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# 脉冲射线源三维中子成像重建算法研究

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**摘 要:**借鉴计算机层析成像方法,通过多角度成像的方式得到氘氘燃烧区三维分布信息。介绍了球谐函数分解、最大期望代数迭代两种具有代表性的极少角度三维图像重建算法,并提出将球谐函数分解重建结果作为最大期望代数迭代初始值的重建约束方法。该方法利用了激光、Z箍缩聚变靶具有一定球对称性的物理先验,既改善了球谐函数在极少角度投影重建表示能力不足、残影较大的缺点,又利用了最大期望算法局部最优的特点,可以将重建结果限制在物理似然域中,从而在一定程度上克服了极少角度投影三维重建解空间过大的问题。利用峰值信噪比、均方根误差、KL偏差等三种指标对重建结果进行了量化评价,结果表明球谐函数初值约束的最大期望代数迭代算法效果最好。

**关键词:**三维图像重建;球谐函数分解;最大期望迭代算法;快中子成像;惯性约束聚变

**中图分类号:**O536;TL81

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## 0 引言

近二十年来,激光、Z箍缩驱动聚变点火技术取得了长足进步<sup>[1-2]</sup>,氘氘燃烧区的空间分布反映了所有点火物理过程的最终状态,精确诊断燃烧区空间结构对于改进压缩对称性技术措施、校验物理模型、评估点火成败等具有重要意义,不同能量的中子图像能够直接给出燃烧区以及周围物质在高压缩比条件下的空间分布,因此中子成像技术是研究脉冲射线源性能的核心技术<sup>[3-5]</sup>。吕敏院士、王奎禄研究员等在20世纪80年代建立了基于厚针孔、射线图像转换屏、选通型像增强器、高速图像记录等关键技术的中子、伽马成像系统,这些基本技术思路几十年来没有发生根本性的变化,但针对测量对象特征的不同进行了一些重要技术改进,如针对低强度脉冲中子源,开展了半影孔、环形孔成像与图像复原技术研究<sup>[6-8]</sup>,发展建立了多幅相机、条纹相机、光纤-光电倍增管阵列等三种不同的时空分布信息记录系统<sup>[9,10]</sup>。单光轴中子成像获得的二维图像是辐射区域中子的沿着特定方向线积分投影,无法直接反映氘氘燃烧区域内部的三维空间分布,如氘氘燃烧区体积等重要物理参数需要根据旋转对称假设给出。最近在高能量密度物理研究领域,数值模拟程序由二维向三维发展,美国国家点火装置(National Ignition Fusion, NIF)团队的研究结果表明,三维模拟程序计算的中子产额与实验结果最为接近,三维程序给出的燃烧区分布需要直接的实验结果验证<sup>[11]</sup>。借鉴计算机层析成像思想,通过多角度成像的方式可以给出氘氘区三维分布信息,但由于脉冲射线源一般具有自辐射强、持续时间短、射线强度变化范围大等特点,这些特点使得不同光轴之间像素级精确匹配、高精度时间关联、相机灵敏度差异校正等存在较大困难,一般建立的光轴数量非常有限( $\leq 5$ ),如美国国家点火装置上历经十余年,完成了三条中子成像光轴建设<sup>[12]</sup>。这种基于非常有限光轴中子图像重建三维空间分布是一件极具挑战性的工作,有解空间大、存在伪影、无法保持边缘特征的问题。中国工程物理研究院江少恩等在1997年进行了ICF聚变等离子体的X射线三维成像技术探索,建立了基于代数重建(Algebraic Reconstruction Technique, ART)的三维图像复原算法<sup>[13]</sup>;美国VOLEGOV P L等建立了最大期望(expectation

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Maximization, EM)算法图像重建方法<sup>[14]</sup>,以及三维中子图像球谐函数分解(Spherical Harmonic Decomposition, SHD)、柱谐函数分解重建方法(Cylindrical Harmonic Deomposition, CHD)<sup>[15-16]</sup>,这种函数分解解析方法具有重建速度快、重建结果在某些限定条件下解唯一确定的优点,但其表达能力受投影角度数量的限制较大。

本文以辐射源强度空间分布为高斯函数的椭球源为研究对象,分别介绍并实现了球谐函数分解和EM迭代两种三维图像重建算法。与NIF团队重建算法设计思路不同,根据惯性约束聚变靶燃烧区结构具有一定球对称性这一物理特点,设计了一种将球谐函数分解与EM迭代相结合的重建算法。该方法首先以球谐函数分解对辐射源进行三维重建,然后将其重建结果作为EM算法的初值进行迭代重建,其最终重建结果可以看作是EM迭代算法在球谐函数分解重建结果约束下寻找到的最优解。

## 1 球谐函数分解三维图像重建算法

三维脉冲辐射源的积分投影的成像方程可以表示为<sup>[15]</sup>

$$I(d, \hat{p}) \equiv P_{\hat{p}}\{S\}(d) = \int_{-\infty}^{\infty} S(d + \xi \hat{p}) d\xi = \int_{-\infty}^{\infty} S(u\hat{e}_1 + v\hat{e}_2 + \xi \hat{p}) d\xi \quad (1)$$

式中,  $I(d, \hat{p})$  为积分投影图像,  $d = u\hat{e}_1 + v\hat{e}_2$  为像平面上的点,  $\hat{p}$  为投影方向,  $S$  为源图像。将脉冲射线源  $S(\mathbf{r})$  用球谐函数展开

$$S(\mathbf{r}) = \sum_{l=0}^N \sum_{m=-l}^l s_{l,m}(r) Y_{l,m}(\theta, \varphi) \quad (2)$$

式中,  $\mathbf{r} = u\hat{e}_1 + v\hat{e}_2 + \xi \hat{p}$ ,  $(r, \theta, \varphi)$  为矢量  $\mathbf{r}$  在球坐标系下的表示,  $s_{l,m}(r)$  为球谐函数  $Y_{l,m}(\theta, \varphi)$  的展开系数。

将式(2)代入式(1),通过旋转源  $S$ ,使投影方向  $\hat{p}$  与  $z$  轴平行,根据中心切片定理,有

$$\tilde{I}(\omega_x, \omega_y) = \sum_{l=0}^N \sum_{m=-l}^l \sum_{m'=-l}^l T_{m,m'}^{(l)}(\mathcal{R}) \mathcal{F}\{s_{l,m}(r) Y_{l,m}(\theta, \varphi)\}(\omega_x, \omega_y, 0) \quad (3)$$

