

# Turbulence of different waveform laser propagation in atmosphere

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**Abstract:** When the high energy laser propagates through the atmosphere, many atmospheric turbulence effects appear, including the beam drift, beam jitter, beam expansion and so on. Based on the wave optical theory, a four-dimensional computer code simulating the turbulence effect was established in this work. The influence of turbulence on different waveform laser, such as the Gaussian laser and the rectangle laser, was calculated. The numerical results show that when the turbulence becomes stronger, the phase aberration of laser become larger and the power in the bucket and the focusing ability become weaken. A comparison between the Gaussian beam and rectangle beam indicates that the turbulence effect on the latter is weaker under the same other conditions and it may be a better candidate to reduce the turbulence effect.

**Key words:** turbulence; different waveform; refractive-index structure parameter;  
power in the bucket; numerical simulation

**CLC number:** O436    **Document code:** A    **Article ID:** 1007-2276(2015)S-0041-05

## 不同形状激光波束在大气中传输的湍流效应

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**摘 要:** 当激光在大气传输时, 会引起光束的抖动、光束扩展和漂移等大气湍流效应, 这严重影响了激光传输的效能。根据波动光学理论, 建立了不同波束激光在大气湍流传输的数学模型, 采用龙哥库塔法编写了四维程序。数值结果表明: 大气湍流效应会造成激光光束的相位起伏变大, 光束的聚焦能力减弱, 光斑会发生分裂; 相比高斯光束, 矩形光束可大大减弱湍流效应。

**关键词:** 湍流; 波束形状; 折射率结构常数; 桶中功率; 数值模拟

收稿日期: 2015-04-05; 修订日期: 2015-05-10

基金项目: 国家自然科学基金(11404398)

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## 0 Introduction

In the turbulent motion state all the time is the most significant characteristics of the atmosphere. High Energy Laser (HEL) is not only affected the absorption of atmospheric molecule, scattering, but also affected by atmospheric turbulence<sup>[1]</sup>. When the laser propagates in atmosphere, effect of atmospheric turbulence on laser beam performance for beam jitter, flicker, beam spreading and the image point dithering, which limit the application of HEL. Consequently, developing the thermal blooming research is propitious to the application and the development of HEL<sup>[2,9]</sup>.

At present, many scholars bring forward the corresponding mathematical models of atmospheric turbulence in perfect condition. These theoretical models, which provide theoretical basis for the design of laser radar system, are testified by detailed experimentation<sup>[1-6]</sup>. However, considering different waveform laser propagation in atmosphere, the old model is not appropriate to the problem. So this paper sets up the model of atmospheric turbulence, and analyzes turbulence by four-dimensional computer code. What is more, in order to make a contrast, the paper simulates the model in three different laser waveform, including the gauss laser and the rectangle laser<sup>[3-4]</sup>.

## 1 Academic analysis

On the assumption that atmospheric turbulence brings the influence of laser wave phase, without energy wastage. The stochastic fluctuant phase of the laser is affected by the stochastic refraction change.

The wind blows along  $x$  direction, the laser propagates along  $z$  direction. The scalar wave equation in the Fresnel limit is<sup>[5-7]</sup>:

$$2ik\partial u/\partial z = \nabla_{\perp}^2 u + k^2 \delta \epsilon u \quad (1)$$

Where absorption coefficient is  $\alpha$ , the capacitance change affected by hydrodynamics  $\delta \epsilon = n^2/n_0^2 - 1$ .

If the phase  $S$  is small adequately, which is affected by the stochastic refraction change, so the amplitude of  $z_{n+1}$  plane is as follows:

$$u(r, z_{n+1}) = \exp[-iS(x, y, z_i)] \exp\left[-\frac{i}{2k} \Delta z \nabla_{\perp}^2 u(r, z_n)\right] \quad (2)$$

Accordingly, the numerical simulation of laser propagation in atmosphere turbulence is divided into two parts: transmission numeration in free space and stochastic phase constitution numeration. Laser transmission numeration in free space can be used with the discrete Fourier transform method and the Runge-Kutta method.

