

## Analysis of channel capacity employing circle polarization shift keying over Gamma–Gamma atmospheric turbulence channel

Wang Yi<sup>1,2</sup>, Wang Deli<sup>1</sup>, Du Fan<sup>1</sup>, Ma Jing<sup>2</sup>, Tan Liying<sup>2</sup>

(1. College of Information Engineering, China JiLiang University, Hangzhou 310018, China

2. National Key Laboratory of Tunable Laser Technology, Harbin Institute of Technology, Harbin 150001, China)

**Abstract:** Circular polarization control error caused by system design and intensity fluctuation as well as phase noise induced during the transmission of optical signal in atmospheric turbulence will directly lead to performance impairments of circular polarization shift keying (CPolSK) system in FSO. Therefore, on the basis of roundly considering the three parameters, which are the intensity scintillation, the phase noise and the circular polarization control error, the mathematical statistical model of the CPolSK system channel was established based on the Gamma–Gamma atmospheric channel model, and the closed expression of the average channel capacity was derived. Simulations were given to analyze the effects of intensity scintillation, phase fluctuation and circular polarization control error on channel capacity of CPolSK system. Results show that the increase of both intensity fluctuation and phase noise will cause the degradation of the channel capacity, while raising the average SNR can improve the channel capacity in a certain degree. It is also noted that the average channel capacity will significantly decrease when the circular polarization control error is larger than  $8^\circ$ .

**Key words:** free space optical communication; circular polarization shift keying;  
Gamma–Gamma atmospheric turbulence channel; channel capacity

**CLC number:** TN929.12    **Document code:** A    **Article ID:** 1007–2276(2015)10–3084–08

## 大气 Gamma–Gamma 湍流信道中采用圆偏振位移键控的信道容量分析

王 怡<sup>1,2</sup>, 王德丽<sup>1</sup>, 杜 凡<sup>1</sup>, 马 晶<sup>2</sup>, 谭立英<sup>2</sup>

(1. 中国计量学院 信息工程学院, 浙江 杭州 310018;

2. 哈尔滨工业大学 可调谐激光技术国家级重点实验室, 黑龙江 哈尔滨 150001)

**摘 要:** 在自由空间光通信(FSO)中,由系统设计以及大气湍流所引起的光强闪烁和相位起伏会造成圆偏振控制误差,这严重影响圆偏振位移键控(CPolSK)通信系统的性能。因此,基于 Gamma–Gamma 大气信道模型,在较全面考虑光强起伏、相位噪声和圆偏振控制误差三个参数的基础上,建立了 CPolSK 系统信道的数学统计模型,推导了平均信道容量的闭合表达式,仿真研究了误码率在连续相

收稿日期:2015-02-12; 修订日期:2015-03-16

基金项目:海洋工程国家重点实验室(上海交通大学)开放课题(1418);中国博士后科学基金(2013M540290);  
国家自然科学基金(61379027)

作者简介:王怡(1980–),女,副教授,博士,主要从事自由空间激光通信和星地光通信方面的研究。Email:wcy16@cjlu.edu.cn

位噪声下的性能以及三个参数对信道容量的影响。结果表明,光强起伏的增强和相位噪声的增大均会导致 CPolSK 系统性能的劣化,而提高平均信噪比在一定程度上能够改善系统性能。特别地,当圆偏振控制误差大于  $8^\circ$  时,CPolSK 系统的平均信道容量显著下降。

**关键词:** 自由空间光通信; 圆偏振位移键控; 大气 Gamma-Gamma 湍流信道; 信道容量

## 0 Introduction

Free space optical communications (FSO) have been resurgent studied for its notable advantages of higher transmission rate, larger bandwidth, more excellent security, faster link installation and lower power consumption over radio frequency (RF) communications<sup>[1-3]</sup> in recent years. However, the transmitting channel of FSO is atmospheric channel, the most imperative problem is that FSO links suffer from random change of refractive index caused by the variation of air temperature and pressure, which will directly lead to intensity fluctuation and phase noise to cause severe performance impairments of FSO system, thus seriously affect the stability and reliability of the FSO system and largely restrict the further development of FSO technology<sup>[4-6]</sup>.