式中,  $\tilde{I}(\omega_x, \omega_y)$  为  $I(x, y)$  的傅里叶变换,  $T_{m,m'}^{(l)}(\mathcal{R})$  Wigner-D 函数<sup>[17-18]</sup>,  $\mathcal{R}\{\dots\}$  表示旋转操作。利用平面波球谐函数展开<sup>[17]</sup>,对两边进行积分,式(3)可以写成(4)。

$$\tilde{I}_m(\omega) = \sum_{l=0}^N \sum_{m'=-l}^l \tilde{H}_m^{l,m'}(\mathcal{R}) \tilde{s}_{l,m}(\omega) \quad (4)$$

式中,  $\omega = \sqrt{\omega_x^2 + \omega_y^2}$ ,  $\tilde{s}_{l,m}(\omega)$  为  $s_{l,m}(r)$  的 Hankel 变换。

$$\tilde{s}_{l,m}(\omega) = \sqrt{\frac{2}{\pi}} \cdot i^{-l} \int_0^{\infty} s_{l,m}(r) r^2 J_l(\omega r) dr \quad (5)$$

式中,  $J_l(\cdot)$  为第一类球贝塞尔函数。根据投影的角度数和式(4),得到线性方程组<sup>[17]</sup>

$$\tilde{I} = \tilde{H} \tilde{s} \quad (6)$$

通过解线性方程组(6),得到源的球谐函数展开系数  $s_{l,m}$  的 Hankel 变换  $\tilde{s}_{l,m}$ , 然后对  $\tilde{s}_{l,m}$  进行逆 Hankel 变换,最后可以得到脉冲辐射源的分布  $S(\mathbf{r})$ 。

球谐函数分解对一定球对称性的对象具有良好的适应性,利用解析方法可获得唯一确定的解。源  $S(\mathbf{r})$  在不同的展开阶数  $N$  时,线性方程组的系数矩阵  $\tilde{H}$  的秩受到了成像轴数目  $V$  的限制。在投影轴角度分布合适的情况下,当  $N \leq V - 1$  时,  $\text{rank}(\tilde{H}) = (N + 1)^2$ , 当  $N \geq V$  时,  $\text{rank}(\tilde{H}) < (N + 1)^2$ 。只有当  $N \leq V - 1$  时,矩阵  $\tilde{H}$  满秩,方程组(6)存在唯一解,因此解析方法能够确定辐射源的最大展开阶数为  $N = V - 1$ 。

利用球谐函数分解对5个投影角度的椭球源进行了三维图像重建,5个投影角度如图1所示,在球坐标下的角度分布分别为  $(104^\circ, 20^\circ)$ ,  $(149^\circ, 307^\circ)$ ,  $(61^\circ, 328^\circ)$ ,  $(157^\circ, 132^\circ)$ ,  $(84^\circ, 247^\circ)$ ,选择该投影轴的角度分布可以使得系数矩阵  $\tilde{H}$  的条件数相对较小。设置脉冲射线源的像素数目为  $100 \times 100 \times 100$ ,椭球的三个轴长度分别为70、50和30个像素,球心在源矩阵的中心,源强空间分布为高斯函数,中心强度为1。积分投影的射线间隔为1个像素,即每个成像轴获得的积分投影图像大小为  $100 \times 100$ 。

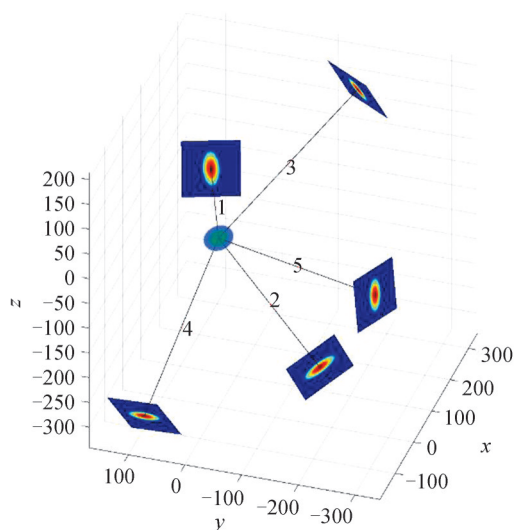
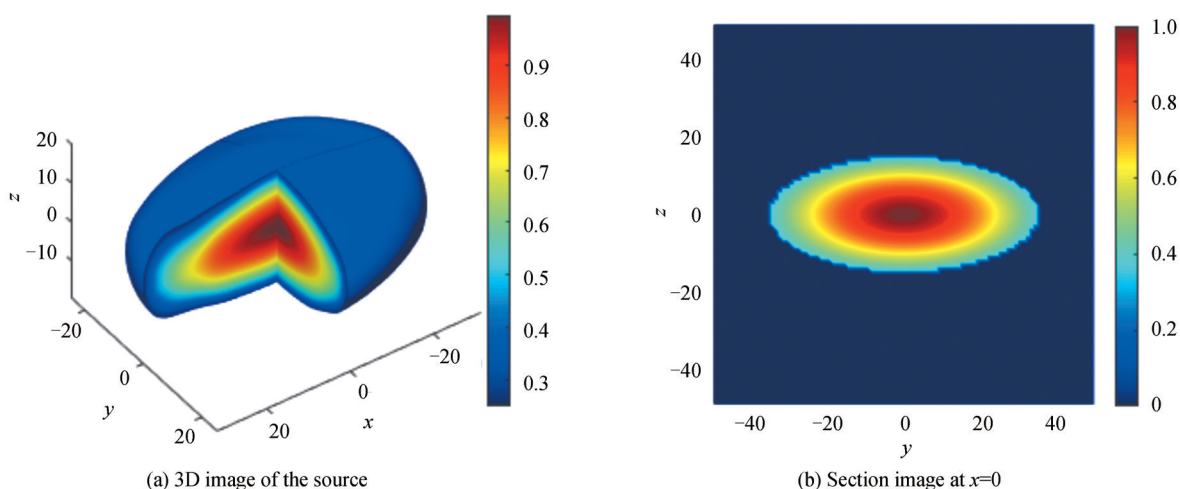
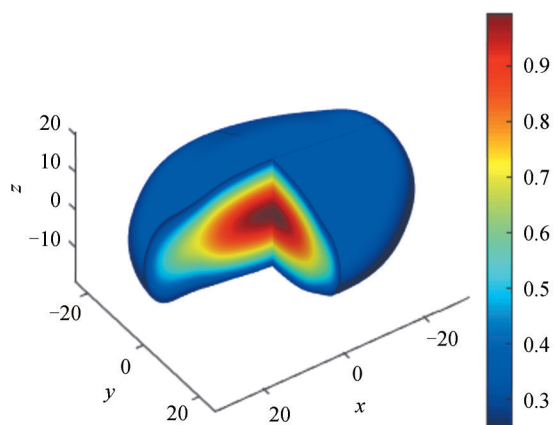