Laser transmission equation in free space is:

$$2ik\partial u/\partial z = \nabla_{\perp}^2 u \quad (3)$$

With the discrete Fourier transform method<sup>[7]</sup>, the equation is:

$$\frac{\partial u}{\partial z} = -\frac{i}{2} (F^{-1}(-K^2 F(u))) \quad (4)$$

Where the two-dimensional Fourier transform is  $F$ ; the two-dimensional Fourier transform wave number is  $K$ . So the equation can change to the normal differential equation:

$$\partial u/\partial z = L_5(u) \quad (5)$$

The computation format of the Runge-Kutta method is as follows<sup>[5]</sup>:

$$\begin{aligned} u^1 &= u^n + \Delta z L_5(u^n) \\ u^2 &= (3u^n + u^1 + \Delta z L_5(u^1))/4 \\ u^{n+1} &= (u^n + 2u^2 + 2\Delta z L_5(u^2))/3 \end{aligned} \quad (6)$$

## 2 Numerical simulation

The statistical orderliness of turbulence is described by modified Von Karman spectrum. the phase screen  $S_1$  is calculated by power spectrum, which is put forward by Glamery.

But the disadvantage of the power spectrum method is that, the low frequency of the phase can not emerge, owing to the limited gridding sampling. In order to describe the turbulent phase accurately, the paper compensate the low frequency by the harmonic method, which is put forward by Lane<sup>[7]</sup>. The low

frequency is  $S_2$ ; the total phase is the summation of  $S_1$ ,  $S_2$ <sup>[8]</sup>.

The phase  $S_1$ , which is put forward by Glamery, is as follows:

$$S_1(x, y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C_{n,m} \exp[i2\pi(xf_{xn} + yf_{ym})] \quad (7)$$

$$\langle |C_{n,m}|^2 \rangle = \phi_s(f_{x_n}, f_{y_m}) \Delta f_{x_n} \Delta f_{y_m} \quad (8)$$

Where the Fourier coefficient is  $C_{n,m}$ , the spatial frequency is  $f_{x_n}, f_{y_m}$ , the modified Von Karman spectrum is:

$$\varphi_s(f) = 0.023 r_0^{-5/3} \frac{\exp(-f^2/f_m^2)}{(f^2 + f_0^2)^{11/6}} \quad (9)$$

Where the frequency  $f = (f_{x_n}^2 + f_{y_m}^2)^{1/2}$ ;  $f_m = 5.92/10/(2 \times \pi)$ ;  $f_0 = 1/L_0$ ; the inner scale  $l_0$ ; the outer scale  $L_0$ ; the coherence length  $r_0 = (0.423 k^2 C_n^2 \Delta z)^{-3/5}$ ; the low frequency  $S_2$  is:

$$S_2(x, y) = \sum_{p=1}^{Np} \sum_{n=-1}^1 \sum_{m=-1}^1 C_{n,m} \exp[i2\pi(xf_{xn} + yf_{ym})] \quad (10)$$

Do these simultaneous equations, the paper gets:

$$u(r, z_{n+1}) = \exp\{-i[S_1(x, y, z_i) + S_2(x, y, z_i)]\} \cdot \exp\left[-\frac{i}{2k} \Delta z \nabla_{\perp}^2 u(r, z_n)\right] \quad (11)$$

The parameters of the model are as follows: the phase screen  $D=1$  m; the inner scale  $l_0=0.01$  m; the outer scale  $L_0=100$  m; the laser wavelength  $\lambda=10.6 \mu\text{m}$ . Atmospheric turbulence is simulated with different turbulence intensity (the refractive index structure constant  $C_n^2=1 \times 10^{-16}, 1 \times 10^{-14}, 1 \times 10^{-12} \text{m}^{-2/3}$ ).