In the wireless optical communication, atmospheric channel would have a little perceptible influence on polarization states, so the polarization modulation technique has a wide potential for application. Based on orthogonal linear polarization modulation PolSK (L-PolSK), proposed CPolSK, which possesses low bits error rate characteristic the same as L-PolSK modulation, meanwhile compared to the L-PolSK scheme, CPolSK also has two advantages: (1) CPolSK modulation no longer requires the alignment of the polarization coordinates of the transmitter and the receiver; (2) circular polarized light intensity distribution more uniform after particle scattering. These two advantages make the circular polarization modulation more suitable for the atmospheric channel of free space optical communication system.

In 1995, America JPL laboratory achieved the

communication data rate of 1.2 Gbps test, which with two different polarization data transmission in satellite optical communication system using light polarization multiplexing technique<sup>[7]</sup>. The Davis group of the United States Maryland University had firstly studied the PolSK technology in wireless optical communication in 2002, optical fiber coupling and EDFA optical amplifier technology were used in the test, the orthogonal polarization states selected was horizontal polarization and vertical polarization, the polarization multiplexing communication experiment realized the data rate up to 1.2 Gbps at a distance of 2 km, verifying the feasibility of atmospheric linear polarization multiplexing technology<sup>[8]</sup>. And in 2006 Davis group made a comparative study on control of polarization (PolSK) and on-off keying (OOK), found that PolSK possess less sensitivity than OOK is about 3 dB<sup>[9]</sup>. In 2005, Jun Zhu from Tsinghua University firstly proposed and proved the feasibility of the polarization multiplexing communication system of a circularly polarized light<sup>[10]</sup>. The Xinhui Zhao research group of Harbin Institute of Technology presented the device model of CPolSK in 2008 and 2009, meanwhile did research on CPolSK only in weak turbulence. Compared with on-off keying (OOK) and pulse position modulation (PPM), CPolSK can suppress background noise and internal noise of photoelectric detector, greatly reducing the bit error rate; At the same time, they performance improvement of CPolSK is 3 dB than OOK. The results show that the SOP keeps unchanged and the DOP slightly increases on the propagation in turbulent atmosphere<sup>[11-14]</sup>. In 2012, Changchun Institute of Optics and Fine Mechanics Peng Yang designed the CPolSK optic communication system, and had carried on the simple CPolSK

communication experiment, proved the feasibility of CPolSK<sup>[15]</sup>.

Previous studies of CPolSK in FSO field have been focused on either the feasibility of CPolSK system or the effects caused by atmospheric turbulence to error performance. However, most research only considered the effect of either a certain turbulent atmosphere (weak, moderate or strong) or some channel parameter caused by the atmospheric turbulence, such as intensity fluctuation or degree of polarization and so on to study the error performance of CPolSK system.

This paper bases on the established mathematical statistical channel model of CPolSK system, Gamma-Gamma distributed channel model was used to research the atmospheric turbulence induced intensity fluctuation and phase noise from weak to strong with closed form expression of average channel capacity derived from the established mathematical statistical channel model of CPolSK system based on circular polarization control error. Simulations were given to analyze the effects of intensity fluctuation, phase noise and circular polarization control error on channel capacity of CPolSK system.

## 1 CPolSK system model

Intensity fluctuation and phase noise caused by atmospheric turbulence to the laser beam would have a serve influence on degrading the channel capacity. Since Gamma-Gamma turbulence channel model can model the irradiance of FSO channels in all the turbulent scenarios from weak to strong, and hence, it is chosen in the paper. Moreover, multipath scattering is also ignored in the paper.