图1 椭球辐射源5个角度投影  
Fig.1 The 5 projection views of the ellipsoidal radiation source

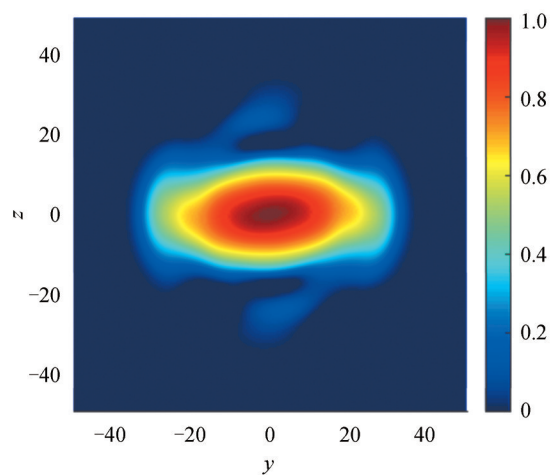
椭球脉冲射线源及 $x=0$ (短轴中心位置)处切片图如图2(a)、(b)所示,本文给出的所有切片图都为同一位置。在球谐函数分解解析求解的过程中,为了减少积分投影图像中高频噪声对结果的影响,在积分投影图像的频域中增加了低通高斯滤波器,重建图像及切片图如图2(c)、(d)所示。为了验证噪声对球谐函数分解解析方法重建结果的影响,对椭球源的积分投影图像添加两种水平的高斯噪声,噪声的标准差分别为积分投影图像最大值的2%和5%,对于噪声投影图像重建结果分别如图2(e)、(f)和图2(g)、(h)所示。通过与无噪声重建结果的切片图比较可以看到,在有噪声的情况下,解析方法重建结果中心部分产生了较大畸变,而边缘部分的重建伪影部分受噪声影响不大。

无噪声重建结果的积分投影图像与理想的椭球源积分投影图像( $149^\circ, 307^\circ$ )的差值 $|\Delta|$ 及 $\Delta$ 的一维分布直方图如图3(a)、(b)所示,其中 $\Delta$ 为重建投影图像与理想投影图像之间的强度差,由于大部分差值数据为 $\Delta=0$ ,为了更好地显示差值分布,在进行直方图统计时没有将 $\Delta=0$ 的结果统计在内。2%和5%噪声水平的重建结果差值及直方图分别如图3(c)、(d)和图3(e)、(f)所示。可以看到随着噪声水平的增加,非零差值的直方图分布变的更宽,理想情况下差值图像的直方图分布的最可几值在 $\Delta=0$ 附近,当噪声为5%时,差值图像的直方图分布较理想情况下出现了较大偏移,高斯拟合结果给出的最可几值出现在 $\Delta=0.38$ 附近。

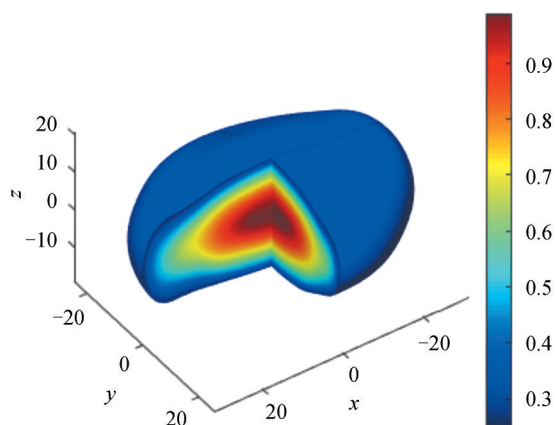




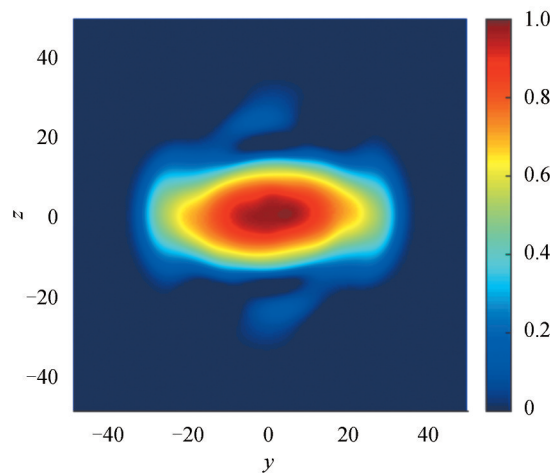
(c) SHD reconstructed 3D image



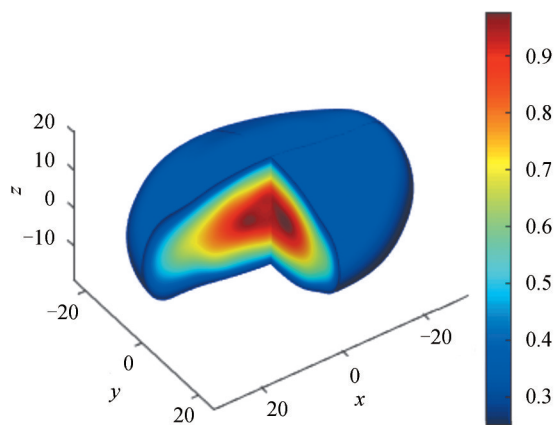
(d) Section image of the reconstructed 3D image



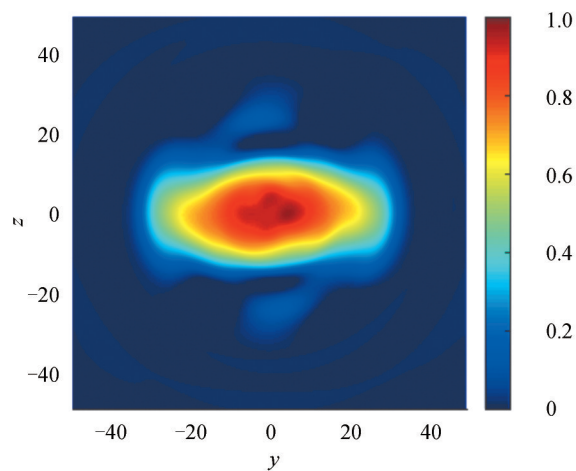
(e) SHD reconstructed 3D image with 2% noise