The detailed numerical results is arranged in Fig.1. When the refractive index structure constant  $C_n^2=1 \times 10^{-16} \text{m}^{-2/3}$ , the phase is 0.2 rad, however, the phase increases to 20 rad when the refractive index structure constant  $C_n^2=1 \times 10^{-12}$ . With the increased turbulence intensity, the phase aberration is much larger.

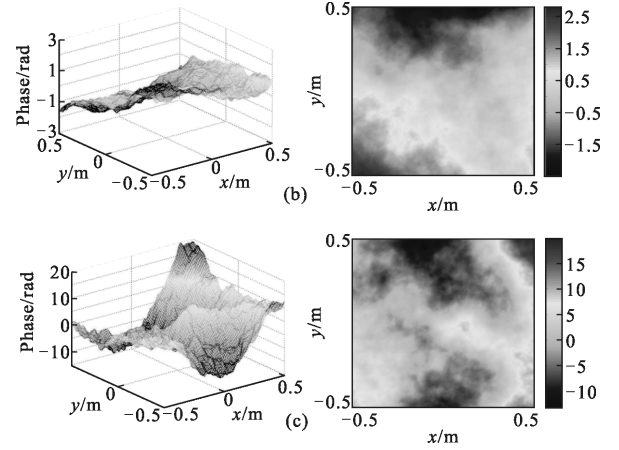
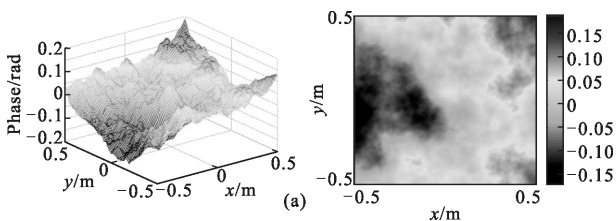


Fig.1 Phase screen and gray image under different turbulent intensity

### 3 Numerical analysis

Considering the problem of beam of light on the Far-Field, the estimation parameters of the beam quality are as follows: the center of beam, (the power in the bucket) PIB and the peak irradiance. The value of PIB is bigger, the focusing extent of the laser power is much more higher.

#### 3.1 Turbulence of the Gauss laser

For initial Gauss lasers, the electric field amplitude at  $z=0$  is given by:

$$u(x, y, 0, t) = u_0(x, y, 0) \times g(t)$$

$$u_0(x, y, 0) = \sum_{i=1}^3 \exp\{[-(x-x_i)^2]/2a^2\}$$

$$g(t) = \begin{cases} 1, & \text{mod}(t, t_s) \leq t_p \\ 0, & \text{mod}(t, t_s) > t_p \end{cases} \quad (12)$$

The 3-d distribution figures and gray image of intensity in initial Gauss beam can be seen in Fig.2.

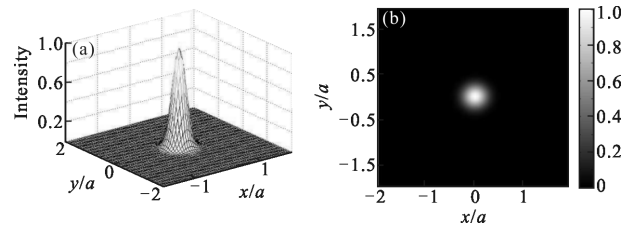


Fig.2 3-d distribution figures and gray image of intensity in initial Gauss beam

The paper give the parameters of the model: the initial beam radius  $a=0.25$  m, the power  $P_0=200$  kW, the wavelength  $\lambda=10.6 \mu\text{m}$ , the wind speed  $v=10$  m/s.

In the model, transmission distance  $z=4.632$  km, radius of the bucket  $R=0.1198$  m, the pulse duration  $t_p=100$  ms, the interval between pulses  $t_s=0.2$  s, The phase screen  $D=0.6$  m, the inner scale  $l_0=0.01$  m, the outer scale  $L_0=100$  m. By Simulating the mode with different refractive index structure constants, the paper gets the gray image and contour map of gauss laser under different turbulent intensity in Fig.3. The result is in Tab.1.

