

五阶容积卡尔曼滤波算法及其应用

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摘 要: 容积卡尔曼滤波(CKF)是一种新型的非线性滤波方法,可获得优于扩展卡尔曼滤波(EKF)和无迹卡尔曼滤波(UKF)的滤波精度和滤波效率。但是,传统的 CKF 基于三阶容积准则而提出,因此滤波精度受到限制,为进一步提高 CKF 滤波性能,文中将容积准则由三阶扩展到五阶,采用两种不同容积点集选择方案,提出一种新型的五阶 CKF 算法。该算法可有效改善传统 CKF 在精度方面的理论局限,并有效改善一般五阶 CKF 计算量大的问题。机动目标跟踪仿真结果表明了新方法的有效性和可行性。

关键词: 容积卡尔曼滤波; 五阶容积卡尔曼滤波; 滤波精度; 目标跟踪

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Fifth degree cubature Kalman filter algorithm and its application

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Abstract: Cubature Kalman Filter (CKF) is one of new nonlinear filters, its accuracy and efficiency are better than Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). But the traditional CKF was proposed based on third order cubature rule and the filter accuracy was restricted. So the spherical-radial cubature rule was expended from third order to fifth order, the fifth order cubature rule based on two kinds of cubature point was used, a new fifth order CKF was proposed. The restriction of traditional CKF in theoretic accuracy was improved by the new fifth order CKF. The simulation results of the maneuvering target tracking show the validity and feasibility.

Key words: cubature Kalman filter; fifth order CKF; filter accuracy; target tracking

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0 引言

为解决非线性系统状态估计问题,通常采用非线性滤波算法^[1-3]。扩展卡尔曼滤波(Extended Kalman Filter, EKF)^[4-5]是一种常用的非线性滤波算法,但是,由于该算法对于强非线性系统线性截断误差,容易产生滤波发散。另外, EKF 需要计算 Jacobian 矩阵,给实际应用造成不便。相比于 EKF,无迹卡尔曼滤波(Unscented Kalman Filter, UKF)^[6-9]滤波精度较高,且滤波过程不用计算 Jacobian 矩阵,容易实现。容积卡尔曼滤波算法(Cubature Kalman Filter, CKF)^[10-12]是一种基于球面径向准则的非线性卡尔曼滤波算法,该算法用一组等权值 Cubature 点集来解决数值积分问题,可以获得优于 EKF 和 UKF 的滤波精度^[13]。传统 CKF 基于三阶容积准则,精度提高受到很大限制,为解决这一问题,文中在分析高阶容积准则的基础上,提出一种新的五阶 CKF 算法,有效提高了 CKF 算法精度,且具有很好滤波效率。

1 容积准则

对于积分:

$$I(f) = \int_{R^n} f(x) \exp(-x^T x) dx \quad (1)$$

假设 $x = ry, y^T y = 1$, 则

$$x^T x = r^2 \quad r \in [0, \infty)$$

转化为径向与超球面的二重积分:

$$I(f) = \int_0^\infty \int_{U_n} f(ry) r^{n-1} \exp(-r^2) d\sigma(y) dr \quad (2)$$

式中: U_n 表示单位超球面, $U_n = \{y \in R^n | y^T y = 1\}$; $\sigma(\cdot)$ 为积分微元。

进一步简化,可得:

$$I = \int_0^\infty S(r) r^{n-1} \exp(-r^2) dr \quad (3)$$

式中: $S(r)$ 为球面积分,且

$$S(r) = \int_{U_n} f(ry) d\sigma(y)$$

采用 Gauss-Hermite 准则、Spherical 准则^[14-16]可得

$$\int_0^\infty S(r) r^{n-1} \exp(-r^2) dr = \sum_{i=1}^{m_r} a_i S(r_i) \quad (4)$$

$$\int_{U_n} f(ry) d\sigma(y) = \sum_{j=1}^{m_r} b_j f(ry_j) \quad (5)$$

于是,积分可转化为:

$$I(f) = \int_{R^n} f(x) \exp(-x^T x) dx \approx \sum_{j=1}^{m_r} \sum_{i=1}^{m_r} a_i b_j f(ry_j) \quad (6)$$

2 三阶容积准则

将容积规则可以写成如下形式:

$$S(r) = \int_{U_n} g(rs) d\sigma(s) \approx W \sum_{i=1}^{2n} g([u]_i) \quad (7)$$