### 1.1 Gamma-Gamma turbulent channel model

Optical wave propagating in turbulent atmosphere suffers from random change of air refractive index will lead to intensity fluctuation, the probability distribution model of which accords with Gamma-Gamma distribution, thus the intensity fluctuation  $I$  can be expressed as<sup>[1]</sup>:

$$f(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta-2)/2} K_{\alpha-\beta}(2\sqrt{\alpha\beta I}), I > 0 \quad (1)$$

where  $\Gamma(\cdot)$  is the Gamma function, and  $K_n(\cdot)$  is the modified Bessel function of the second kind of order  $n$ . Positive parameters  $\alpha$  and  $\beta$  are large-scale and small-scale intensity fluctuation of optical wave, which are given by<sup>[11]</sup>:

$$\alpha = \{ \exp[0.49\delta_R^2 / (1 + 1.11\delta_R^{12/5})^{7/6}] - 1 \}^{-1}$$

$$\beta = \{ \exp[0.51\delta_R^2 / (1 + 0.69\delta_R^{12/5})^{5/6}] - 1 \}^{-1} \quad (2)$$

where  $\delta_R^2$  is the Rytov variance that used to classify intensity fluctuation under different turbulent optical links, that is, the intensity fluctuation is weak when  $\delta_R^2 < 1$ , while the moderate intensity fluctuation is defined by  $\delta_R^2 \cong 1$  and the strong intensity fluctuation comes out when  $\delta_R^2 > 1$ . Moreover, here we use scintillation index ( $SI$ ) according with  $\alpha$  and  $\beta$  to reveal the intensity fluctuation from weak to strong, which is defined as<sup>[5]</sup>:

$$SI = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha\beta} \quad (3)$$

Besides, phase noise caused by the atmospheric turbulence can be described by Gaussian distribution based on perturbation approximation theory<sup>[16]</sup>. Note that the effect of phase noise to laser linewidth also satisfies Gaussian distribution<sup>[5]</sup>, the distribution of phase noise can be described as

$$f_\varphi(\varphi) = \frac{1}{\sqrt{2\pi}\delta} e^{-\varphi^2/2\delta^2} \quad (4)$$

where  $\delta^2$  is variance of phase noise.

### 1.2 CPolSK system model

According to Xinhui Zhao's paper<sup>[11]</sup>, SOP of a beam affected by atmospheric turbulence is little, so we does not take the influence of SOP into consideration, considering the circular polarization control error, only to analyze the influence intensity fluctuation and phase noise on the system performance.

An optical communication system with CPolSK modulation based on Gamma-Gamma atmospheric turbulence channel is illustrated in Fig.1.