(f) Section image of the reconstructed 3D image with 2% noise



(g) SHD reconstructed 3D image with 5% noise

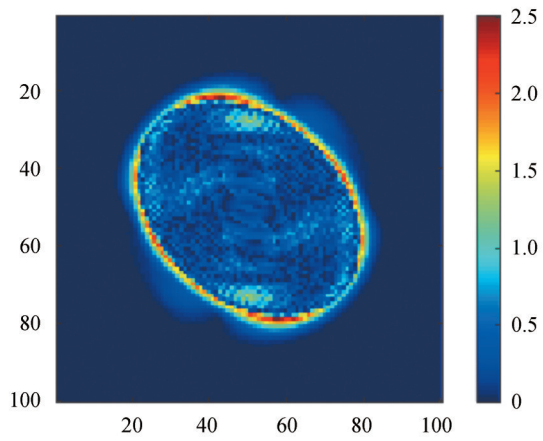


(h) SHD reconstructed 3D image with 5% noise

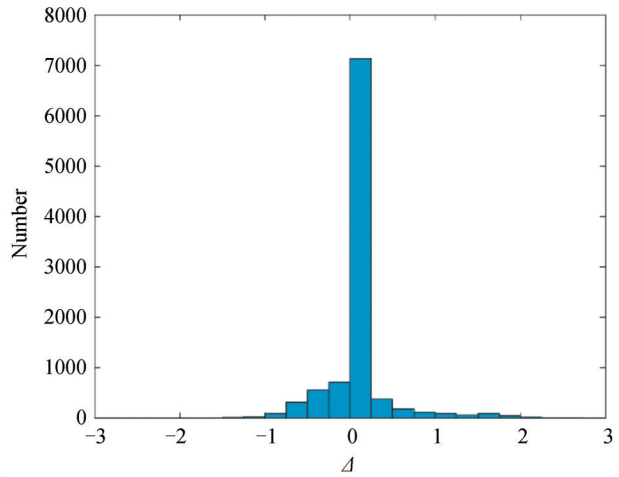
图2 球谐函数分解三维图像重建结果

Fig. 2 The three-dimensional image reconstructed results by spherical harmonic decomposition

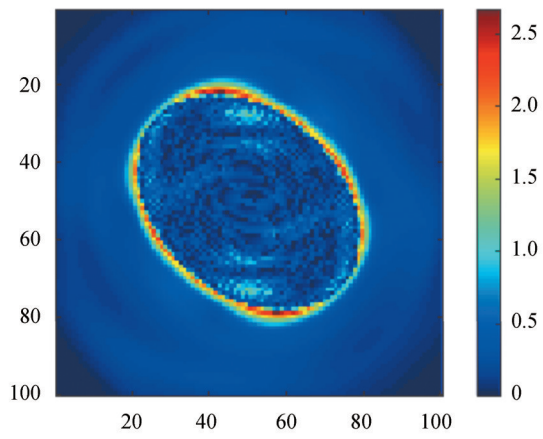




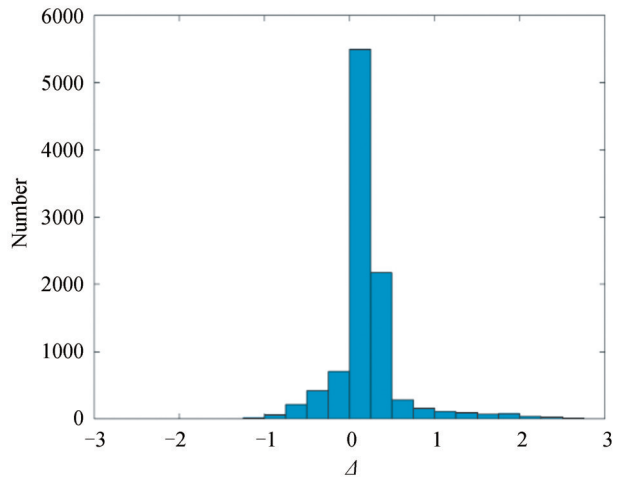
(a) Difference image between the projection image (149°,307°) of reconstructed 3D image and ideal 3D image



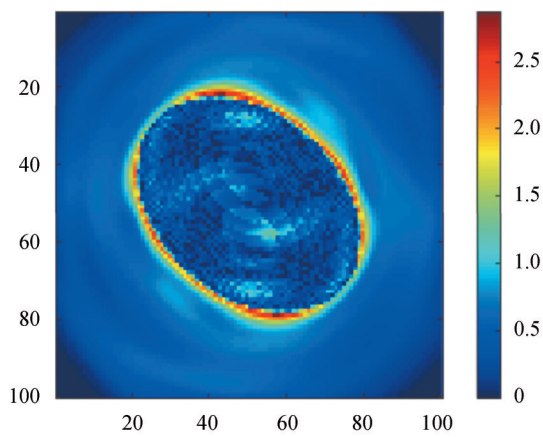
(b) Statistical histogram of the difference between the projection image (149°,307°) of reconstructed 3D image and ideal 3D image



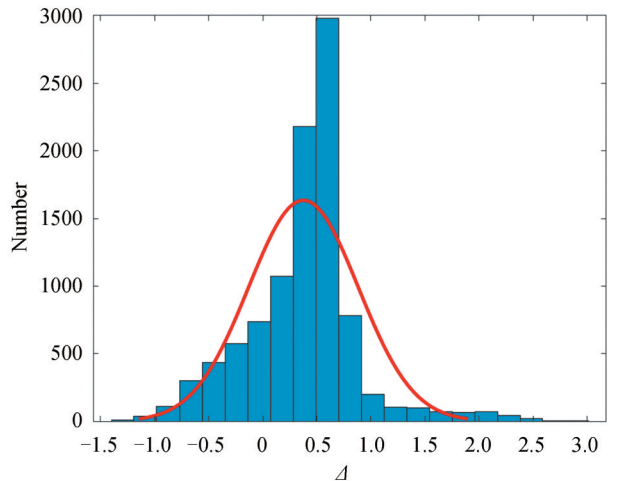
(c) Difference image between the projection image (149°,307°) of reconstructed 3D image and ideal 3D image with 2% noise



(d) Statistical histogram of the difference between the projection image (149°,307°) of reconstructed 3D image and ideal 3D image with 2% noise



(e) Difference image between the projection image (149°,307°) of reconstructed 3D image and ideal 3D image with 5% noise



(f) Statistical histogram of the difference between the projection image (149°,307°) of reconstructed 3D image and ideal 3D image with 5% noise

图3 不同噪声水平下重建三维图像投影图像与理想投影图像之间的差值及其直方图

Fig. 3 The difference image and its histogram between the projection image of reconstructed 3D image and ideal 3D image with different noise