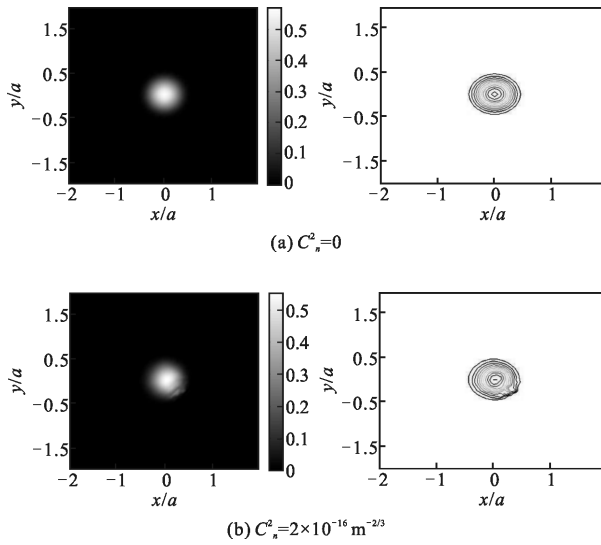


Fig.3 Gray image and contour map of Gauss beam under different turbulent intensity

**Tab.1 Result of Gauss beam under different turbulent intensity**

Item	Parameter	
$C_n^2/\text{m}^{-2/3}$	0	$2 \times 10^{-16}$
Center of beam( $\bar{x}/a, \bar{y}/a$ )	(0,0)	(0.019 1, 0.003 0)
PIB	0.116 0	0.127 8
Peak irradiance	0.566 1	0.550 1

As shown above in Fig.3 and Tab.1, we can clearly see when the refractive index structure constants  $C_n^2=1 \times 10^{-16}$ , the center of Gauss beam can not change, the peak irradiance reduces from 1 – 0.560 1; the PIB of the gauss beam reduces to 11.60%. Besides, we can clearly see when the refractive index structure constants, the peak irradiance reduces to 0.550 1, the irradiance distorts much more severely, at the same time, the beam distribution distorts severely, that is to say, the PIB of the Gauss beam

reduces to 12.78%, what is more, the size of spot enlarges much more, especially, the center of Gauss beam in  $x$ –height changes from 0–0.019 1 in the upwind direction, the center of gauss beam in  $y$ –height changes from 0–0.003 0, the spot in an vertical wind direction expands much severely.

In a word, turbulence affects laser propagation in Atmosphere, If the refractive index structure increases, the center of Gauss beam distorts much severely; the PIB decreases, that is to say, the focus ability of the beam is smaller; the peak irradiance reduces; the size of spot enlarges much more, the turbulence is much more serious. When turbulence intensity increases to a certain extent, the beam intensity fluctuation and the spot dancing that caused by atmospheric turbulence will come forth. Consequently, the turbulence should be compensated in some effective measures.

### 3.2 Turbulence of the rectangle laser

For initial rectangle laser, the electric field amplitude at  $z=0$  is given by:

$$\begin{aligned}
 u(x,y,0,t) &= u_0(x,y,0) \times g(t) \\
 u_0(x,y,0) &= \begin{cases} 1, & |x| \leq a \text{ \& \& } |y| \leq a \\ 0 & \end{cases} \\
 g(t) &= \begin{cases} 1, & \text{mod}(t, t_s) \leq t_p \\ 0, & \text{mod}(t, t_s) > t_p \end{cases} \quad (13)
 \end{aligned}$$

The 3–d distribution figures and gray image of intensity in initial rectangular beam can be seen in Fig.4. The parameters of the rectangular laser model are the same as those of the Gauss laser model.

