Theoretical study on damage thresholds for elastic stress fracture in laser-irradiated optical glass

GUO Shao-feng , LU Qi-sheng , YIN Yan , CHENG Xiang-ai , JIANG Hou-man , ZHOU Ping

(Institute of Science , National University of Defense Technology , Changsha 410073 , China)

Abstract : Based on thermo-elastic theory , an analytic model is developed for determining the laser fluence required to damage a series of thin glass disks which serve as transmissive optical components. In particular , the dependence of threshold fluence on laser beam diameter and mechanical boundary conditions of samples is investigated. Study shows that beam diameter effect in material response results from the energy loss of the irradiated area. As for structural response , different mechanical boundaries result in different relationships of LIDT (laser induced damage threshold) with radius of laser beam. It is also found that mechanical boundaries have great influence on LIDT even if beam diameter is fixed , and optimal boundary under which tensile stress is eliminated is available theoretically when particular compressive boundary is applied.

Key words : Lase induced damage ; Optical materials ; Damage thresholds ; Optimal boundary

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激光辐射下光学玻璃应力破坏的阈值理论

郭少锋 ,陆启生 ,银 燕 ,程湘爱 ,江厚满 ,周 萍

(国防科技大学 理学院 ,湖南 长沙 410073)

摘 要 :基于热弹理论 ,建立了计算光学材料激光诱导破坏阈值的解析模型。特别研究了能量阈值与激光光斑大小以及光学材料力学边界条件的依赖关系。研究表明 :在材料响应情形下 ,光斑效应体现在光斑能量向外的弥散 ,而在结构响应情形下 ,不同的边界条件导致不同的破坏阈值与光斑直径之间的函数关系。提出了一种最优边界 ,使激光诱导产生的拉应力场最小。

关键词 :激光诱导破坏 ; 光学材料 ; 破坏阈值 ; 最优边界

0 Introduction

Recently there has been increasing interest in LIDTs (laser induced damage thresholds) in optical components[1,2] , which play a significant role in determining the maximum output power of high energy laser systems . The CW lasers easily induce heat

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作者简介 :郭少锋(1975-) ,男 ,河南平顶山人 ,博士生 ,研究方向为激光与物质相互作用。

deposition in optical materials, which in turn results in macroscopic damage. It is important to investigate the CW lasers induced damage mechanisms and failure modes and improve the damage resistance of optical materials. In general, laser induced damage in optical glasses arises either from elastic fracture or from phenomena associated with surface relaxation[3]. In this paper, we focus on damage induced by elastic stress which often happens in transmissive optical components of high energy laser systems. Although a lot of measurements have been conducted to obtain the intrinsic LIDT in optical window materials[4~6], the work, so far, is not adequate in the absence of theoretical study which can show the inherent relationship of LIDT with laser parameters and other factors. Here we describe a method to estimate the threshold laser fluence to initiate elastic fracture and apply this model to study the influence of beam diameter and mechanical constraints on LIDT.

1 Elastic failure model

We assume that glass behaves as isotropic, homogeneous and linear elastic solid. This behavior is uniquely described by heat conduction equation and thermo-elastic equations together with proper initial and boundary conductions[7]. Our major goal is to develop analytic procedures which are relatively simple to apply, so uncoupled quasi-static theory is introduced, in which the thermal stress problem is solved in sequence after the heat conduction problem is solved for temperature distribution.

The basic situation we considered is that a uniform, stationary and axially symmetric beam normally is incident on a thin glass disk (see Fig. 1). For CW laser and thin target disk, the heat flux might be expected to become essentially radial after some initial period and the disk volume under beam become heated to an essentially uniform temperature through the thickness. Consequently one dimensional radial heat conduc

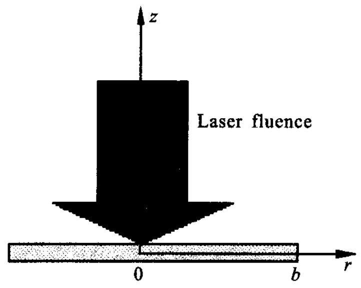

Fig. 1 Model for laser irradiating a glass disk

tion equation is introduced here to obtain temperature profile.

