Joint reconstruction algorithm for distributed compressed sensing

Cui Ping, Ni Lin

(Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei 230061, China)

Abstract: Distributed compressed sensing is concerned with representing an ensemble of jointly sparse signals using as few linear measurements as possible. Joint reconstruction algorithm for distributed compressed perception was based on the idea of using one of the signals as side information, and then reconstruct other signals by the correlation between the side information and other signals. To resolve the complexity of reconstruction algorithms and reduce the measurements, two novel joint reconstruction algorithms for distributed compressed sensing based on joint sparse models were presented in this paper. Its application in signals and images processing was presented which are on the basis of demonstrating its feasibility. The result represent that the two novel joint reconstruction algorithms need fewer measurements for getting the same quality.

Key words: distributed compressed sensing; joint reconstruction algorithm; joint sparse model
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分布式压缩感知联合重构算法

崔 平,倪 林

(中国科学技术大学 信息科学技术学院,安徽 合肥 230061)

摘 要: 分布式压缩感知是用尽可能少线性测量值来表示一个联合稀疏信号。分布式压缩感知联合重构算法是以信号集中的某个信号为边信息,根据信号集中信号之间的相关关系来重构信号的算法。为了解决已有重构算法的复杂性以及减少重构算法所需的测量值数,提出了两种新的分布式压缩感知联合重构算法。对提出的两种新算法在信号和图像处理上进行了实验,验证了其可行性与先进性。结果表示,这两种联合重建算法在获取相同的图像质量时需要测量值更少。

关键词: 分布式压缩感知; 联合重构算法; 联合稀疏模型

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作者简介: 崔平(1990-),女,硕士生,主要研究方向为信号与图像处理。Email: cuiapple@mail.ustc.edu.cn

导师简介: 倪林(1965-),男,副教授,博士,主要研究方向为信号与图像处理。Email: nilin@ustc.edu.cn

0 Introduction

In recent years, as an effective new technology in obtaining data, distributed compressed sensing gains more and more attention in the distributed scenario [1-2]. DCS relies on the theory of compressed sensing (CS) to reduce the dimensionality of the signal acquired by each node of the distributed network supposed to be sparse under some basis by means of random projections [2]. It also exploits the inter-correlation among the different signals in the ensemble to lower the number of measurements that each node needs to acquire without requiring cooperation among nodes.

With the deepening of the research and the extension of application, distributed compressed sensing is finding wider and wider application in the actual scene. And its research and application in wireless sensor network is favored by scholars and technicians.

More and more reconstruction algorithms have been proposed [2-4]. In distributed compressed sensing, the joint sparse models provide a way of signal sparse, and applied to their corresponding signal processing scenario.

In this paper, on the basis of introducing the joint sparse models, we leverage the difference of innovation to design two novel joint reconstruction algorithms which are based on JSM1 and JSM3, and apply the algorithms to image processing, and verify the performance of the two joint reconstruction algorithms.

1 Background

1.1 Compressed sensing

Compressed sensing is a novel theory for measurement coding and reconstruction encoding of sparse signals[1]. If the signal itself is sparse, you can directly to measurement coding; Unless the signal itself is not sparse, you need to find a sparse basis of the signal, and find the sparse representation under the basis, then measurement coding for the sparse representation.

Let us consider a signal $X\in R^N$, having a sparse representation under basis $\Psi_{N\times N}$:

$$X=\Psi\Theta_0 \quad \|\Theta\|_0=K\ll N \tag{1}$$

Among the formulation, $\|\Theta\|_0$ is the l_0 norm of Θ. K is the number of its nonzero entries for Θ.

In the coder, We acquire measurements as a vector of random projections:

$$Y=\Phi X=\Phi\Psi\Theta=A^{CS}\Theta \tag{2}$$

Which, $\Phi\in R^{M\times N}$ is a sensing matrix, and $K<M\ll N$.