容积准则对于所有三阶以下(包括三阶)的单项式都可以准确描述,且对于满足 $\sum_{i=1}^n d_i = 0, 2$ 的所有单项式(即 $g(s) = 1$ 和 $g(s) = s_1^2$)也可以精确描述。根据全对称容积准则,未知参数 u, w 满足:

$$2nW = \int_{U_n} d \sigma(s) = A_n$$

$$2Wu^2 = \int_{U_n} s_1^2 d\sigma(s) = A_n/n$$

式中: $A_n = 2\Gamma(1/2)^n / \Gamma(n/2) = 2\sqrt{\pi} / \Gamma(n/2)$ 为单位圆表面积。

通过 $t = r^2$ 变换,对积分式进行转换,得到:

$$\int_0^\infty S(r) r^{n-1} \exp(-r^2) dr = \frac{1}{2} \int_0^\infty \tilde{S}(t) t^{\frac{1}{2}n-1} \exp(-t) dt \quad (8)$$

式中: $\tilde{S}(t) = S(\sqrt{t})$ 。

若使得球面-径向容积规则对于 $x \in R^n$ 中所有三阶多项式均可精确表示,推广高斯-拉盖尔规则应满足:

$$\int_0^\infty \tilde{S}_i(t) t^{\frac{1}{2}n-1} \exp(-t) dt = W_i \tilde{S}_i(t_i) \quad i = 0, 1$$

式中: $\tilde{S}_0(t) = 1, \tilde{S}_1(t) = t$ 。

于是,上述积分可以使用三阶球面-径向容积准则表示为:

$$I(g) \approx \frac{\sqrt{\pi}^n}{2n} \sum_{i=1}^{2n} g\left(\sqrt{\frac{n}{2}} [1]_i\right) \quad (9)$$

3 五阶容积准则

令 $s = [s_1, s_2, \dots, s_n]^T, p = [p_1, p_2, \dots, p_n], |p| = p_1 + p_2 + \dots + p_n$, 下标 p_i 为非负整数, $c(u_p)$ 表示 $u_p = (u_{p_1}, u_{p_2}, u_{p_n})$ 中非零元素的个数。对于球面积分:

$$I_{U_n}(g) = \int_{U_n} g(s) d\sigma(s) \quad (10)$$

其 $2m+1$ 阶球面准则可描述为:

$$I_{U_n, 2m+1}(g) = \sum_{|p|=m} W_p G\{u_p\} (m \geq 1) \quad (11)$$

$$W_p = I_{U_n} \left(\prod_{i=1}^n \prod_{j=0}^{p_i-1} \frac{s_i^2 - s_j^2}{u_{pi} - u_j} \right) \quad (12)$$

$$G\{u_p\} = 2^{-c(u_p)} \sum_v g(v_1 u_{p_1}, v_2 u_{p_2}, \dots, v_n u_{p_n}) \quad (13)$$

式中: I_{U_n} 表示球面积分; $I_{U_n, 2m+1}$ 表示用于近似该球面积分的球面规则, 它由容积点集 $[v_1 u_{p_1}, v_2 u_{p_2}, \dots, v_n u_{p_n}]^T$ 确定, $v_i = \pm 1$ 。为使得容积点最少, 可选择 $u_p = \sqrt{p/m}$ ($p_i = 0, 1, \dots, m$), $[v_1 u_{p_1}, v_2 u_{p_2}, \dots, v_n u_{p_n}]^T$ 的权值为 $2^{-c(u_p)}$ W_{p_0} 。若 q 为 p 的一种排列, 此时有 $W_q = W_{p_0}$ 。对于一个特定的 p , 如 $p = [1, 0, \dots, 0]$, 则:

$$W_p = I_{U_n} \left(\prod_{i=1}^n \prod_{j=0}^{p_i-1} \frac{s_i^2 - s_j^2}{u_{pi} - u_j} \right) = \int_{U_n} s_1^2 d\alpha(s) \quad (14)$$