$$\begin{cases} \rho C_p \dfrac{\partial T}{\partial t} - K \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r \dfrac{\partial T}{\partial r}\right) = \dfrac{\eta(1-R) I(r,t)}{h} \\ \left.\dfrac{\partial T}{\partial r}\right|_{r \to b} = 0, \quad T(r,0) = 0 \end{cases} \tag{1}$$

where ρ, C_p, K represent the density, heat capacity and thermal conductivity of the glass respectively, h and b are the thickness and radius of glass disk respectively, and η and R denote the absorption coefficient for the glass at the laser wavelength and surface reflectance respectively. The intensity profile $I(r,t)$ is assumed to be uniform in irradiation region and square during irradiation time, written as

$$I(r,t) = \begin{cases} I_0, & r \leqslant a \\ 0, & r > a \end{cases}$$

where a is the beam radius.

In all problems we discuss here, the thermal boundary condition is assumed to be adiabatic and initial temperature is assumed to be zero.

Closed form analytic solution can be derived from Hankel transformation method[8], consisting of an infinite Bessel function series

$$T(r,t) = \frac{\eta I_0 (1-R) b^2}{Kh}\left(\frac{a^2 \kappa t}{b^4} + 2\frac{a}{b}\sum_{1}^{\infty} J_0(\mu_m r/b)\, w_m s_m(t) \right) \tag{2}$$

where $s_m(t) = 1 - \exp(-\mu_m^2 \kappa t / b^2)$, $w_m = \dfrac{J_1(\mu_m a/b)}{\mu_m^3 [J_0(\mu_m)]^2}$, κ is the thermal diffusivity of glass and μ_m denotes the mth positive root of one order Bessel function $J_1(x)$.

we consider two extreme cases here

1) Small beam radiation ($a \ll b$). This case happens when we focus the laser beam on the large optical components. In this case, the temperature distribution can be written as integral form

$$T(r,t) = \frac{\eta I_0(1-R)a}{Kh} \int_0^\infty J_0(\beta r) J_1(\alpha\beta)(1 - e^{-\kappa t \beta^2}) \frac{1}{\beta^2} d\beta \tag{3}$$

In section 3, we mainly discuss this case.

2) Large beam radiation ($a \gg (\kappa\tau_{ir})^{1/2}$, where τ_{ir} denotes the total irradiation time). This case happens when CW laser interacts with the output window. In this case, the heat loss from the beam area to "cool area" can be neglected during the radiation period. The temperature profile then can be simplified as

$$T(r,t) = \begin{cases} \dfrac{\eta I_0(1-R)}{\rho C_p h} t & r \leqslant a \\ 0 & r > a \end{cases} \tag{4}$$

We will discuss this case later in section 4.

One dimensional radial model for heat conduction, together with the fact that the thickness of the specimens h is much less than the radius b, develops a plane stress state which is particularly amenable to analyze. In plane stress problem, the only non-zero stresses are radial stress component σ_r and tangential stress component σ_θ, and they must satisfy the equilibrium equation

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{5}$$

From the stress-strain equations and the strain-displacement equations, we can easily get the analytic form of displacement and stress distribution

$$u_r = \alpha(1+\mu)\frac{1}{r}\int_0^r T(r) r dr + Cr \tag{6}$$

$$\sigma_r = -\alpha E \frac{1}{r^2}\int_0^r T(r) r dr + \frac{E}{1-\mu}C \tag{7}$$

$$\sigma_\theta = \alpha E \frac{1}{r^2}\int_0^r T(r) r dr + \frac{E}{1-\mu}C - \alpha E T \tag{8}$$

where u_r and μ denote radial displacement of particles in optical material and the Poisson's ratio respectively. C is the integral constant and can be determined by mechanical boundary condition.

The glass is brittle material and its tensile strength σ_t is far less than the compressive one σ_c, so as for damage problem, we only consider the tensile stress produced in the glass. In the following problem, the peak tensile stress appears in the tangential stress field, so only σ_θ is considered later. By substituting Eq. (2) into (8), we can get the tangential stress distribution in terms of laser parameters such as intensity I_0, irradiation time t_{ir} etc. Then the threshold intensity for tensile fracture is determined from the following manner. The tensile stress profile $\sigma_\theta(r,t)$ is evaluated as a function of position and time with the help of boundary condition to determine constant C. The peak tensile stress $(\sigma_\theta)_{max}$ is supposed to appear when $t = t_{ir}$ and where $r \approx a + 2(\kappa t_{ir})^{1/2}$. Setting $(\sigma_\theta)_{max} = \sigma_t$, we can obtain threshold intensity I_{th}, and threshold power P_{th} can be obtained consequently using relationship $P_{th} = \pi a^2 I_{th}$. The detailed expressions of thresholds under particular conditions will be presented in the following.