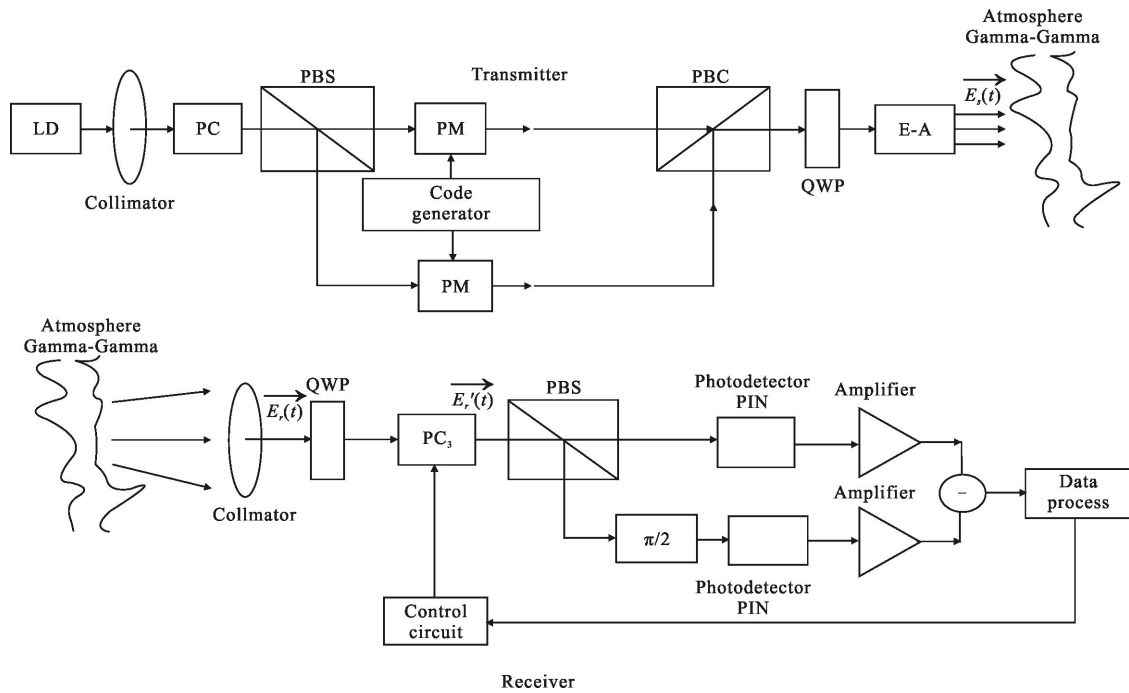


Fig.1 Block diagram of the CPolSK system over FSO links with Gamma-Gamma channel. LD(laser diode); PM(LiNbO<sub>3</sub> device-based external phase modulator); PC(polarization controller); PBS(polarization beam splitter); PBC(polarization beam combiner); QWP(quarter-wave plate); E-A(emission antenna)

Here, we assume that the PMs are both linear as reported in Ref.[17] and that the channel is the ideal additive white Gauss noise (AWGN) channel. The emitted optical field  $\vec{E}_s(t)$  is thus given as:

$$\vec{E}_s(t) = \sqrt{\frac{P_s}{2}} e^{j(w_s t + \varphi_s)} \{ e^{j\phi_x} \cdot \hat{x} + j e^{-j\phi_y} \cdot \hat{y} \} \quad (5)$$

where  $P_s$ ,  $w_s$  and  $\varphi_s$  are the optical power, angular frequency and the laser phase noise of the transmitted optical carrier,  $\phi_x$  and  $\phi_y$  are the phases modulated to the different polarization components of the beam with  $\{\phi_x, \phi_y\} = \{0, \pi\}$  respectively. According to the rules of the codes fed into the modulators, namely, the phase does not change when entering "0" while the phase changes when entering "1", there by producing the right-and left-handed circular polarized light by changing  $\Delta\phi = \phi_y - \phi_x = \pi$  or  $0$  so that the phase becomes  $\pi/2$  or  $-\pi/2$  corresponding to the  $\pi/2$  phase delay of QWP.

The received optical signal  $\vec{E}_r(t)$  can be expressed as:

$$\vec{E}_r(t) = S_r(t) \sqrt{\frac{P_s}{2}} e^{j(w_s t + \varphi_s(t))} \{ e^{j\phi_x} \cdot \hat{x} + j e^{-j\phi_y} \cdot \hat{y} \} \quad (6)$$

where  $S_r(t)$  is the intensity scintillation of the optical signal induced in atmospheric turbulence and  $\varphi_r(t)$  is the phase noise of the optical carrier propagating in atmospheric turbulence with the phase noise to laser line width also included, which is assumed to be a stationary Gaussian process with a zero-mean and a variance of  $\delta^2$ . Both the parameters  $S_r(t)$  and  $\varphi_r(t)$  are time-variant statistics because of atmospheric turbulence.