由此, 三阶、五阶球面容积规则分别可描述为:

$$I_{U_n, 3}(g) = \frac{A_n}{2n} \sum_{j=1}^n (g(e_j) + g(-e_j)) \quad (15)$$

式中: e_j 为单位矢量, 且第 j 个元素为 1。

$$I_{U_n, 5}(g) = W_{p,1} \sum_{j=1}^{n(n-1)/2} (g(s_1^{(j)}) + g(-s_1^{(j)}) + g(s_2^{(j)}) + g(-s_2^{(j)})) + W_{p,2} \sum_{j=1}^n (g(e_j) + g(-e_j)) \quad (16)$$

式中: $W_{p,1} = \frac{A_n}{n(n+2)}$, $W_{p,2} = \frac{(4-n)A_n}{2n(n+2)}$

$$\{s_1^{(j)}\} = \left\{ \sqrt{\frac{1}{2}} (e_k + e_l) : k < l, k, l = 1, 2, \dots, n \right\}$$

$$\{s_2^{(j)}\} = \left\{ \sqrt{\frac{1}{2}} (e_k - e_l) : k < l, k, l = 1, 2, \dots, n \right\}$$

于是, 五阶球面-径向容积规则为:

$$I_1(g) = \int_{R^n} g(x) \times N(x|0, I) dx \approx \frac{1}{\pi^{n/2}} \sum_{j=1}^{N_s} \sum_{i=1}^{N_r} W_{r,i} W_{s,j} g(\sqrt{2} R_s j) = \frac{2}{n+2} g(0) + \frac{1}{(n+2)^2} \sum_{j=1}^{n(n-1)/2} [g(\sqrt{n+2} \cdot S_j^+) + g(-\sqrt{n+2} \cdot S_j^+)] + \frac{1}{(n+2)^2} \sum_{j=1}^{n(n-1)/2} [g(\sqrt{n+2} \cdot S_j^-) + g(-\sqrt{n+2} \cdot S_j^-)] + \frac{4-n}{2(n+2)^2} \sum_{j=1}^n [g(\sqrt{n+2} \cdot e_j) + g(-\sqrt{n+2} \cdot e_j)] \quad (17)$$

式中: $N_r = 2, N_s = 2n^2$ 。

$$I_1(g) = \int_{R^n} g(x) \times N(x|0, I) dx \approx \frac{1}{\pi^{n/2}} \sum_{j=1}^{N_s} \sum_{i=1}^{N_r} W_{r,i} W_{s,j} \cdot g(\sqrt{2} R_s j) = \frac{2}{n+2} g(0) + \frac{1}{(n+2)^2}$$

$$\sum_{j=1}^{n(n-1)/2} [g(\sqrt{n+2} \cdot S_j^+) + g(-\sqrt{n+2} \cdot S_j^+)] + \frac{1}{(n+2)^2} \sum_{j=1}^{n(n-1)/2} [g(\sqrt{n+2} \cdot S_j^-) + g(-\sqrt{n+2} \cdot S_j^-)] + \frac{4-n}{2(n+2)^2} \sum_{j=1}^n [g(\sqrt{n+2} \cdot e_j) + g(-\sqrt{n+2} \cdot e_j)] \quad (18)$$

五阶球面-径向容积规则点集为:

$$\begin{cases} \xi_{i=0} = 0, w_0 = \frac{2}{n+2} \\ \xi_i = \sqrt{n+2} [1]_i, w_i = \frac{4-n}{2(n+2)^2}, i=1, 2, \dots, 2n \\ \xi_{2n+i} = \sqrt{n+2} [1, 1]_i, w_{2n+i} = \frac{1}{(n+2)^2}, i=1, 2, \dots, 2n(n-1) \end{cases} \quad (19)$$

4 五阶 CKF 仿真验证

选择目标跟踪问题为例, 对 UKF、高斯-厄米特求积分滤波(GHQF)、三阶 CKF 和五阶 CKF 几种不同滤波算法的性能进行分析和对比。假设目标跟踪动力学方程为:

$$\begin{cases} \dot{x}_1(t) = x_3(t) \\ \dot{x}_2(t) = x_4(t) \\ \dot{x}_3(t) = D(t)x_3(t) + G(t)x_1(t) + w_1(t) \\ \dot{x}_4(t) = D(t)x_4(t) + G(t)x_2(t) + w_2(t) \\ \dot{x}_5(t) = x_5(t) \end{cases} \quad (20)$$