In the encoder, the best way to recover the original signal from its measurements is by solving an optimization problem trying to minimize the l_0 norm of the signal in the sparsity domain. That is:

$$\hat{\Theta}=\mathrm{argmin}_{\Theta}\|\Theta\|_0 \text{ subject to } Y=\Phi\Psi\Theta$$

However, this problem is computationally intractable due to its NP-hard complexity, so it is common to consider a relaxed form using the l_1 norm, which can be solved by means of linear programming techniques:

$$\hat{\Theta}=\mathrm{argmin}_{\Theta}\|\Theta\|_1 \text{ subject to } Y=\Phi\Psi\Theta$$

and greedy algorithm, i.e., orthogonal matching pursuit (OMP) [5]. For OMP, it can compute the support of the sparse signal X iteratively, where the support denotes the index of those columns in $\Phi_{M\times N}$ that have the largest inner product with signal X. Once the support of the signal is computed correctly, the pseudo-inverse of the measurement matrix restricted to the corresponding columns can be used to reconstruct the actual signal X. OMP can recover an K sparse signal when the number of measurements N is nearly proportional to K. In this paper, two novel joint reconstruction algorithms for distributed compressed sensing based on OMP are presented[6-7].

1.2 Distributed compressed sensing

In a distributed scenario, an ensemble of signals with both intra and inter-sensor correlations is considered. The notion of joint sparse has been introduced in Reference [2] for the framework of DCS. Among the joint sparse models discussed in Reference [2], we focus on the JSM-1 and JSM-3 models, according to which the J signals in the ensemble have sparse innovation components and sparse or non-sparse common component, respectively.

$$\Theta_j=\Theta_C+\Theta_{I,j} \tag{3}$$

$\| \Theta_C \|_0 = K_C$ and $\| \Theta_{I,j} \|_0 = K_{I,j}, j \in [1,J]$

A joint reconstruction algorithm can leverage the structure of the joint sparsity model to improve performance, namely to achieve higher quality for the same number of measurements or decrease the number of measurements needed to achieve the same quality.

2 Joint reconstruction algorithm

In the joint sparse models which are discussed in References[8–9], each of an ensemble of signals can be divided into common support and innovations two parts. The innovations is difference and can be sparse, so we can just use the difference of innovations to modify the existing algorithm without thinking about the common support of an ensemble of signals. In this section, we present two novel of joint reconstruction algorithm, which used the difference of innovations based on OMP algorithm.

2.1 The first joint reconstruction algorithm

Consider an ensemble of signals $X_j, j \in \{1,2,\cdots,J\}$, which are sparse under the same basis Ψ. Here, we choose signal X_1 as the side information.

Algorithm 1 steps

(1) Input: $A=\Phi\Psi$, K_I;

(2) Compute: $Y_{d,j}=Y_j-Y_1 \ \ j \in [2,J]$;

(3) Recover $\Theta_{d,j}$ from $Y_{d,j}$ by OMP algorithm:

a. Initialize

Let the support $I=\Phi$ and the residual $r_1=Y_{d,j}$ and the iteration counter $t=1$.

b. Identify

Select the largest coordinate λ_t of $Y_t=Ar_t$ in absolute value.

c. Update

Add the coordinate λ_t to support, $I \leftarrow I \cup \{\lambda_t\}$, update the residual:

$$\hat{\theta}_t = \arg\min_z \| Y_{d,j} - A|_I \cdot z \|_2$$

$$r_{t+1} = r_t - A \cdot \hat{\theta}_t$$

and increase $t=t+1$. Return to the identify procedure if $t< K_I+1=T$, where T is the maximum iteration times, and $A|_I$ denotes the submatrix whose columns are selected from depending on the index set I.

d. Terminate

If $t \geqslant T$, stop the iterations. Once the support I of the signal $X_j, j \in \{2,\cdots,J\}$ is found, the estimate can be reconstructed as $\Theta_{d,j}=\bar{A}|_I \cdot Y_{d,j}$, where define the pesudoinverse by $\bar{A}=(A^*A)^{-1}. A^*$, $A^{\wedge *}$ is the conjugate gradient projection of A.