## 2 EM 迭代算法

在中子成像中,物像距远大于辐射源自身尺寸,受厚针孔视角限制,射线经过厚针孔的有效发射角度远小于1。成像系统获取的中子图像是具有三维空间结构的脉冲辐射源沿着某一方向的积分投影图像,其矩阵形式为

$$\boldsymbol{p} = \boldsymbol{A}\boldsymbol{f} \quad (7)$$

式中,投影数据 $\boldsymbol{p}$ 为 $M \times 1$ 的向量,源强度 $\boldsymbol{f}$ 为 $N \times 1$ 的向量,投影矩阵 $\boldsymbol{A}$ 为 $M \times N$ 的矩阵。EM算法是一种统计类代数迭代重建方法,可用于处理投影角度少、数据不完备的问题,较ART、MART等代数类迭代算法具有更好的重建效果。受中子成像探测数量限制和投影噪声的影响,EM算法随着迭代次数增加,迭代噪声增强,易引起图像质量退化<sup>[20]</sup>。

EM迭代算法的基本思想为:假设射线源放射强度 $\boldsymbol{f}$ 服从泊松分布,利用观测投影数据 $\boldsymbol{p}$ ,通过EM迭代寻找使似然函数的期望最大化的参量 $\boldsymbol{f}$ ,似然函数定义为

$$P(\boldsymbol{p}|\boldsymbol{f}) = \prod_{i=1}^M e^{-A_i \boldsymbol{f}} \frac{(A_i \boldsymbol{f})^{p_i}}{p_i!} \quad (8)$$

式中, $A_i$ 是 $\boldsymbol{A}$ 矩阵第 $i$ 行。在EM算法中,依赖于一组完全观测数据 $\{z_{ij}\}$ ,数据的每个元素表示探测像素 $i$ 探测到第 $j$ 个辐射源体素内发射的光子数,服从Poisson分布

$$z_{ij} \sim \text{Poisson}(a_{ij} f_j) \quad (9)$$

式中, $a_{ij}$ 是 $\boldsymbol{A}$ 矩阵第 $i$ 行 $j$ 列元素, $f_j$ 表示向量 $\boldsymbol{f}$ 第 $j$ 列的元素值,投影数据 $\boldsymbol{p}$ 与观测数据 $z$ 的关系为

$$p_i = \sum_{j=1}^N z_{ij} \quad (10)$$

式中, $p_i$ 表示 $\boldsymbol{p}$ 第 $i$ 行的元素值。结合式(8)和(10),目标函数为

$$L(\boldsymbol{f}) = \ln(P(\boldsymbol{z}|\boldsymbol{f})) = \sum_{i=1}^M \sum_{j=1}^N (-a_{ij} f_j + z_{ij} \ln(a_{ij} f_j)) \quad (11)$$

采用EM算法分为两个步骤。

E-step:已知当前源图像 $\boldsymbol{f}^k$ ,投影值 $\boldsymbol{p}$ ,观测数据的条件期望为

$$z'_{ij} = E(z_{ij} | p_i, \boldsymbol{f}^k) = \frac{a_{ij} f_j^k}{\sum_{j=1}^N a_{ij} f_j^k} p_i \quad (12)$$

将式(11)的目标函数的随机变量 $z_{ij}$ 用期望值代替,则有

$$L(\boldsymbol{f}) = \sum_{i=1}^M \sum_{j=1}^N (-a_{ij} f_j + z'_{ij} \ln(a_{ij} f_j)) \quad (13)$$

M-step:使条件期望最大化,对目标函数求偏导。

$$\frac{\partial L(\boldsymbol{f})}{\partial f_j} = \sum_{i=1}^M \left( \frac{z'_{ij}}{f_j} - a_{ij} \right) \quad (14)$$

令式(14)为零,得到经典的EM算法迭代格式<sup>[21-22]</sup>

$$f_j^{(k+1)} = \frac{f_j^k}{\sum_{i=1}^M a_{ij}} \sum_{i=1}^M \frac{a_{ij} p_i}{\sum_{j=1}^N a_{ij} f_j^k} \quad (15)$$

将每个投影方向数据作为一个子集,采用有序子集最大期望重建算法(Ordered Subset Expectation Maximization, OSEM)可以加速算法收敛速度<sup>[21-22]</sup>,本文中的算例迭代5次即可收敛至两次迭代差别可忽略的程度。在利用EM算法进行图像重建时易产生高频噪声,可以利用中值滤波(Median Filter, MF)、全变差最小(Total Variation Minimum, TVM)等约束方法对迭代数据进行平滑约束。文献[14]引入图像空间分布的先验信息,使用Gibbs能量分布函数对EM算法的迭代过程进行约束,提高了算法的噪声抑制能力。EM算法获得的是局部最优解,通常在利用EM迭代算法时,初值设为0或者1。如果能够将EM算法的初值限

定在一定范围内,就有可能获得与受初值约束的局部最优解。大多数对EM算法解空间的约束依赖于投影图像数据的先验假设,如高斯先验、脉冲噪声先验、结构化先验等<sup>[23]</sup>。激光、Z箍缩驱动惯性约束聚变研究中,为了获得更高的能量转换效率,靶丸的内爆压缩过程尽可能为对称压缩,其结构上具有一定的球对称性,基于此类物理先验信息可以帮助我们限定EM类算法的初值,将重建结果限定在物理结构先验约束范围内,将球谐函数分解重建结果作为EM迭代算法的初值是ICF中子图像三维重建的一种自然选择。当然,根据重建对象的不同,EM迭代算法的初值约束可以有不同的选择,比如辐射源具有一定柱对称性,可以利用阿贝尔变换( Abel Transform)或者文献[16]提出的柱谐函数分解重建结果作为EM迭代算法的初值。