The received circularly polarized beam is first converted into linearly polarized light through a QWP after collimation. It is noted that the linearly polarized beam will generate a polarization control error  $\xi$ , which means the SOP difference between the beam after PBC at the transmitter and before the PBS at the receiver while its passing through PC3 that is derived by the feedback control circuit. Here, the optical field before PBS can be described as  $\vec{E}_r'(t) = \vec{E}_r(t) \cdot e^{j\xi}$ . The linearly polarized beam is then split into  $\hat{x}$  and  $\hat{y}$  polarization light with the optical fields as follows:

$$\vec{E}_x(t) = \sqrt{\frac{P_s}{2}} \cdot \{ S_x \cdot e^{j(w_s t + \phi_x + \varphi_s(t))} \cos\xi - S_y \cdot$$

$$\vec{E}_y(t) = \sqrt{\frac{P_s}{2}} \cdot \{S_x \cdot e^{j(w_x t + \phi_x + \varphi_x(t))} \sin \zeta + S_y \cdot e^{j(w_y t + \phi_y + \varphi_y(t))} \cos \zeta\} \hat{x} \quad (7)$$

where  $\varphi_x(t)$  and  $\varphi_y(t)$  are the phase noises of the two orthogonal polarization components during their propagation through the atmospheric turbulence,  $S_x$  and  $S_y$  are the scintillations of the two orthogonal polarization components without losing the generality of  $S_x^2 = S_y^2 = I(t)$ . There after, the signals carried by the two lights are demodulated using a dual differential method with the direction of one light rotating  $90^\circ$  by setting a phase retarder at  $\pi/2$ . Thus, the current signal detected can be written as

$$i(t) = \eta R P_s I(t) \cos(\Delta\phi) \cos\varphi + n_{th}(t) \quad (8)$$

where  $\eta = \cos^2 \xi - \sin^2 \xi$  is the detection efficiency,  $R$  is the responsivity of the photo detector,  $\varphi = \varphi_x(t) - \varphi_y(t)$  is the received phase noise, and  $n_{th}(t)$  is the detector noise that satisfies the Gaussian process with a zero-mean and a variance of  $\delta_n^2/2$  on account of the noise terms  $n_x(t)$  and  $n_y(t)$  being assumed to be statistically independent, and a stationary Gaussian processes with a zero-mean and a variance of  $\delta_n^2/2$ . Subsequently, the transmitting signal can be detected from  $i(t)$  using the principle that the output differential signal represents signal "1" when  $\cos(\Delta\phi) = 1$  and the output differential signal represents signal "0" when  $\cos(\Delta\phi) = -1$ . On this basis, the modulation and demodulation processes of CPolSK system are accomplished.

## 2 Performance analysis of average channel capacity

The average capacity, which is also called ergodic capacity, is an important performance index to estimate the communication link. It can not accurately predict how the changes of atmospheric turbulence affect CPolSK system when the laser beam propagating in the atmosphere, moreover, the influence is changing with the time, so the atmospheric

turbulence channel can be regarded as a system that is unstable, stochastic and time-varying. Assuming that ergodic information of the ideal channel at the transmitter and the receiver iterate the average channel capacity, which is denoted as  $\langle C \rangle$

$$\langle C \rangle = \int_0^\infty \int_{-\infty}^\infty C_{AMGN} f(I) f_\varphi(\varphi) dI d\varphi \quad (9)$$

where  $C_{AMGN} = B \log_2(1 + \eta^2 \bar{\gamma} I \cos^2 \varphi)$  is Gauss channel capacity. By submitting formula (4) into (9) and using the Gaussian-Hermite quadrature formula<sup>[18]</sup> with  $x = \varphi / \sqrt{2} \delta$ , thus average channel capacity can be expressed as

$$\langle C \rangle = \frac{B}{\pi^{1/2}} \sum_{i=1}^N w(x_i) \int_0^\infty \log_2(1 + \tau) f_i(I) dI \quad (10)$$

where  $\tau = \eta^2 \bar{\gamma} I \cos^2(\sqrt{2} \delta x_i)$ ,  $\log_2[\cdot]$  and  $K_v[\cdot]$  can be expressed by Merger-G function as

$$\log_2(1 + \tau) \times \ln 2 = \ln(1 + \tau) = G_{2,2}^{1,2}(\tau_{1,0}^{1,1}) \quad (11)$$

$$K_{\alpha-\beta}(2\sqrt{2\alpha\beta I}) = \frac{1}{2} G_{0,2}^{2,0}[\alpha\beta I]_{\frac{\alpha-\beta}{2}, \frac{\alpha-\beta}{2}} \quad (12)$$