(4) Compute: $\hat{\theta}_j=\Theta_1+\Theta_{d,j}$; $\hat{X}_j=\Psi\hat{\theta}_j$;

The sparsity of common support of the signals is not considered in the algorithm discussed above. We just consider the difference of innovations of each signal to recover the other signals by using the side information, here is signal X_1. In this way, the correlation between X_1 and X_j, $j \in [2,J]$ can be adequately exploited. Consequently, the information of common part from X_1 contributes to the decoding process of $X_j, j \in \{2, \cdots, J\}$ converges faster than ever before. As a result, the algorithm could considerably decrease the total number of measurements and save large amount of running time. Therefore, power consumption at both encoding and decoding sensor will fall down to an proper level that could be acceptable and easy to be implemented in practice.

2.2 The second joint reconstruction algorithm

Consider an ensemble of signals $X_j, j \in \{2, \cdots, J\}$, which are sparse under the same basis Ψ. Here, we choose signal X_1 as the side information.

Algorithm 2 steps

(1) Input: $A=\Phi\Psi, K_I$;

(2) Compute: $\hat{Y}_C=\frac{1}{J}\sum_{J=1}^{J} Y_j$; $\hat{Y}_{I,1}=Y_1-\hat{Y}_C$;

(3) Recover $\hat{\Theta}_{I,1}$ from $\hat{Y}_{I,1}$ by OMP algorithm (the same procedures as Recover $\Theta_{d,j}$ from $Y_{d,j}$ in section A)

(4) Compute: $Y_{d,j}=Y_j-Y_1 \ j[2,J]$

$$\hat{Y}_{I,j}=Y_{d,j}-\hat{Y}_{I,1}$$

(5) Recover $\hat{\Theta}_{d,j}$ from $\hat{Y}_{I,j}$ by OMP algorithm (the same procedures as Recover $\Theta_{d,j}$ from $Y_{d,j}$ in section A)

(6) Compute: $\hat{\theta}_j=\Theta_1-\hat{\Theta}_{I,1}+\hat{\Theta}_{I,j}$; $\hat{X}_j=\Psi\hat{\theta}_j$;

The algorithm discussed above is similar to the first joint reconstruction algorithm. The difference between them is the method of computing the difference of innovations. In the first joint reconstruction algorithm, proceeding pairwise by using the side information X_1 and each of the other signals $X_j, j\in\{1,2,\cdots,J\}$ in the ensemble to compute the difference between the measurements of the side information Y_1 and those of signal $Y_j, j\in\{2,\cdots J\}$. But in the second joint reconstruction algorithm, we first compute the average measurements of all the signals, that is $\hat{Y}_C$; then compare the difference between the measurements of the side information Y_1 and $\hat{Y}_C$ and the difference between the measurements of the side information Y_1 and those of signal $Y_j, j\in\{2,\cdots J\}$. Finally, add the two kinds of difference as the innovations of the other signals $\hat{Y}_{I,j}$. So it is more accurate to compute the difference of innovations in the second joint reconstruction algorithm than in the first joint reconstruction algorithm. And it gains better performance to recover the signals and needs fewer measurements to achieve the same quality.

3 Experimental results and analysis

In this section, we will compare the two novel joint reconstruction algorithm with the OMP algorithm and the exsited joint reconstruction algorithm.

In test 1, we generate two signals x_1 and x_2 in RN, which are in accord with JMS−1 model, the signal length N=256, K_c=10, K_i = 2, to do 50 times cycle for each point in the curve and obtain the average.

In Figure 1, it displays the average recovery performance at different number ofmeasurements with the separate OMP algorithm and the two novel jointreconstruction algorithm. Signals for the three algorithms are generated in thesame way. We draw an Gaussian measurement matrix ϕ and execute thethree algorithm while decoding x_2. For one thing, considering saving of numberof measurements, our two novel joint

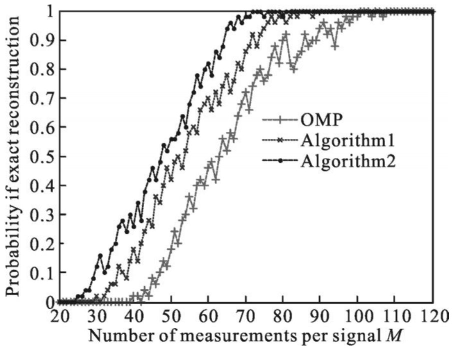

Fig.1 Joint vs separation

reconstruction algorithm with obvious improvement. For another, at the same number of measurements, the two joint reconstructionalgorithm provided in this paper achieve more performance than the separate OMP algorithm, especially the second joint reconstruction algorithm.