式中: x_1, x_2 为目标位置; x_3, x_4 为目标速度; x_5 为空气动力参数; $w(t)$ 为过程噪声向量; $D(t), G(t)$ 分别为阻力相关力和重力相关力, 表示如下:

$$D(t) = \beta(t) \exp\left\{ \frac{[R_0 - R(t)]}{H_0} \right\} V(t)$$

$$G(t) = -\frac{Gm_0}{R^3(t)}, \beta(t) = \beta_0 \exp(x_5(t))$$

式中: $R(t) = \sqrt{x_1^2(t) + x_2^2(t)}$ 为目标到地球球心的距离; $V(t) = \sqrt{x_3^2(t) + x_4^2(t)}$ 表示目标速度; 且 $\beta_0 = -0.59783$; $H_0 = 13.406$; $Gm_0 = 3.9860 \times 10^5$; $R_0 = 6374$ 。

由 Euler 积分法将上述目标跟踪动力学方程转化为:

$$\begin{cases} x_1(k+1) = x_1(k) + x_3(k) \cdot \Delta t & x_2(k+1) = x_2(k) + x_4(k) \cdot \Delta t \\ x_3(k+1) = x_3(k) + (D(k)x_3(k) + G(k)x_1(k)) \cdot \Delta t + w_1(k) \\ x_4(k+1) = x_4(k) + (D(k)x_4(k) + G(k)x_2(k)) \cdot \Delta t + w_2(k) \\ x_5(k+1) = x_5(k) + w_3(k) \end{cases}$$

假设观测点为 $(s_x, s_y) = (R_0, 0)$, r_k, θ_k 分别为目标相对观测点的距离和角度, 表示为:

$$\begin{cases} r_k = \sqrt{(x_1(k) - s_x)^2 + (x_2(k) - s_y)^2} + v_1(k) \\ \theta_k = \arctan\left(\frac{x_2(k) - s_y}{x_1(k) - s_x} \right) + v_2(k) \end{cases} \quad (21)$$

式中： $v_1(k)$ 、 $v_2(k)$ 为量测噪声， $v_1(k) \sim N(0, \sigma_r^2)$ ， $v_2(k) \sim N(0, \sigma_\theta^2)$ ， $\sigma_r = 10^{-3} \text{ km}$ ， $\sigma_\theta = 0.17 \text{ mrad}$ 。

设置参数为：

$$\Delta t = 0.1 \text{ s}$$

$$Q(k) = \begin{bmatrix} 2.4064 \times 10^{-5} & 0 & 0 \\ 0 & 2.4064 \times 10^{-5} & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix}$$

$$X_0 = [6500.4 \quad 349.14 \quad -1.8093 \quad -6.7967 \quad 0.6932]^T$$

$$P_0 = \begin{bmatrix} 10^{-6} & 0 & 0 & 0 & 0 \\ 0 & 10^{-6} & 0 & 0 & 0 \\ 0 & 0 & 10^{-6} & 0 & 0 \\ 0 & 0 & 0 & 10^{-6} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

