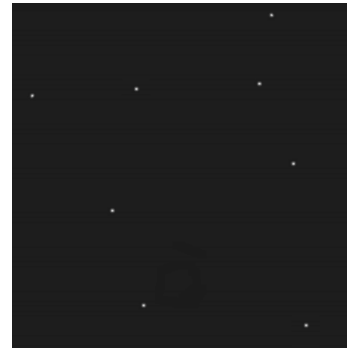

Fig.3 Simulated star map

In Table 2 the star centroid extraction results were given. Meanwhile the x and y coordinate error were shown in Figure 4 and the horizontal axis coordinates representation the 8 stellar number. The average error of moment mass center is 0.032 pixel, Square weighted error is 0.039, and the Gaussian surface fitting is 0.027. The minimum energy difference is proposed by this paper which average error is 0.013 which surpasses to other methods.

Tab.2 Simulation results of star spot coordinates by different algorithms

Right ascension/(°)	Declination/(°)	Mapping coordinates		Pixel coordinates	
		x_0	y_0	x_p	y_p
241.485 6	23.475 8	284.642 9	13.201 3	285	13
239.340 1	26.395 7	22.017 8	102.658 0	22	102
238.982 1	28.653 7	135.688 2	94.739 1	136	95
238.314 9	24.071 0	271.028 7	89.430 3	271	89
236.094 5	26.457 2	308.887 1	176.794 4	309	177
235.884 5	20.601 7	109.732 9	228.894 5	109	229
233.209 3	25.352 8	142.621 0	334.712 9	143	335
232.824 0	23.692 3	322.810 9	357.102 2	323	357

Moment mass center		Square weighted		Gaussian fitting		Minimum energy	
284.672 8	13.215 3	284.622 9	13.218 3	284.624 8	13.219 3	284.627 9	13.194 3
22.054 8	102.621 0	22.053 7	102.633 0	21.997 8	102.678 0	22.024 8	102.670 1
135.748 2	94.706 1	135.658 2	94.766 1	135.718 1	94.753 1	135.673 2	94.732 1
271.048 7	89.388 3	271.053 7	89.461 3	271.010 7	89.448 3	271.015 7	89.419 3
308.927 1	176.829 4	308.847 1	176.773 4	308.857 1	176.810 4	308.902 1	176.782 4
109.766 9	228.928 5	109.712 9	228.874 5	109.749 9	228.911 5	109.720 9	228.906 5
142.651 0	334.746 9	142.656 0	334.694 9	142.596 0	334.714 9	228.901 3	334.720 7
322.785 9	357.075 2	334.684 9	357.130 2	334.692 9	357.084 2	334.729 9	357.114 2

(a) Comparison of *x* coordinate error

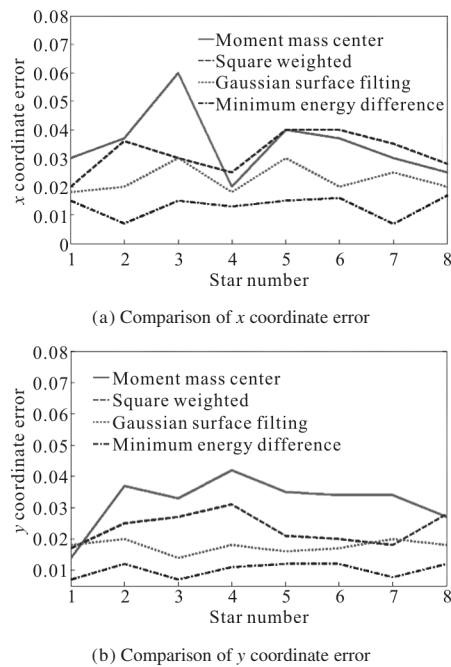(b) Comparison of *y* coordinate error

Fig.4 Centroid error comparison of 8 star image spots

5 Conclusion

The positioning accuracy of star centroid, which plays a vital role in the astronomical navigation, directly affects star matching and positioning accuracy. Firstly, according to the characteristics of Gaussian distribution, we analyze the energy distribution of star image. Secondly, we take advantage of the linear interpolation to improve the accuracy of centroid positioning. Thirdly, a method based on multi−step minimum energy difference is proposed. Moreover, the experimental results prove that the method can effectively improve the position precision. The advantage of this theory is that we can get the precise centroid position of star in the condition of a low SNR with simple calculation. In the simulation, we use the proposed method to calculate the centroid of real star image for 2 magnitudes Polaris and compare it with the theoretic centroid. We do not test the angle measuring system error when calculating the theoretic centroid. Correcting the error, we use the factory index instead of the error. Although there are some deviations, the effect is negligible. The proposed method by comparison can effectively calculate the sub−pixel centroid.

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