2 Beam diameter effects

In this section, we use the model described above to derive damage thresholds for K9 glass subjected to CO_2 laser whose beam radius is so small that the boundary condition could be neglected. Tab. 1 shows a list of the properties of K9 glass.

Tab. 1 Material properties of K9 glass

Properties	Value
$K/\mathrm{W}(\mathrm{m}\ ^\circ\mathrm{C})^{-1}$	1.207
$\kappa/\mathrm{m^2 s^{-1}}$	6.8×10^{-7}
$\rho/\mathrm{kg\ m^{-3}}$	2.5×10^{3}
$C_p/\mathrm{J}(\mathrm{kg}\ ^\circ\mathrm{C})^{-1}$	710
$\sigma_t/\mathrm{Ntm^{-2}}$	4.9×10^{7}
$\alpha/\ ^\circ\mathrm{C}$	8.5×10^{-6}
$E/\mathrm{Ntm^{-2}}$	80×10^{9}
μ	0.206
R(to 3.8 μm)	0.027
η(to 3.8 μm)	0.15

Substituting Eq. (3) into (8), setting $C = 0$ for neglecting boundary and letting $t = t_{ir}$ and $r = r_m = a + 2(\kappa t_{ir})^{1/2}$, the peak tensile stress can be written as

$$(\sigma_\theta)_{max} = \sigma_\theta(r_m, t_{ir}) = \alpha E \frac{\eta I_0 (1 - R) a}{Kh} [G(a, r_m, \kappa, t_{ir}) - H(a, r_m, \kappa, t_{ir})] \quad (9)$$

where

$$G(a, r, \kappa, t) = \int_0^\infty r J_1(r\beta) J_1(a\beta)(1 - e^{-\kappa t\beta^2}) \frac{1}{\beta^3} d\beta$$

$$H(a, r, \kappa, t) = \int_0^\infty J_0(\beta r) J_1(a\beta)(1 - e^{-\kappa t\beta^2}) \frac{1}{\beta^2} d\beta$$

setting $(\sigma_\theta)_{max} = \sigma_t$, we take the threshold intensity for tensile failure to be

$$I_{th} = \frac{\sigma_t Kh}{\alpha E \eta (1 - R) a} [G(a, r_m, \kappa, \tau_0) - H(a, r_m, \kappa, \tau_0)]^{-1} \quad (10)$$

and threshold power thus can be written as

$$P_{th} = \pi a^2 I_{th} = \frac{\pi \sigma_t Kha}{\alpha E \eta (1 - R)} [G(a, r_m, \kappa, t_{ir}) - H(a, r_m, \kappa, t_{ir})]^{-1} \quad (11)$$

Fig. 2 and Fig. 3 show the predicted threshold intensity and power threshold as the function of beam radius, respectively. The thresholds are calculated based on a glass thickness of 0.5 cm. According to Fig. 2, when beam radius is less than 1 mm, the curve of I_{th} has an abrupt decreasing from 10^6 W/cm^2 to 10^4 W/cm^2, but when beam radius is greater than 4 mm, I_{th}

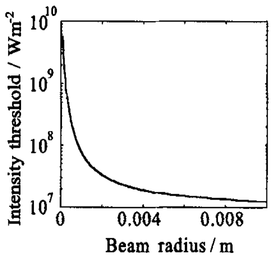

Fig. 2 Spot size effect of intensity thresholds in material response

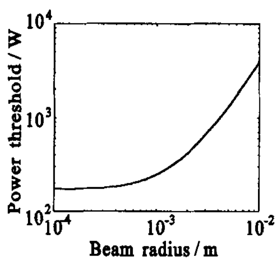

Fig. 3 Spot size effect of power thresholds in material response

varies only slightly. This result is qualitatively consistent with the experiment which Gong etc conducted with K2 glass under the irradiation of CO_2 laser. Fig. 3 indicates that when beam radius is less than 1 mm, the threshold power is nearly constant, but when it is larger, P_{th} begins to increase linearly with a sharp slope. At the same time, I_{th} varies slightly shown in Fig. 2.