不同滤波算法对 x_1 、 x_3 、 x_5 的估计均方根误差 (RMSE) 和均方差如图 1 所示。其中图(a1)~(a3)为UKF，

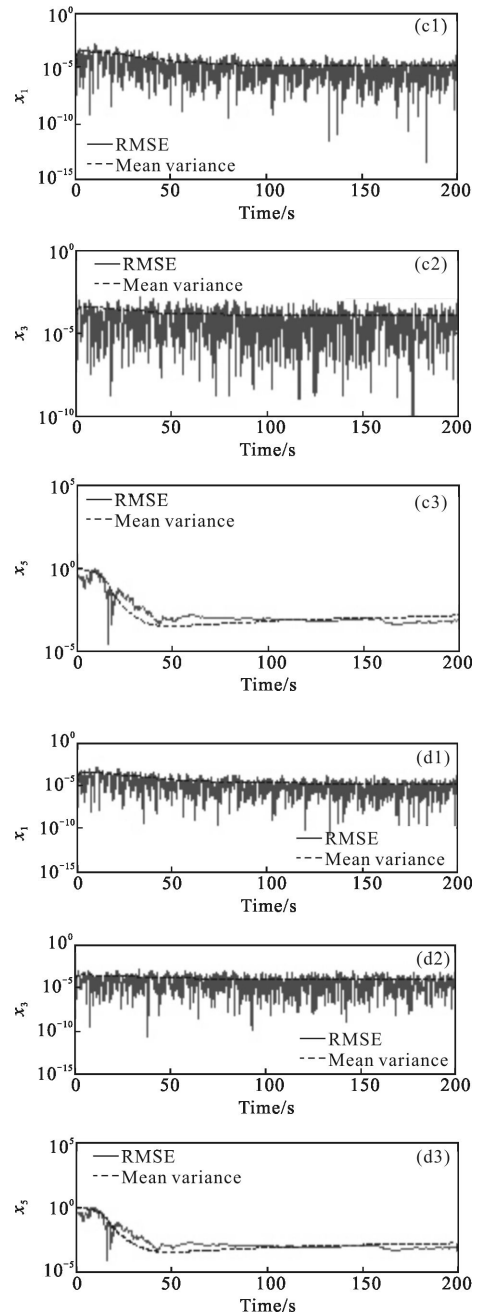
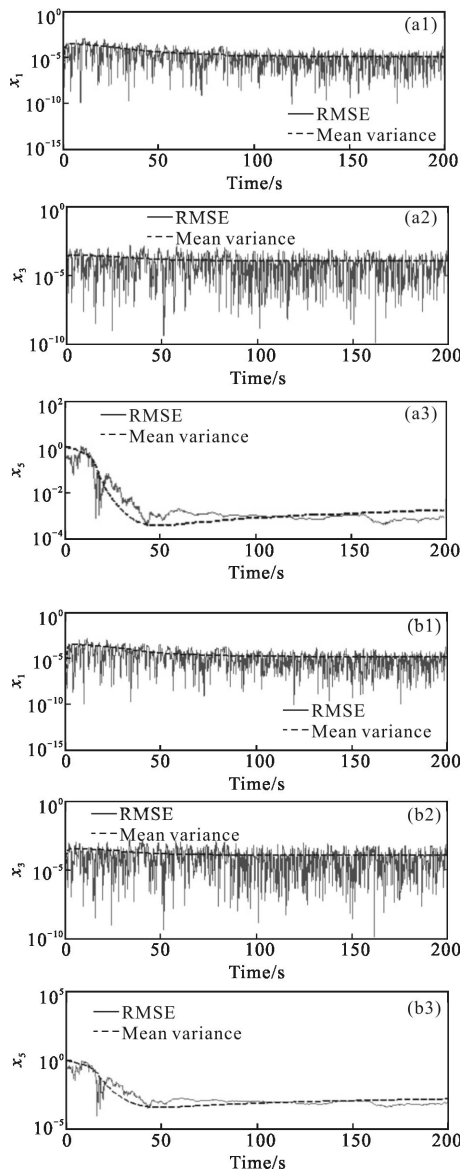


图 1 不同滤波算法的估计误差

Fig.1 Estimating errors of different filters

图(b1)~(b3)为 CKF3, 图(c1)~(c3)为 GHQF, 图(d1)~(d3)为 CKF5。经 Monte Carlo 仿真, 得到的 RMSE 值如图 2 所示。

由图 1 和图 2 可以看出, CKF3 和 CKF5 均可获得明显优于 UKF 的滤波精度, 相比于 CKF3, CKF5 的滤波精度更高; 相比于 GHQF, CKF5 可获得更高的滤波效率。因此, 综合考虑滤波精度和滤波效率, CKF5 滤波方法性能相对更好。

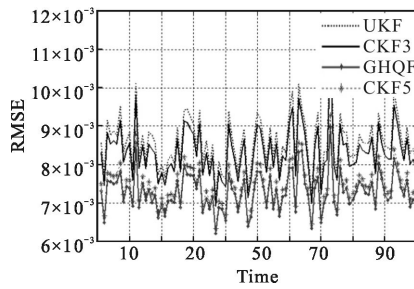


图2 Monte Carlo 仿真中各算法的 RMSE 值

Fig.2 RMSE of different filters in Monte Carlo simulation

5 结论

CKF 滤波算法是一种新型的非线性滤波方法,但是由于采用三阶容积准则,所以其精度受限。文中从滤波算法理论出发,基于五阶容积准则,提出一种新的五阶 CKF 算法,该算法从理论层面提高了 CKF 算法精度,并具有很好的滤波效率。目标跟踪非线性系统仿真结果表明,相比于 UKF 和三阶 CKF,五阶 CKF 具有更优的综合滤波性能,因此可以获得更高的跟踪精度。

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