By using the operational characteristics of Meijer-G function<sup>[16]</sup>, the closed form expression for average channel capacity available can be written as

$$\langle C \rangle = \frac{B 2^{\alpha+\beta-2}}{\pi^{3/2} \ln 2 \cdot \Gamma(\alpha) \Gamma(\beta)} \sum_{i=1}^N w_i \times G_{62}^{1,6} \left[ \frac{2^4 \eta^2 \bar{\gamma} \cos(\sqrt{2} \delta x_i)}{(\alpha\beta)^2} \right]_{1,1, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-\beta}{2}, \frac{2-\beta}{2}, 1,0} \quad (13)$$

where  $N$  is the order of approximation,  $x_i (i=1, 2, \dots, N)$  and  $w_i (i=1, 2, \dots, N)$  are the zeros and weight factors of the  $N$ th-order Hermite polynomial, respectively, which can be obtained by checking table.

## 3 Numerical results and discussions

In this section, we conduct both simulation and numerical analysis to illustrate that the intensity fluctuation, phase noise and circular polarization control error have considerable influences on average channel capacity of CPolSK system. In both the simulation and analysis, we select 0.49, 1.21 and 4 as the Rytov variances of light intensity to represent the

weak, moderate and strong turbulent scenarios, respectively. On this basis, the parameters of intensity fluctuation in Gamma-Gamma atmospheric turbulence channel become  $\{\alpha=6.05, \beta=4.47, SI=0.43\}$ ,  $\{\alpha=4.19, \beta=2.26, SI=0.79\}$ ,  $\{\alpha=4.34, \beta=1.31, SI=1.17\}$ , respectively. Besides, phase noises are taken into consideration with  $0.085\pi$ ,  $0.25\pi$ ,  $0.625\pi$  for different turbulent scenarios, accordingly.

Fig.1 shows the average channel capacity ( $\langle C \rangle / B$ ) versus average SNR for different intensity fluctuations and phase noise based on the premise that circular polarization control error is neglected. As can be seen, atmospheric turbulence causes the decline of average channel capacity, namely, both enhance the intensity fluctuation and enlarge the phase noise will degrade the average channel capacity. For example, the average channel capacity of weak fluctuation ( $SI=0.43$ ) can reach 3.95 conditioned on phase noise  $\delta=0.25\pi$  and averaged SNR of 45 dB, while the average channel capacities of moderate fluctuation ( $SI=0.79$ ) and strong fluctuation ( $SI=1.17$ ) can only obtain 3.64 and 3.39. Moreover, when  $SI=0.79$ , if average SNR is 45 dB, the average channel capacities for phase noises  $0.25\pi$  and  $0.625\pi$  are 3.61 and 4.35 compared to that of  $0.085\pi$  is 3.14 in terms of average channel capacity, respectively. Noted that, with all the considered  $SI$  and phase noise, it exerts a significant improvement of channel capacity for CPolSK system in a certain range of SNR.

As shown in Fig.2, the analytic curve of average channel capacity as a function of circular polarization control error that also accounts for the intensity fluctuation and phase noise at 45 dB of SNR. the average channel capacity keep decreasing with the increase of circular polarization control error. With respect to circular polarization control error more than  $8^\circ$ , with the rise of which, the average channel capacity will drop dramatically. Simulation result of Fig.3 also reveals that the channel capacity of CPolSK system gets smaller and smaller as the turbulent

scenario that both intensity fluctuation and phase noise included changes from weak to strong with the same average SNR, which is consistent with theoretical analysis. Therefore, through raising average SNR properly and reducing the circular polarization control error can effectively increase the average channel capacity of system, so as to improve the performance of the CPolSK system.