图4(a)~(j)为无噪声情况下图1所示5个投影角度椭球源OSEM、MF-OSEM、球谐函数分解初值约束

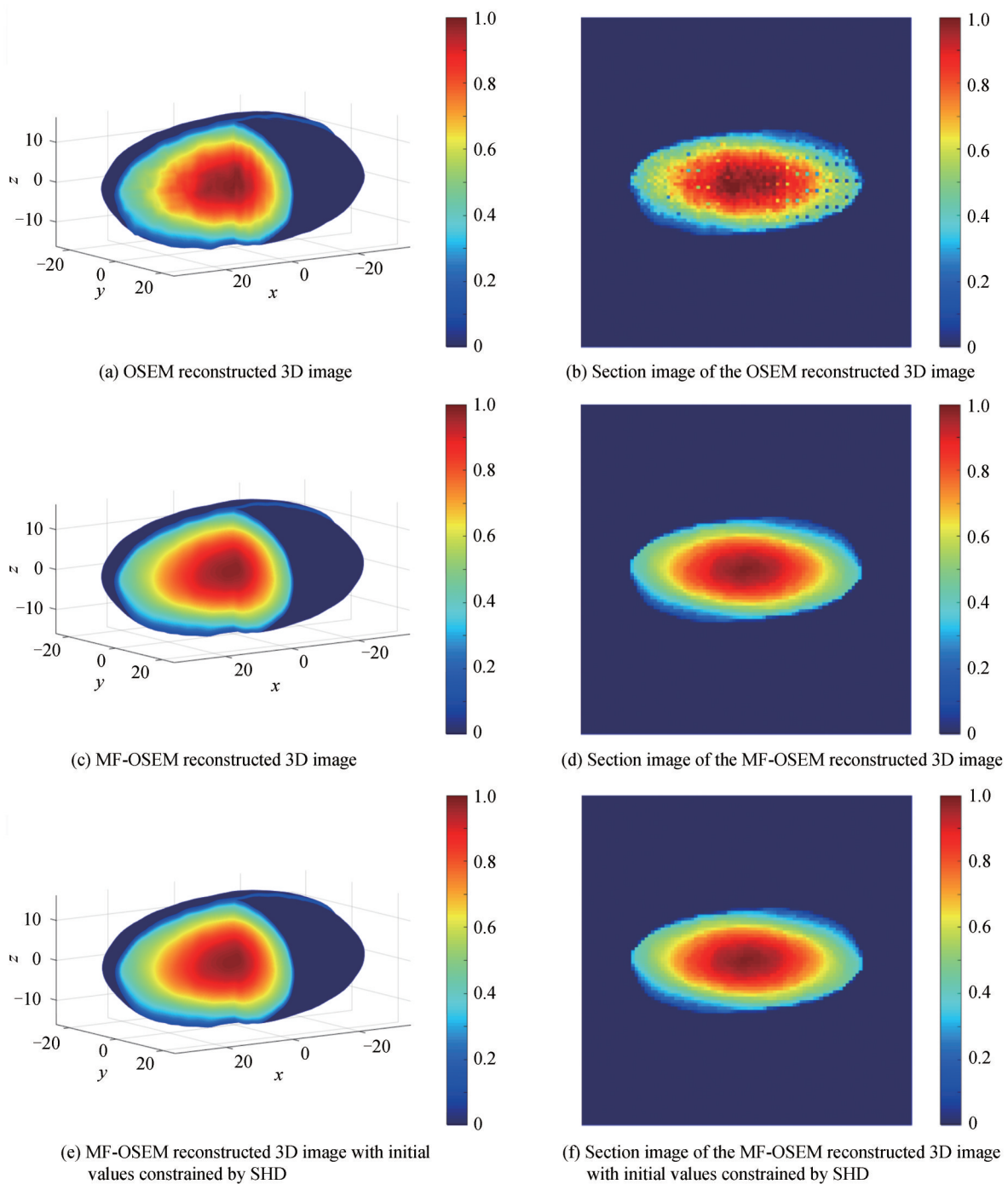
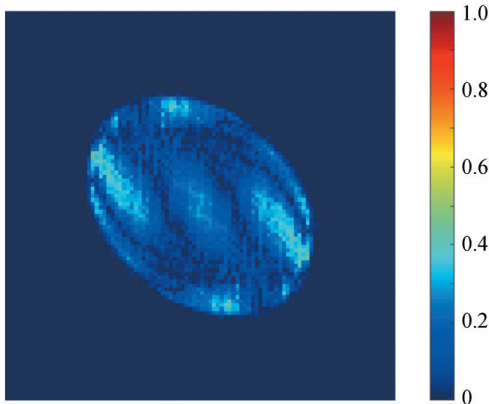
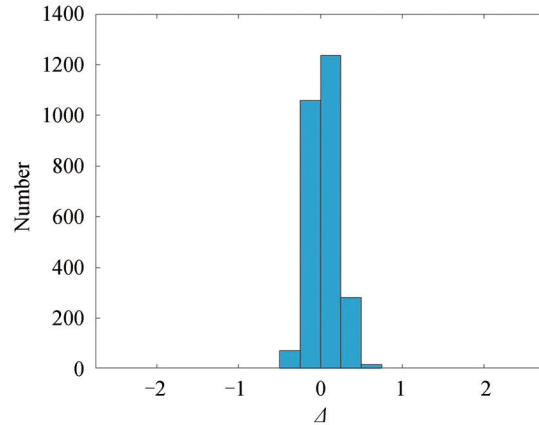


图4 EM类算法三维图像重建结果  
Fig.4 The EM algorithm reconstructed results

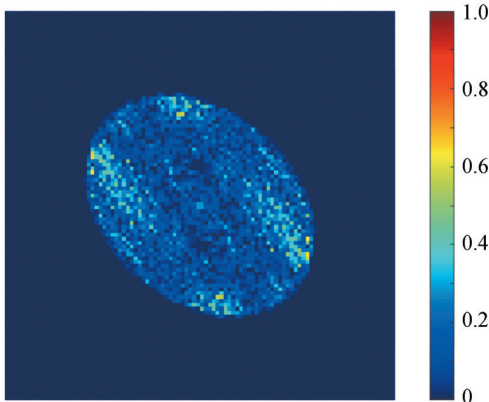
的MF-OSEM算法三维图像重建结果其切片图。图4(b)中OSEM算法重建图像给出的切片图存在较多迭代伪影,包含辐射源内部的高频噪声和边缘噪声,图4(d)中MF-OSEM算法的结果对高频噪声进行了较好的抑制;MF-OSEM、球谐函数初值约束的MF-OSEM重建三维图在与理想的椭球源积分投影图像(149°, 307°)的差值及其直方图分别如图5(a)、(b)和图5(c)、(d)所示。从直方图中看球谐函数约束的MF-OSEM算法差值图像较MF-OSEM算法绝对值更小,分布也更为均匀,受投影角度的影响也更小一些,直方图的强度分布都集中在 $\Delta=0$ 附近。图5(e)、(f)和图5(g)、(h)分别为添加2%和5%噪声之后,球谐函数初值约束