In test 2, we generate two signals x_1 and x_2 in RN, which are in accord with JMS−3 model, the signal length N=256, K_i=25, to do 50 times cycle for each point in the curve and obtain the average.

In Figure 2, the two novel joint reconstruction algorithms have been tested on the JSM −3 model of distributed compressed sensing and compared against the TECC algorithm presented in Ref. [2]. The two proposed algorithms rely on the usage of the same sensing matrix for all nodes, while TECC requires different matrices. Moreover, the TECC algorithm must compute the commom compartment of signals, while the two proposed algorithms just need to compute the difference of the innovations of signals. Figure 2 shows the MSE as a

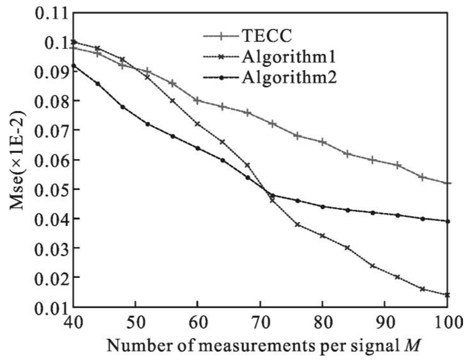

Fig.2 Mean square error vs number of measurements

function of the number of measurements acquired by each node. The two proposed algorithms are able to outperform TECC algorithm and also be suitable for the JSM−1 model.

4 Application in image processing

We use news video images photograph a part of the production line, then the computer system collect and analyze the contents of the photograph and ensure the production line running well, which is in order to achieve the purpose of quality control. The contents of every picture are complex, but a collection of all the news video images will have high correlation, because the news video images only have a small difference (which is sparse). In the same news video, each video frame may not be sparse, but the small differences between the video frames can be sparse. Such image data compression, all can apply joint reconstruction algorithm to reconstruct.

Test: two news images of 256 ×256, which have different parts. We use image *a* as the side information, reconstructing image *b* by the two novel joint reconstruction algorithm, and compared the quality with the two novel joint reconstruction algorithm. We just need to recover the difference of innovations to reconstruct image *b*, which reduce the number of iterations and narrow the computing time. Especially, the second joint reconstruction algorithm gains a better quality.

The original image *a* and *b* is shown in Fig.3 (a) and Fig.3(b) respectively.

Fig.3 Original images

Using the first joint reconstruction algorithm to reconstruct image *b* under different sampling rates is shown in Fig.4.

Fig.4 The first joint reconstruction algorithm

Using the second joint reconstruction algorithm to reconstruct image *b* under different sampling rates is shown in Fig.5.

Fig.5 The second joint reconstruction algorithm

Contrast the performance of the two joint reconstruction algorithms:

Figure 6 shows the relationship between measurements and PSNR. From the figure, we know that the two joint reconstruction algorithms allow to decrease the number of measurements needed to achieve a target quality in the reconstruction or to improve quality for the same number of measurements, which the separate OMP algorithm cannot reach. The second joint reconstruction algorithm perform more obvious.

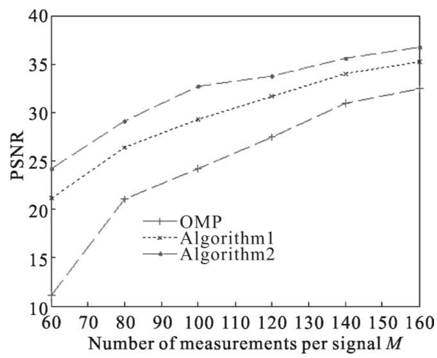

Fig.6 Joint vs separate

5 Conclusion

We proposed two novel joint reconstruction algorithms for the JSM1 and JSM3 models in distributed compressed sensing. Thanks to the use of side information, it is possible to devise methods that avoid the need to reconstruct the common component, thus allowing to deal with the case of a non-sparse common component in a straightforward manner, especially in reconstructing video frames.

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