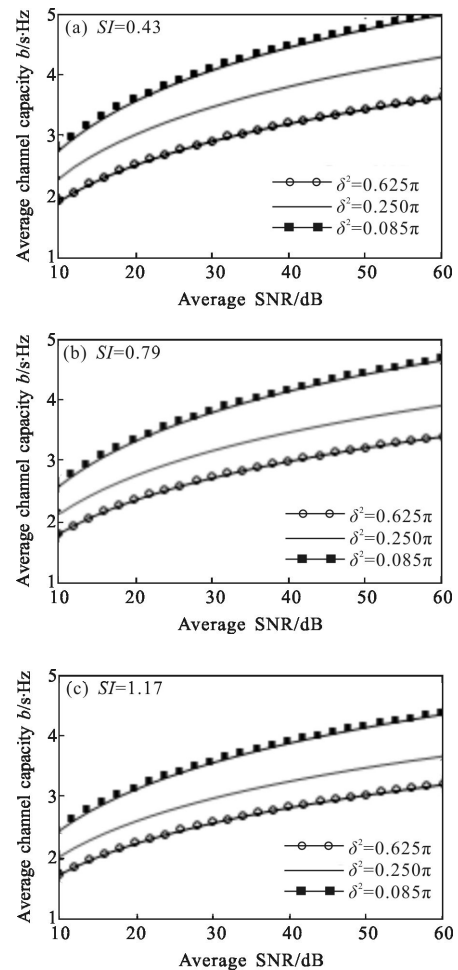
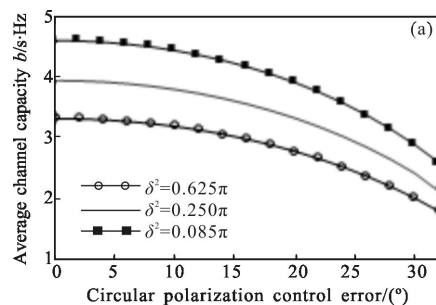


Fig.2 Average channel capacity of the CPolSK system as a function of average SNR for different phase noise at the transmission rate of 16 Gb/s in different intensity fluctuation



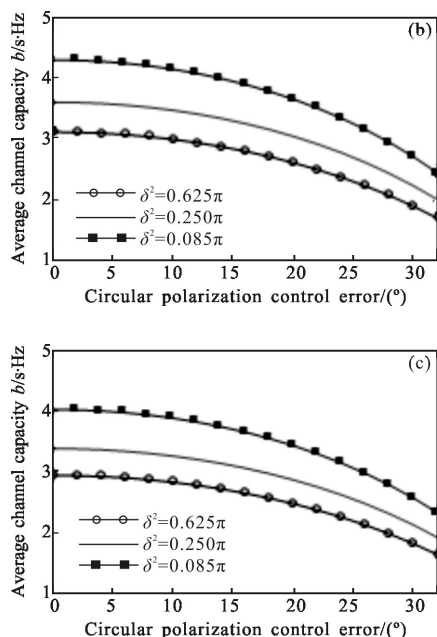


Fig.3 Average channel capacity of the CPolSK system as a function of circular polarization control error for different phase noise at the transmission rate of 16 Gb/s in different intensity fluctuation with average SNR 45 dB

#### 4 Conclusion

Using Gamma -Gamma distributed channel model, with consideration of circular polarization control error caused by system design, we then theoretically analyzed the average channel capacity of CPolSK system that was assisted with communication channel capacity to highlight the effects of intensity fluctuation and phase noise under different atmospheric turbulent scenarios. Simulations were given to demonstrate the effects of intensity fluctuation, phase noise and circular polarization control error to channel capacity of CPolSK system. Results show that the increase of both intensity fluctuation and phase noise will cause the degradation of the channel capacity, while raising the average SNR properly could improve the channel capacity in a certain degree. The change of average bit error rate is not obvious, but for the average channel capacity change greatly when phase noises are  $\delta=0.085\pi$  and  $\delta=0.25\pi$ , Furthermore, BER performance, communication interrupt and the average

channel capacity significantly decreased when the circular polarization control error is larger than  $8^\circ$ . The results of our study will help to practically obtain modulation of high performance and to advance the design of CPolSK system in FSO links.