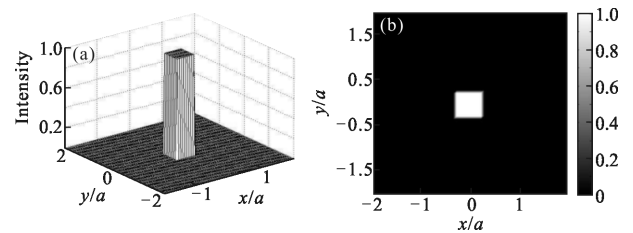


Fig.4 3–d distribution figures and gray image of intensity in initial rectangular beam

The paper gets the gray image and contour map of rectangular beam under different turbulent intensity in Fig.5 by Simulating the mode with different refractive index structure constants, the result is in Tab.2.

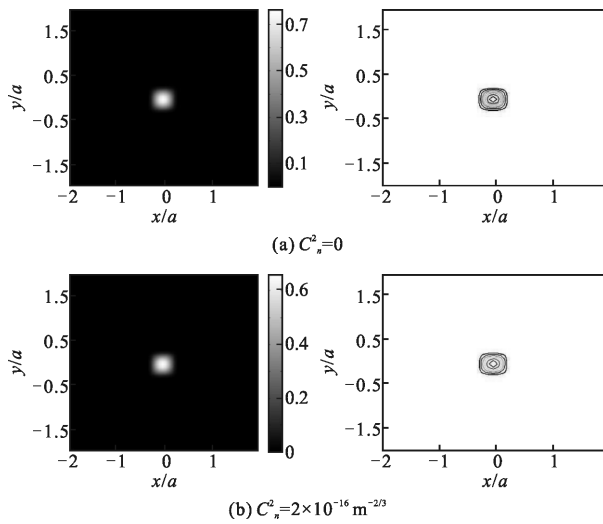


Fig.5 Gray image and contour map of rectangular beam under different turbulent intensity

**Tab.2 Result of rectangular beam under different turbulent intensity**

Item	Parameter	
$C_n^2/m^{-2/3}$	0	$2 \times 10^{-16}$
Center of beam ( $\bar{x}/a, \bar{y}/a$ )	(-0.062 5, -0.062 5)	(-0.063 0, -0.060 1)
PIB	0.281 9	0.278 2
Peak irradiance	0.758 4	0.655 2

Fig.5 and Tab.2 indicates when the refractive index structure constants  $C_n^2=0$ , the center of Gauss beam can not change, the irradiance distribution of rectangle laser and the spot are almost the same as the initial beam. Tab.2 suggests the center of rectangle beam in  $x$ -height and in  $y$ -height can not change; the PIB of the rectangle beam reduces to 28.19%; the peak irradiance reduces from 1-0.758 4.

Besides, when the refractive index structure constants  $C_n^2=2 \times 10^{-16}$ , the center in  $x$ -height and in  $y$ -height moves little, which distorts less severely than that of Gauss laser; the PIB of the rectangle beam reduces to 27.82%, which is bigger than that of Gauss beam, that is to say, the focus ability of the beam is bigger than that of Gauss laser; the relative peak irradiance is about to fall to 0.655 2, which is bigger than that of Gauss laser; the shape of the irradiance distribution and the spot expands less severely than that of Gauss laser; In conclusion, the laser waveform affects turbulence. In order to transmit higher peak

power at long lengths with good quality, we had better use rectangle beam.

## 4 Conclusion

This article analyzes the influence of atmospheric turbulence effect chiefly, simulates up the model of turbulence for different waveforms with the Runge-Kutta method, and gets the results as follows: If the refractive index structure constants increases, the peak irradiance of laser distorts severely, PIB becomes smaller, and the spot expands seriously. That is to say, turbulence becomes more severely. What is more, compared with the Gauss beam, we can clearly see the center of the Gauss laser moves further; the peak irradiance of the rectangular laser distorts less severely; the PIB of the rectangle laser is the bigger, that is to say, the focusing ability of the rectangular laser is better. In order to decrease the atmospheric turbulence effect, we had better use rectangle laser.

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