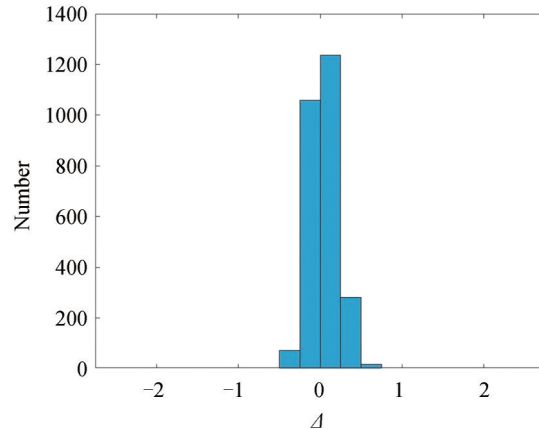
(a) Difference image between the projection image (149°,307°) of the MF-OSEM reconstructed 3D image and ideal 3D image



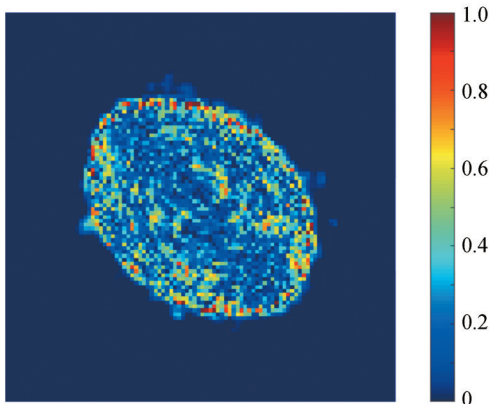
(b) Statistical histogram of the difference between the projection image (149°,307°) of the MF-OSEM reconstructed 3D image and ideal 3D image



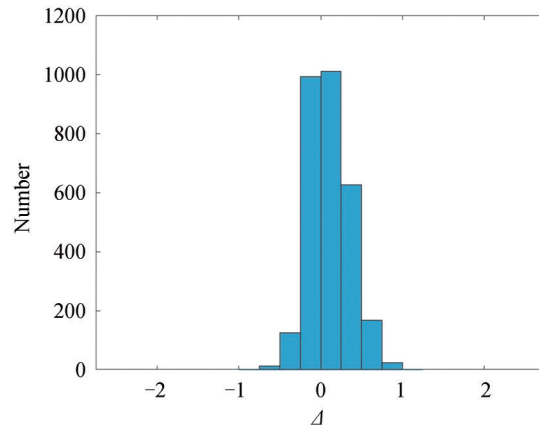
(c) Difference image between the projection image (149°,307°) of the MF-OSEM reconstructed 3D image with initial values constrained by SHD and ideal 3D image



(d) Statistical histogram of the difference between the projection image (149°,307°) of the MF-OSEM reconstructed 3D image with initial values constrained by SHD and ideal 3D image

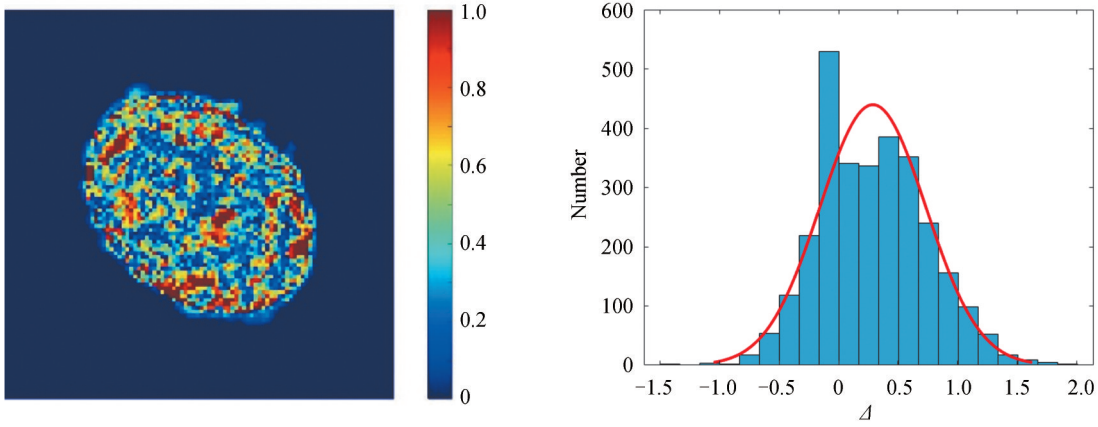


(e) Difference image between the projection image (149°,307°) of the MF-OSEM reconstructed 3D image with initial values constrained by SHD and ideal 3D image with 2% noise



(f) Statistical histogram of the difference between the projection image (149°,307°) of the MF-OSEM reconstructed 3D image with initial values constrained by SHD and ideal 3D image with 2% noise





(g) Difference image between the projection image (149°,307°) of the MF-OSEM reconstructed 3D image with initial values constrained by SHD and ideal 3D image with 5% noise

(h) Statistical histogram of the difference between the projection image (149°,307°) of the MF-OSEM reconstructed 3D image with initial values constrained by SHD and ideal 3D image with 5% noise

图5 不同噪声水平下球谐函数初值约束的MF-OSEM算法重建三维图像投影图像与理想投影图像之间的差值及其直方图  
Fig. 5 The difference image and its histogram between the projection image of the MF-OSEM reconstructed 3D image with initial values constrained by SHD and ideal 3D image with different noise

的MF-OSEM算法重建投影图像与理想投影图像之间的差值及其直方图,与球谐函数分解结果类似,随着噪声水平的提高,差值图像的绝对值也变大,直方图分布也变的更宽,与图3(e)差值图像边缘与理想图像差别较大不同,差值图像图5(g)分布较为均匀,图5(h)的直方图高斯拟合给出的最可几值出现在 $\Delta=0.29$ 附近。

### 3 重建结果量化比较

在重建图像结果评价中,通常采用一些量化指标进行评价。本文采用峰值信噪比(Peak-Signal to Noise Ratio, PSNR)、均方根误差(Root Mean Squared Error, RMSE)、KL偏差(Kullback-Leibler divergence)<sup>[24]</sup>等3种重建图像质量评价方法对几种图像重建结果进行定量比较。

设理想图像为 $X$ ,重建图像为 $Y$ , $P \times Q$ 为图像大小,PSNR定义为

$$\text{PSNR} = 10 \lg \frac{X_{\max}^2 PQ}{\sum_{i=1}^P \sum_{j=1}^Q [X(i,j) - Y(i,j)]^2} \quad (16)$$

$X_{\max}$ 为图像 $X$ 的最大值(本文中为1.0)。RMSE定义为

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^P \sum_{j=1}^Q [X(i,j) - Y(i,j)]^2}{PQ}} \quad (17)$$