#### References:

- [1] Ijaz M, Ghassemlooy Z, Ansari S, et al. Experimental investigation of the performance of different modulation techniques under controlled FSO turbulence channel [C]// Proceedings of IST 2010. Thessaloniki: IEEE, 2010: 59-64.
- [2] Sandalidis H G. Coded free-space optical links over strong turbulence and misalignment fading channels [J]. *IEEE Transactions on Communications*, 2011, 59(3): 669-674.
- [3] Wang Yi, Du Fan, Ma Jing, et al. Research on collimating characteristics of the emitted laser beam in free space optical communication system [J]. *Infrared and Laser Engineering*, 2015, 44(3): 1008-1014. (in Chinese)
- [4] Zhu Xiaoming, Kahn J M. Free-space optical communication through atmospheric turbulence channels [J]. *IEEE Transactions on Communications*, 2002, 50(8): 1293-1300.
- [5] Lim W, Yun C, Kim K. BER performance analysis of radio over free-space optical systems considering laser phase noise under Gamma -Gamma turbulence channels [J]. *Optics Express*, 2009, 17(6): 4479-4484.
- [6] Wang Yi, Zhang Ao, Ma Jing, et al. Effects of phase fluctuation and intensity scintillation caused by weak atmospheric turbulence to BER employing distributed antenna array [J]. *Infrared and Laser Engineering*, 2015, 44(2): 758-763. (in Chinese)
- [7] Tan Liying, Ma Jing, Lin Weiqiu. development of intersatellite optical communications technology [J]. *Laser Technology*, 1999, 23(5): 99-303.
- [8] Smolyaninov I I, Wasiczko L, Cho K. Long-distance 1.2 Gb/s optical wireless communication link at 1 550 nm [C]//Free-Space Laser Communication and Laser Imaging, SPIE, 2002: 241-250.
- [9] Trisno S, Davis C C. Performance of free space optical communication systems using polarization shift keying modulation [C]//Free-Space Laser Communications VI, SPIE, 2006: 63040-63049.
- [10] Zhu Jun, Zhang Shulian. Circularly polarized light polarization multiplexing laser communication system [J]. *Infrared and Laser Engineering*, 2005, 35(2): 78-79. (in Chinese)

- [11] Zhao X H, Yao Y, Sun Y X, et al. Circle polarization shift keying with direct detection for free-space optical communication [J]. *Journal of Optical Communications and Networking*, 2009, 1(4): 307–312.
- [12] Liu C, Sun Y X, Yao Y, et al. Analysis of direct detection and coherent detection in wireless optical communication with polarization shift keying [C]//8th Pacific Rim Conference on Lasers and Electro-Optics, 2009: 1–2.
- [13] Zhao Xinhui, Yao Yong, Sun Yunxu, et al. A new modulation method in free space optical communication: circle polarization shift keying[J]. *Acta Optica Sinica*, 2008, 28(S2): 223–226. (in Chinese)
- [14] Zhao Xinhui, Yong Yao, Sun Yunxu, et al. Condition of keeping polarization property unchanged in the circle polarization shift keying system [J]. *Optical Communications and Networking*, 2010, 2(8): 570–575.
- [15] Yang Peng, Ai Hua. Design of laser communication system with circular polarization modulation [J]. *Chinese Journal of Lasers*, 2012, 39(9): 210–216. (in Chinese)
- [16] Andrews L C, Phillips R L. *Laser Beam Propagation Through Random Media*[M]. Bellingham: SPIE Spress, 2005.
- [17] Li Y, Wang R, Ding G, et al. Novel phase modulator linearity measurement[J]. *IEEE Photon Technol Lett*, 2009, 21(19): 1405–1407.
- [18] Jeffrey Alan, Daniel Zwillinger. *Table of Integrals, Series, and Products* [M]. US: Academic Press, 2007.