Therefore, we present here that the ability of optical materials to resist the laser irradiation should be characterized by P_{th}, instead of I_{th}, when laser beam radius is less than 1 mm, nevertheless the ability should be characterized by I_{th}, when beam is much larger than 1 mm.

3 Influence of mechanical boundary conditions

Two kinds of boundary condition are considered in this section. One is that the radial stress is zero at each point on the edge of the glass, described as

$$\sigma_r |_{r=b} = 0 \quad (12)$$

and we call this case free boundary. The other is "fixed boundary", described as

$$u_r |_{r=b} = 0 \quad (13)$$

By substituting Eq. (12) into (8), we can determine the integral constant C for free boundary condition

$$C = \alpha(1 - \mu) \frac{1}{b^2} \int_0^b T(r) r\,dr \quad (14)$$

then the tensile stress profile is obtained from Eq. (8), with the help of Eq. (14) and (4), written as

$$\sigma_\theta(r,t) = \begin{cases} -\dfrac{1}{2}\dfrac{\alpha E\eta(1-R)I_0 t}{\rho C_p h}\left(1-\left(\dfrac{a}{b}\right)^2\right) & r \leqslant a \\ \dfrac{1}{2}\dfrac{\alpha E\eta(1-R)I_0 t}{\rho C_p h}\left(\left(\dfrac{a}{b}\right)^2+\left(\dfrac{a}{r}\right)^2\right) & r > a \end{cases} \quad (15)$$

Apparently, the maximum tensile stress appears at the edge of the beam when the laser is terminated, namely

$$(\sigma_\theta)_{\max} = \frac{1}{2}\frac{\alpha E\eta(1-R)I_0 t_{ir}}{\rho C_p h}\left(1+\left(\frac{a}{b}\right)^2\right) \quad (16)$$

and the threshold intensity can be readily written as

$$I_{th} = \frac{2\sigma_t \rho C_p h}{\alpha E\eta(1-R)t_{ir}}\frac{b^2}{b^2+a^2} \quad (17)$$

Similarly, we can get the threshold intensity for fixed boundary condition, described as

$$I'_{th} = \frac{2\sigma_t \rho C_p h}{\alpha E(1-R)t_{ir}}\frac{b^2}{b^2-\dfrac{1+\mu}{1-\mu}a^2} \quad (18)$$

Comparing I_{th} with I'_{th}, we can draw some useful conclusions as follows. I'_{th} is dependent on the Poisson's ratio μ of optical glass, but I_{th} is independent on it. From

$$\frac{I'_{th}}{I_{th}} = \frac{b^2+a^2}{b^2-\dfrac{1+\mu}{1-\mu}a^2} > 1 \quad (19)$$

we infer that an optical window with fixed boundary will undergo more laser impact than that with free boundary. When we carefully examine Eq. (18), an interesting result is also presented. When laser beam radius a approaches a critical dimension, described as $r_c = [(1-\mu)/(1+\mu)]^{1/2} b$, I'_{th} approaches infinity, and when a exceeds r_c, I'_{th} becomes negative. The reason for the above result is that infinite or negative I'_{th} means tensile stress induced in material is eliminated, then the threshold intensity should be calculated by means of maximum compressive stress exceeding compressive strength. Any how, when a exceeds r_c, the threshold intensity will increase abruptly since σ_c is much larger than σ_t.

4 Conclusions

A model has been developed to predict elastic fracture thresholds for CW laser irradiated glass. We apply this model to investigate the effects of laser beam radius and mechanical boundary on the damage thresholds. It is obvious that beam radius effects result from the energy loss from irradiated area into "cool area", so when beam dimension is large enough, the energy loss is only a little fraction of total laser energy, then beam effects become negligible. When we study beam dimension effect, the boundary conditions aren't considered, so the thresholds obtained represent the ability to resist laser impact of material itself. When boundary condition comes into question, the thresholds represent the ability of a structure. We have seen that different structures have different damage thresholds. As for the output window of high energy laser system, the structure with fixed boundary may be appropriate.

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