KL偏差 $D_{\text{KL}}$ 定义为

$$D_{\text{KL}} = \frac{\sum_{i=1}^P \sum_{j=1}^Q X(i,j) \log \frac{X(i,j)}{Y(i,j)}}{\sum_{i=1}^P \sum_{j=1}^Q X(i,j)} \quad (18)$$

从式(16)和(17)中可以看出,重建结果评价指标中PSNR越大, RMSE越小。KL偏差越小,表示重建图像与理想图像之间越接近。文中涉及到的几种重建算法量化比较结果如表1所示。

从表1的结果可以看出,随着噪声水平的提高,所有算法重建结果都会变差,其中OSEM算法由于在重建过程中没有对噪声进行抑制,其重建结果对噪声最为敏感。球谐函数分解方法虽然依赖于投影角度数量,在本文5个投影角度情况下,对椭球源依然保持了较好的重建精度。MF-OSEM算法和基于球谐函数分解的MF-OSEM算法整体效果较好,采用球谐函数重建结果作为MF-OSEM的迭代初值可以把EM类算法的迭代结果限制在初值附近。在本文的量化评价指标下,该方法的重建结果与理想图像之间的差别最小,对噪声适应性也较好。球谐函数分解算法为解析方法,占用计算资源较小,在处理器intel(R) core(TM)

表1 几种三维重建算法效果比较  
Table 1 The comparison of these three-dimensional image reconstruction algorithms

	Algorithm	PSNR	RMSE	$D_{KL}$
Noise-free	SHD	30.185 0	0.031 0	0.038 6
	OSEM	33.782 0	0.020 5	0.043 7
	MF-OSEM	36.437 8	0.015 1	0.031 8
	MF-OSEM with SHD	36.545 4	0.014 9	0.031 5
With 2% noise	SHD	30.091 6	0.031 3	0.151 5
	OSEM	21.732 7	0.081 9	0.102 8
	MF-OSEM	35.017 5	0.017 7	0.051 8
	MF-OSEM with SHD	35.134 7	0.017 5	0.049 5
With 5% noise	SHD	29.586 3	0.033 2	0.186 2
	OSEM	18.857 8	0.199 0	0.199 0
	MF-OSEM	33.048 8	0.022 3	0.084 1
	MF-OSEM with SHD	33.245 4	0.021 8	0.079 8

i7-4790 CPU@3.6GHz,内存为24GB的计算资源上完成一次计算的时间约为6分钟。EM类迭代算法占用了大部分计算时间,几种EM类迭代算法所需时间差别不大。本文中EM算法迭代5次左右即可收敛,所需时间约为1455 s(~24 min)。需要说明的是,本文计算的三维结构总像素数为 $100 \times 100 \times 100$ ,如果要重建更多像素数图像,则需要较高性能服务器上进行计算以缩短计算时间。

## 4 结论

三维图像数据是校验惯性约束聚变靶物理设计的重要结果,极少角度投影三维图像重建存在解空间过大的问题。本文建立了强度空间分布为高斯函数理想椭球源5个投影角度球谐函数分解、EM迭代等三维重建算法。球谐函数分解解析重建方法表达能力受投影角度数量的限制,EM迭代算法只具有局部最优解,基于惯性约束聚变靶为具有一定球对称性的射线源的认识先验,将球谐函数分解解析重建结果作为EM迭代算法的初始值,从而把EM迭代算法重建结果约束在球谐函数分解解析结果附近。重建结果及量化评价指标表明,该算法具有良好的效果,对噪声的适应性也较好,可作为类似辐射源极少角度投影三维重建的基准比较算法。下一步,将在实验室构建相应的多光轴成像系统,通过实际实验结果来进一步校验算法的有效性。

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## Three-dimensional Neutron Imaging Reconstruction Algorithms Research of Pulsed Radiation Sources

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**Abstract:** Three-dimensional (3D) neutron imaging is the cutting-edge diagnostic method for pulsed radiation sources driven by laser or Z-pinch. Neutron images with different energies can accurately diagnose the spatial structure of the burning Deuterium-Tritium (DT) plasma and surrounding materials under high compression ratio, which is of great significance for improving the technical measures of compression symmetry, verifying the physical model and evaluating the ignition performance. Utilized the computed

tomography method, 3D information of the target internal structure can be acquired by multi-view neutron imaging method. Along with the limitations of current technology, there are great difficulties in pixel level accurate matching, high-precision time correlation and camera sensitivity difference correction between different imaging axes due to the high radiation intensities, short durations, and wide energy ranges of the pulsed radiation sources. Thus, only a limited number of axis can be implemented. Reconstructing 3D images from severely incomplete two-dimensional data is an ill-posed problem, which suffers from large solution space, artifacts and inability to maintain edge features. JIANG Shaoen et al developed a 3D reconstruction algorithm based on algebraic reconstruction techniques. VOLEGOV P L et al used an iterative Expectation-Maximization (EM) algorithm to reconstruct 3D neutron or X-ray sources. The algorithm reconstructed the main geometrical features of the source, but introduced artifacts affected by the angular distribution of projection directions. VOLEGOV P L et al later adopted Spherical Harmonic Decomposition (SHD) and Cylindrical Harmonic Decomposition (CHD) methods to the reconstruction of 3D radiation sources. While these analytical methods based on basis functions expansion were fast and could acquire the unique solution under certain conditions, their representation abilities were limited by the number of imaging axes and would produce artifacts related to the truncation of expansion order. In this paper, we introduce and realize SHD reconstruction algorithm and EM iterative algorithm. Considering that the inertial confinement fusion radiation sources have some spherical symmetry and the EM algorithm can find local optimum solution, we exploit the SHD reconstruction results as the initial value of EM iterative algorithm. This method actually utilizes a physical prior based on the target physical characters overcoming the question of large solution space for the limited-view 3D reconstruction algorithm in some sense. To evaluate the performance of the proposed method, an ellipsoidal radiation source with a Gaussian intensity distribution is designed as the target source to be reconstructed. Compared with the results of SHD, OSEM and MF-OSEM algorithms, the reconstruction results of this method are closest to the ideal ellipsoid source. Our method also achieves the best results in terms of three quantitative evaluating indices, which are 36.545 4 (PSNR), 0.014 9 (RMSE) and 0.031 5 ( $D_{KL}$ ), respectively. The results indicate that the EM iterative algorithm with initial values constrained by SHD algorithm exhibits better performances and better adaptability to noise compared with the SHD or EM algorithm. Our method can be used as a benchmark comparison algorithm for the reconstruction of similar 3D radiation sources with few-view projections.

**Key words:** Three-dimensional image reconstruction; Spherical harmonic decomposition; Expectation maximization iterative algorithm; Fast neutron imaging; Inertial confinement fusion

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